Information Acquisition and Provision in School Choice: A Theoretical Investigation*

Yan Chen Yinghua He

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Abstract

When participating in school choice, students may incur information acquisition costs to learn about school quality. This paper investigates how two popular school choice mechanisms, the (Boston) Immediate Acceptance and the Deferred Acceptance, incentivize students' information acquisition. Specifically, we show that only the Immediate Acceptance mechanism incentivizes students to learn their own cardinal and others' preferences. We demonstrate that information acquisition costs affect the efficiency of each mechanism and the welfare ranking between the two. In the case where everyone has the same ordinal preferences, we evaluate the welfare effects of various information provision policies by education authorities.

Keywords: information acquisition, information provision, school choice

JEL Classification Numbers: D47, C78, D82

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1 Introduction

When choosing a school, students often have imperfect information on their own preferences over candidate schools, partly because it is difficult to assess the potential educational outcomes for each school (Dustan, de Janvry and Sadoulet 2015). More importantly, acquiring this information can be costly, if a student faces too many choices, or must acquire information on a large number of factors, such as academic performance, teacher quality, school facilities, extra-curricular activities offered, and peer quality.

The literature on matching and school choice, however, typically assumes that all students have perfect knowledge about their own preferences, at least their ordinal ones. Relaxing this assumption, our study extends the literature by investigating how mechanisms incentivize student information acquisition in school choice and how information provision by educational authorities affects efficiency. Specifically, we focus on two widely used mechanisms, the (Boston) Immediate-Acceptance (hereafter IA) and the Gale-Shapley Deferred-Acceptance (hereafter DA) mechanisms. By taking into account both the benefits and costs of information acquisition, this study provides a more comprehensive evaluation of the mechanisms and as a result provides guidance for the design of school choice or other matching markets.

Our first contribution is to show that IA and DA provide heterogeneous incentives for students to acquire information. In a setting with unknown preferences and costly information acquisition, we prove that both the strategy-proof DA and the non-strategy-proof IA incentivize students to acquire information on their own ordinal preferences. However, we find that only the non-strategy-proof mechanism induces students to learn their own cardinal preferences with which IA can sometimes be more efficient than DA (Abdulkadiroğlu, Che and Yasuda 2011, Troyan 2012). IA's lack of strategy-proofness also implies that information on others' preferences can be useful for the purpose of competing with other students. As such, the acquisition of information on others' preferences may be individually rational but socially wasteful, a disadvantage of a non-strategy-proof mechanism.

Although the above results may seem obvious, to the best of our knowledge, they have not yet been formalized in the literature. More importantly, they lead implications for the study of the mechanisms. For example, the welfare comparison of the two mechanisms is sensitive to costly information acquisition. Taking into account endogenous information acquisition, we provide a numerical example showing that the cost of information acquisition affects student welfare in equilibrium. In the example, IA achieves higher student welfare than DA when students' cardinal preferences are private information (i.e., zero information acquisition cost), a finding similar to Abdulkadiroğlu et al. (2011). As the cost of acquiring information on own preferences increases, student welfare monotonically decreases under both mechanisms; more importantly, when it passes a certain level, the welfare advantage of IA disappears.

Extending these findings, our second contribution is to present some implications for the design of information provision policies. A possible policy intervention to reduce information acquisition cost is to provide information freely. We investigate the welfare effects of information provision by education authorities. Specifically, we consider four sets of policies with increasing information provision. The least informative policy forbids everyone from acquiring any information beyond the distribution of preferences. The second policy informs everyone about her own ordinal preferences, and the third reveals one's own cardinal preferences. The most informative policy makes everyone's cardinal preferences common knowledge. The information on a student's own preferences might be provided through presentation materials on schools (Hastings and Weinstein 2008) or by targeting disadvantaged population (Hoxby and Turner 2015). The information on others' preferences can be (indirectly) provided by publishing everyone's applications and allowing one to revise her own application upon observing others' strategies, as has been done in the school choice context in Amsterdam (De Haan, Gautier, Oosterbeek and Van der Klaauw 2015) and North Carolina (Dur, Hammond and Morrill 2018), as well as in the college admissions context in Inner Mongolia, China (Gong and Liang 2017).

In a setting where students have the same ordinal preferences, we analyze symmetric equilibrium under the four information provision policies. We show that the ex ante student welfare under DA is invariant to the policies, while providing information on one's own cardinal preferences improves welfare under IA. Interestingly, we find that provision of information on others' preferences has ambiguous effects under IA, implying that sometimes providing more information on others' preferences can be welfare-decreasing under IA. The reason is that, knowing there is fierce competition for a school, students who prefer that school may shy away from applying to it; as a result, other students may be assigned that school with a high probability, an efficiency loss.

The paper proceeds as follows. Section 2 reviews the information acquisition and school choice literature. Section 3 presents the theoretical results on information acquisition, and Section 4 discusses those on information provision. Section 5 discusses possible extensions and concludes.

2 Literature Review

This study contributes to the matching literature. Typically, these studies assume that agents know their preferences (Gale and Shapley 1962, Roth and Sotomayor 1990, Abdulkadiroğlu and Sönmez 2003). One exception is Chade, Lewis and Smith (2014), who consider the case where colleges observe signals of students' ability but do not have the possibility to acquire information. Allowing this possibility, Lee and Schwarz (2012) and Rastegari, Condon and Immorlica (2013) study settings where firm preferences over workers are not completely known and are revealed only through interviews.

To our knowledge, the only theoretical papers that address endogenous information acquisition in matching are those of Bade (2015) and Harless and Manjunath (2015). In the context of house allocations, Bade finds that the unique *ex ante* Pareto optimal, strategy-proof and non-bossy allocation mechanism is that of serial dictatorship. However, in their study, Harless and Manjunath (2015) prove that the top-trading-cycles mechanism dominates the serial dictatorship mechanism under progressive measures of social welfare. Both papers focus on ordinal mechanisms.[1] As we show below, in any strategy-proof *ordinal* mechanism, students have no incentives to learn their cardinal preferences beyond the ordinal ones, while information on cardinal preferences can be welfare-improving, especially when students have similar ordinal preferences (Abdulkadiroğlu et al. 2011). Lastly, in an ongoing study, Artemov (2016) considers an environment similar to our experimental setting to compare the performance of IA and DA.

Another unique feature of our study is the acquisition of information on others' preferences, which is in contrast with other studies that focus on the acquisition of information on one's own preferences. One exception in this body of literature is Kim (2008), who considers a common-value first-price auction with two bidders, one of whom learns her opponent's signal.

In addition to the matching literature, information acquisition is considered in other fields, e.g., bargaining (Dang 2008), committee decisions (Persico 2004, Gerardi and Yariv 2008), contract theory (Crémer, Khalil and Rochet 1998, Crémer and Khalil 1992), finance (Barlevy and Veronesi 2000, Hauswald and Marquez 2006, Van Nieuwerburgh and Veldkamp 2010), and law and economics (Lester, Persico and Visschers 2009). In particular, there is a large theoretical literature on the role of information acquisition in mechanism design, especially in auction design, e.g., Persico (2000), Compte and Jehiel (2007), Crémer, Spiegel and Zheng (2009), Shi (2012), surveyed in Bergemann and Valimaki (2006). Notably, Bergemann and Valimaki (2002) show that the Vickrey-Clark-Groves mechanism guarantees both *ex ante* and *ex post* efficiency in every private value environment.

3 Information Acquisition

In this section, we outline a theoretical model of endogenous information acquisition for one's own and others' preferences under two common school choice mechanisms, the Immediate and Deferred Acceptance mechanisms.

[1] An ordinal mechanism only requires agents to reveal their ordinal preferences.

3.1 The Setup

Our model begins with a finite set of students, I, to be assigned to a finite set of schools, S, through a centralized school choice mechanism. S is supplemented by a "null school" or outside option, s^0, and $\overline{S} \equiv S \cup s^0$. For each $s \in S$, there is a finite supply of seats, $q_s \in \mathbb{N}$, and the total capacity is no more than the total number of students, $\sum_{s \in S} q_s \leq |I|$, while $q_s > 0$ for all s. By assumption, $q_{s^0} \geq |I|$. Moreover, schools rank students using a common uniform random lottery (single tie-breaking) whose realization is unknown to students when they enter the mechanism.

Student i's valuations of schools an i.i.d. vector draw from a distribution, F, denoted by a vector $V_i = [v_{i,s}]_{s \in S}$, where $v_{i,s} \in [\underline{v}, \overline{v}]$, $0 < \underline{v} < \overline{v}$, is i's von Neumann-Morgenstern utility of school s. For notational convenience, we assume that $v_{i,s^0} = 0$ for all i, which implies that every school in S is acceptable to everyone. Therefore, this is an independent-private-value model, and we discuss how our results generalize to common- and interdependent-value models in Section 5.

Furthermore, student preferences are strict: For any pair of distinct schools s and t in S, $v_{i,s} \neq v_{i,t}$ for all i. We therefore define strict ordinal preferences P on S such that $sP_i t$ if and only if $v_{i,s} > v_{i,t}$. We also augment the set of all possible strict ordinal preferences $\mathcal{P}$ with a "null preference" $P^{\phi} \equiv \emptyset$ denoting that one has no information on her ordinal preference, expressed as $\bar{\mathcal{P}} = \mathcal{P} \cup \emptyset$. The distribution of V conditional on P is denoted by $F(V|P)$, while the probability mass function of P implied by F is $G(P|F)$ ($\mathcal{P}$ is finite). We impose a full-support assumption on $G(P|F)$, i.e., $G(P|F) > 0, \forall P \in \mathcal{P}$, indicating that every strict ordinal preference ranking is possible given the distribution of cardinal preferences. Necessarily, $G(P^{\phi}|F) = 0$.

In our model, the value of the outside option and the distribution of preferences, $F(V)$ and thus $G(P|F)$, are always common knowledge. However, in contrast to previous models of school choice, we introduce an information-acquisition stage for each i to learn her own preferences (P_i and/or V_i) or others' preferences (V_{-i}) before entering the mechanism. Because of the independent-private-value nature, learning about others' preferences is only for the purpose of gaming or competing with other students.

3.2 School Choice Mechanisms

We focus on two mechanisms popular in both research literature and practice: the Boston Immediate Acceptance and the Gale-Shapley Deferred Acceptance mechanism.

The **Immediate Acceptance** mechanism (IA) asks students to submit rank-ordered lists (ROL) of schools. Together with the pre-announced capacity of each school, IA uses pre-defined rules to determine the school priority ranking over students and consists of the following rounds:

Round 1. Each school considers all students who rank it first and assigns its seats in order of their priority at that school until either there is no seat left at that school or no such student left.

Generally, in:

Round $(k > 1)$. The kth choice of the students who have not yet been assigned is considered. Each school that still has available seats assigns the remaining seats to students who rank it as kth choice in order of their priority at that school until either there is no seat left at that school or no such student left.

The process terminates after any round k when either every student is assigned a seat at some school, or the only students who remain unassigned have listed no more than k choices.

The **Gale-Shapley Deferred Acceptance** mechanism (DA) can be either student-proposing or school-proposing. We focus on the student-proposing DA mechanism in this study. Specifically, the mechanism collects school capacities and students' ROLs for schools. With strict rankings of schools over students that are determined by pre-specified rules, it proceeds as follows:

Round 1. Every student applies to her first choice. Each school rejects the least ranked students in excess of its capacity and temporarily holds the others.

Generally, in:

Round $(k > 1)$. Every student who is rejected in Round $(k-1)$ applies to the k^{th} choice on her list. Each school pools together new applicants and those on hold from Round $(k-1)$. It then rejects the least ranked students in excess of its capacity. Those who are not rejected are temporarily held.

The process terminates after any Round k when no rejections are issued. Each school is then matched with those students it is currently holding.

In the following, for simplicity, we assume that schools rank students by a post-application uniform lottery without pre-defined priorities.[2]

3.3 Acquiring Information on Own Preferences

We first investigate the incentives to acquire information on one's own value. The timing of the game and the corresponding information structure are described as follows and also in Figure 1:

(i) Nature draws individual valuation V_i, and thus ordinal preferences P_i, from $F(V)$ for each i, but i knows only the value distribution $F(V)$;

(ii) Each individual i decides whether to acquire a signal on her ordinal preferences; If yes, she decides how much to invest in information acquisition, denoted by $\alpha \in [0, \bar{\alpha}]$.

[2] This restriction is imposed in the model studied by Abdulkadiroğlu et al. (2011) and is implemented in Beijing's middle school choice (He 2018). By imposing this assumption, we do not consider a student's learning about her own or others' pre-determined priorities at schools. We leave this generalization for future work. In principle, an education authority can effectively inform a student about her priorities, while informing her about her preferences over schools is less straightforward and more costly.

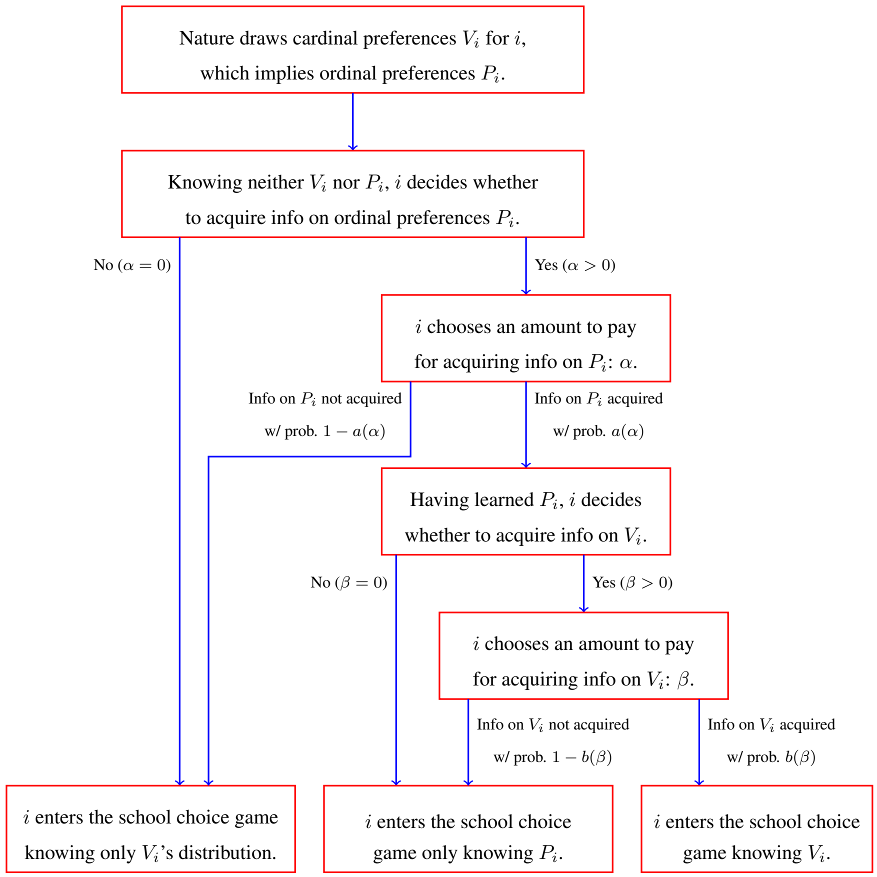

Figure 1: Acquiring Information on One's Own Preferences.

(iii) If ordinal preferences are learned, she then chooses the investment, $\beta \in [0, \bar{\beta}]$, to acquire a signal on her cardinal preferences.

(iv) Regardless of the information acquisition decision or outcome, every student plays the school choice game under either IA or DA.

We differentiate between the learning of ordinal and cardinal preferences, as the former represents acquiring coarse information about the schools, whereas the latter represents obtaining more detailed information and therefore is more costly. In a similar vein, the literature on one-sided and two-sided matching usually assumes that agents know their own ordinal preferences (Roth and Sotomayor 1990), while cardinal preferences being possibly unknown due to "limited rationality" (Bogomolnaia and Moulin 2001).

3.3.1 Technology of Information Acquisition

Information acquisition in our model is covert. That is, i knows that others are engaging in information acquisition, but does not know what information they have acquired.

The information acquisition process consists of two stages (see Figure 1): i first pays a cost α

to acquire a signal on the ordinal preference, $\omega_{1,i} \in \bar{\mathcal{P}}$. With probability $a(\alpha)$, she learns perfectly, $\omega_{1,i} = P_i$; by contrast, with probability $1-a(\alpha)$ she learns nothing, $\omega_{1,i} = P^{\phi}$. In the second stage, having learned ordinal preferences P_i, i may pay another cost, β, to learn her cardinal preferences by acquiring a signal $\omega_{2,i} \in \bar{\mathcal{V}}$, where $\bar{\mathcal{V}} \equiv [\underline{v}, \overline{v}]^{|S|} \cup V^{\phi}$. Here, with probability $b(\beta)$, she learns her cardinal preferences, $\omega_{2,i} = V_i$; by contrast, with probability $1 - b(\beta)$, she learns nothing, $\omega_{2,i} = V^{\phi}$, where V^{ϕ} denotes no cardinal preference information.

The technologies $a(\alpha)$ and $b(\beta)$ are such that $a(0) = b(0) = 0$, $\lim_{\alpha\to\infty} a(\alpha) = \lim_{\beta\to\infty} b(\beta) = 1$, $a', b' > 0$, $a'', b'' < 0$, and $a'(0) = b'(0) = +\infty$.[3] The cost of information acquisition is $c(\alpha, \beta)$, where $c(0,0) = 0$, $c_{\alpha}, c_{\beta} > 0$, $c_{\alpha\beta}, c_{\alpha\alpha}, c_{\beta\beta} \geq 0$ for all (α, β) and $c_{\alpha}(0,0), c_{\beta}(\alpha, 0) < +\infty$ for all $\alpha \geq 0$. Given these restrictions, we limit our attention to $\alpha \in [0, \bar{\alpha}]$ and $\beta \in \left[0, \bar{\beta}\right]$, where $c(\bar{\alpha}, 0) = c(0, \bar{\beta}) = \overline{v}$, so that $c(\alpha, \beta)$ does not exceed the maximum possible payoff ($\overline{v}$).

After the two-stage information acquisition, the information i has is summarized by signals $\omega_i = (\omega_{1,i}, \omega_{2,i}) \in \bar{\mathcal{P}} \times \bar{\mathcal{V}}$. If i pays (α, β), the distribution of signals is $H(\omega_i|\alpha, \beta)$, as outlined below:

$$\begin{aligned}
H\left(\omega_i = \left(P^{\phi}, V^{\phi}\right)|\alpha, \beta\right) &= 1 - a(\alpha), && \text{(learning nothing)} \\
H\left(\omega_i = \left(P_i, V^{\phi}\right)|\alpha, \beta\right) &= a(\alpha)(1 - b(\beta)), && \text{(learning ordinal but not cardinal)} \\
H\left(\omega_i = (P_i, V_i)|\alpha, \beta\right) &= a(\alpha) b(\beta), && \text{(learning both ordinal and cardinal).}
\end{aligned}$$

Together, they imply that $H(\omega_i = (P, V)|\alpha, \beta) = 0$, if $(P, V) \notin \{(P^{\phi}, V^{\phi}), (P_i, V^{\phi}), (P_i, V_i)\}$. In other words, an agent cannot receive anything other than the three types of signals.

Upon observing signal ω_i, the posterior distributions of cardinal and ordinal preferences are:

$$F(V|\omega_i) = \begin{cases} F(V) & \text{if } \omega_i = \left(P^{\phi}, V^{\phi}\right), \\ F(V|P_i) & \text{if } \omega_i = \left(P_i, V^{\phi}\right), \\ 1_{V_i} & \text{if } \omega_i = (P_i, V_i); \end{cases} \quad G(P|\omega_i) = \begin{cases} G(P|F) & \text{if } \omega_i = \left(P^{\phi}, V^{\phi}\right), \\ 1_{P_i} & \text{if } \omega_i = \left(P_i, V^{\phi}\right), \\ 1_{P_i} & \text{if } \omega_i = (P_i, V_i); \end{cases}$$

where 1_{V_i} (or 1_{P_i}) is the probability distribution placing probability 1 on point V_i (or P_i).

3.3.2 Game of School Choice with Information Acquisition

In our model, after observing the signal ω_i, students enter the school choice game under either DA or IA. Each student i submits an ROL denoted by $L_i \in \mathcal{P}$ such that sL_it if and only if s is ranked

[3] The infinite marginal productivity at zero input is consistent with, for example, the Cobb-Douglas function. When necessary, we define $0 \cdot \infty = 0$.

above t.[4] When i submits L_i and others submit L_{-i}, the payoff is represented by:

$$u\left(V_i, L_i, L_{-i}\right) = \sum_{s\in S} a_s\left(L_i, L_{-i}\right) v_{i,s} \equiv A\left(L_i, L_{-i}\right)\cdot V_i,$$

where $a_s\left(L_i, L_{-i}\right)$ is the probability that i is accepted by s, given $\left(L_i, L_{-i}\right)$, and $A\left(L_i, L_{-i}\right)$ is the vector of the probabilities determined by the mechanism. We further distinguish between two types of mechanisms: strategy-proof and non-strategy-proof. A mechanism is **strategy-proof** if:

$$u\left(V_i, P_i, L_{-i}\right) \geq u\left(V_i, L_i, L_{-i}\right), \forall L_i, L_{-i}, \text{and } \forall V_i;$$

i.e., reporting true ordinal preferences is a dominant strategy. It is well-known that the student-proposing DA is strategy-proof (Dubins and Freedman 1981, Roth 1982), while IA is not (Abdulkadiroğlu and Sönmez 2003).

Under either mechanism, a **symmetric Bayesian Nash equilibrium** is defined by a tuple $\left(\alpha^*, \beta^*\left(P, \alpha^*\right), \sigma^*\left(\omega\right)\right)$ such that, for all i:

(i) A (possibly mixed) strategy $\sigma^*\left(\omega\right) : \bar{\mathcal{P}} \times \bar{\mathcal{V}} \to \Delta\left(\mathcal{P}\right)$,

$$\sigma^*\left(\omega\right) \in \arg\max_{\sigma}\left\{\int\int\int u\left(V, \sigma, \sigma^*\left(\omega_{-i}\right)\right) dF\left(V|\omega\right) dF\left(V_{-i}|\omega_{-i}\right) dH\left(\omega_{-i}|\alpha^*_{-i}, \beta^*_{-i}\right)\right\}.$$

With her own signal ω, everyone plays a best response, recognizing that others have paid $\left(\alpha^*_{-i}, \beta^*_{-i}\right)$ to acquire information. This leads to a value function given $\left(\omega, \alpha^*_{-i}, \beta^*_{-i}\right)$:

$$\Pi\left(\omega, \alpha^*_{-i}, \beta^*_{-i}\right) \equiv \max_{\sigma}\left\{\int\int\int u\left(V, \sigma, \sigma^*\left(\omega_{-i}\right)\right) dF\left(V|\omega\right) dF\left(V_{-i}|\omega_{-i}\right) dH\left(\omega_{-i}|\alpha^*_{-i}, \beta^*_{-i}\right)\right\}.$$

(ii) Acquisition of information on cardinal preferences $\beta^*\left(P, \alpha^*\right) : \mathcal{P} \times [0, \bar{\alpha}] \to \left[0, \bar{\beta}\right], \forall P$,

$$\beta^*\left(P, \alpha^*\right) \in \arg\max_{\beta}\left\{\begin{array}{l} b\left(\beta\right)\int \Pi\left(\left(P, V\right), \alpha^*_{-i}, \beta^*_{-i}\right) dF\left(V|P\right) \\ +\left(1 - b\left(\beta\right)\right)\Pi\left(\left(P, V^{\phi}\right), \alpha^*_{-i}, \beta^*_{-i}\right) - c\left(\alpha^*, \beta\right) \end{array}\right\}.$$

Here, $\beta^*\left(P, \alpha^*\right)$ is the optimal decision given that one has learned her ordinal preference (P) after paying α^* to acquire P.

[4] We restrict the set of actions to the set of possible ordinal preferences, $\mathcal{P}$. In other words, students are required to rank all schools in S. The analysis can be straightforwardly extended to allowing ROLs of any length.

(iii) Acquisition of information on ordinal preferences $\alpha^* \in [0, \bar{\alpha}]$,

$$\alpha^* \in \arg\max_{\alpha} \left\{ \begin{array}{l} a(\alpha) \int \left[\begin{array}{l} b(\beta^*(P,\alpha)) \int \Pi\left((P,V), \alpha^*_{-i}, \beta^*_{-i}\right) F(V|P) \\ +(1-b(\beta^*(P,\alpha))) \Pi\left(\left(P, V^{\phi}\right), \alpha^*_{-i}, \beta^*_{-i}\right) \\ -c(\alpha, \beta^*(P,\alpha)) \end{array} \right] dG(P|F) \\ +(1-a(\alpha)) \left[\Pi\left(\left(P^{\phi}, V^{\phi}\right), \alpha^*_{-i}, \beta^*_{-i}\right) - c(\alpha, 0)\right] \end{array} \right\}.$$

The above expression has already taken into account that the optimal β equals zero if one obtains a signal $\omega_1 = P^{\phi}$ in the first stage: $\beta^*(P^{\phi}, \alpha) = 0$ for all α.

Given the above, we can now state our existence result in Lemma 1:

Lemma 1. *Under DA or IA, a symmetric Bayesian Nash equilibrium exists.*

This also leads to our first proposition:

Proposition 1 (Information acquisition incentives: own preferences)**.** *In any symmetric Bayesian Nash equilibrium* $(\alpha^*, \beta^*(P, \alpha^*), \sigma^*(\omega))$ *under DA or IA, the following is true:*

(i) $\alpha^* > 0$*, i.e., students always have an incentive to learn their ordinal preferences;*

(ii) under DA, $\beta^*(P, \alpha^*) = 0\ \forall P, \alpha^*$*, i.e., there is no incentive to learn cardinal preferences;*

(iii) under IA, there exists a preference distribution F *such that* $\beta^*(P, \alpha^*) > 0$ *for some* P.

Remark 1. *Similar to the results for DA, students have no incentive to learn their own cardinal preferences under a strategy-proof mechanism that elicits only ordinal preferences.*[5]

3.4 Acquiring Information on Others' Preferences

We now consider a student's incentive to acquire information on others' preferences. Here, we assume that everyone knows exactly her own cardinal preferences (V_i) but not others' preferences (V_{-i}), and that the distribution of V_i, $F(V_i)$, is common knowledge with the same properties as before. The purpose of such a setting is to highlight the incentive to collect information for strategic purposes, above and beyond the incentive to learn one's own preferences. The process and technology for information acquisition are depicted in Figure 2.

To acquire information, student i may pay δ to acquire a signal of V_{-i}, $\omega_{i,3} \in \bar{\mathcal{V}}^{(|I|-1)}$. With probability $d(\delta)$, she learns perfectly, $\omega_{3,i} = V_{-i}$; with probability $1 - d(\delta)$, $\omega_{3,i} = V^{\phi}_{-i}$, i.e., she

[5] On the contrary, a mechanism that directly elicits and uses information on cardinal preferences, e.g., Kovalenkov (2002), incentivizes students to learn about their own cardinal preferences, even if the mechanism is strategy-proof.

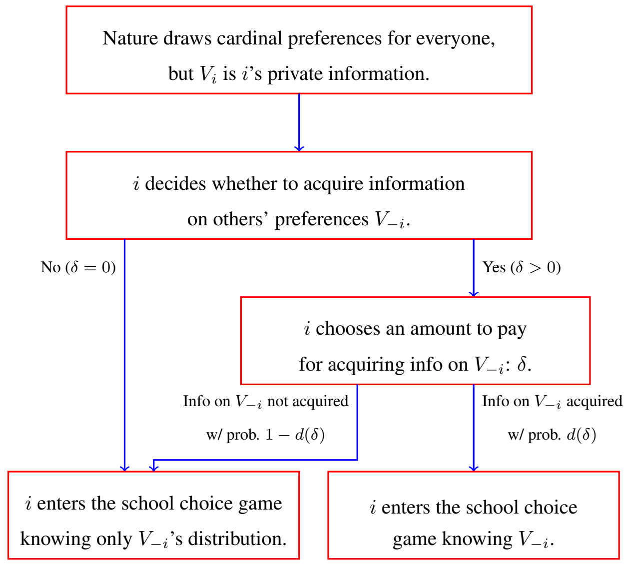

Figure 2: Acquiring Information on Others' Preferences.

learns nothing. The distribution of signals and the posterior distribution of preferences are:

$$\begin{aligned} &K\left(\omega_{3,i} = V^{\phi}_{-i}|\delta\right) = 1 - d\left(\delta\right), \\ &K\left(\omega_{3,i} = V_{-i}|\delta\right) = d\left(\delta\right), \\ &K\left(\omega_{3,i} = V'_{-i}|\delta\right) = 0 \text{ if } V'_{-i} \notin \{V_{-i}, V^{0}_{-i}\}; \end{aligned} \qquad F\left(V_{-i}|\omega_{3,i}\right) = \begin{cases} F\left(V_{-i}\right) & \text{if } \omega_{3,i} = V^{\phi}_{-i}; \\ 1_{V_{-i}} & \text{if } \omega_{3,i} = V_{-i}. \end{cases}$$

The technology has the following properties: $d\left(0\right) = 0$, $\lim_{\delta \to \infty} d\left(\delta\right) = 1$, $d' > 0$, $d'' < 0$, and $d'\left(0\right) = \infty$. The cost for information acquisition is $e\left(\delta\right)$ such that $e\left(0\right) = 0$, $e', e'' > 0$ and $e'\left(0\right) < \infty$. Similarly, we restrict our attention to $\delta \in \left[0, \bar{\delta}\right]$, where $e\left(\bar{\delta}\right) = \bar{v}$.

Information acquisition is again covert. We focus on a symmetric Bayesian Nash equilibrium, $\left(\delta^*\left(V\right), \bar{\sigma}^*\left(\omega_3, V\right)\right)$, where:

(i) A (possibly mixed) strategy $\bar{\sigma}^*\left(\omega_3, V\right) : \bar{\mathcal{V}}^{(|I|-1)} \times \mathcal{V} \to \Delta\left(\mathcal{P}\right)$, such that

$$\bar{\sigma}^*\left(\omega_{3,i}, V_i\right) \in \arg\max_{\bar{\sigma}} \left\{ \int \int u\left(V_i, \bar{\sigma}, \bar{\sigma}^*\left(\omega_{3,-i}, V_{-i}\right)\right) dF\left(V_{-i}|\omega_{3,i}\right) dK\left(\omega_{3,-i}|\delta^*_{-i}\right) \right\}.$$

That is, given one's own signal $\omega_{3,i}$, everyone plays a best response, recognizing that everyone has paid δ^* to acquire information (denoted as δ^*_{-i}). We further define the value function given $\left(\omega_{3,i}, \delta^*_{-i}\right)$ and V_i as:

$$\Phi\left(V_i, \omega_{3,i}, \delta^*_{-i}\right) = \max_{\bar{\sigma}} \left\{ \int \int u\left(V_i, \bar{\sigma}, \bar{\sigma}^*\left(\omega_{3,-i}, V_{-i}\right)\right) dF\left(V_{-i}|\omega_{3,i}\right) dK\left(\omega_{3,-i}|\delta^*_{-i}\right) \right\}.$$

(ii) Acquisition of information on others' preferences $\delta^*(V) : \bar{\mathcal{V}} \to [0, \bar{\delta}], \forall V$:

$$\delta^*(V_i) \in \arg\max_{\delta} \left\{ d(\delta) \int \Phi\left(V_i, V_{-i}, \delta^*_{-i}\right) dF(V_{-i}) + (1 - d(\delta))\, \Phi\left(V_i, V^{\phi}_{-i}, \delta^*_{-i}\right) - e(\delta) \right\}.$$

Here, $\delta^*(V_i)$ is the optimal information acquisition strategy.

The existence of such an equilibrium can be proven by similar arguments in the proof of Lemma 1, and the properties of information acquisition in equilibrium is summarized as follows:

Proposition 2 (Information acquisition incentives: others' preferences)**.** *Suppose* $(\delta^*(V), \sigma^*(\omega_3, V))$ *is an arbitrary symmetric Bayesian Nash equilibrium under a given mechanism. We have:*

(i) $\delta^*(V) = 0$ *for all* V *under DA;*

(ii) There always exists a preference distribution F *such that* $\delta^*(V) > 0$ *under IA for* V *in some positive-measure set.*

Remark 2. *Similar to the results for DA, students have no incentive to learn others' preferences under a strategy-proof mechanism that elicits either ordinal or cardinal information from students.*

In short, this result provides another perspective on strategy-proofness as a desideratum in market design: a strategy-proof mechanism makes the school choice game easier to play by reducing the incentive to acquire information on others' preferences to zero.

3.5 Welfare Effects of Information Acquisition

By considering the cost of information acquisition, our setting delivers different results on the welfare comparison between the two mechanisms. Below, we provide a numerical example showing that student welfare under each mechanism is sensitive to the cost of information acquisition.

Specifically, let us consider an example in which IA dominates DA when cardinal preferences are private information. We investigate how the cost of acquiring information on one's own preferences affects the welfare performance of the two mechanisms. There are two schools. Each school has one seat, and student preference distribution is described in Table 1. There are three students, and each student's preferences are an i.i.d. draw from the distribution.

Table 1: Distribution of Student Preferences (F)

Probability	Preferences: $(v_{i,1}, v_{i,2})$
$p_1 = \frac{1.7}{3}$	$(1, 0.15)$
$p_2 = \frac{0.85}{3}$	$(1, 0.7)$
$p_3 = 0.15$	$(0.15, 1)$

Without any additional information other than the preference distribution, the expected utility for every student of being assigned school 1 is $E(V_{i,1}) = 0.8725$ and $E(V_{i,2}) = 1.3/3$. Without acquiring any information, students will submit (s_1, s_2) under DA; under IA, everyone submitting (s_1, s_2) is also the unique equilibrium. Under either mechanism, every student gets 0.44 in terms of expected utility.

The technology of information acquisition is the same as discussed in Section 3.3 (in particular, Figure 1), and there is no possibility of acquiring information on others' preferences. We further specify that $a(\alpha) = \frac{\sqrt{\alpha}}{k}$ (for ordinal information) and $b(\beta) = \frac{\sqrt{\beta}}{10k^2}$ (for cardinal information).[6] The cost function is $c(\alpha, \beta) = \alpha + \beta + 10k\alpha\beta$. To see how welfare changes with information acquisition, we let k be one of the 17 values $\{0, 0.05, 0.09, 0.15, \ldots, 100, \infty\}$. Between 0.05 and 100, k increases on a logarithmic scale. When $k = 0$, there is no cost to acquire information on either ordinal or cardinal preferences; when $k = \infty$, it is impossible to acquire any information.

For a given k under a mechanism, we solve for a symmetric Bayesian Nash equilibrium as defined in Section 3.3 and calculate ex ante equilibrium payoffs (net of information acquisition costs). Figure 3 depicts how the efficiency of each mechanism is affected by information acquisition costs.

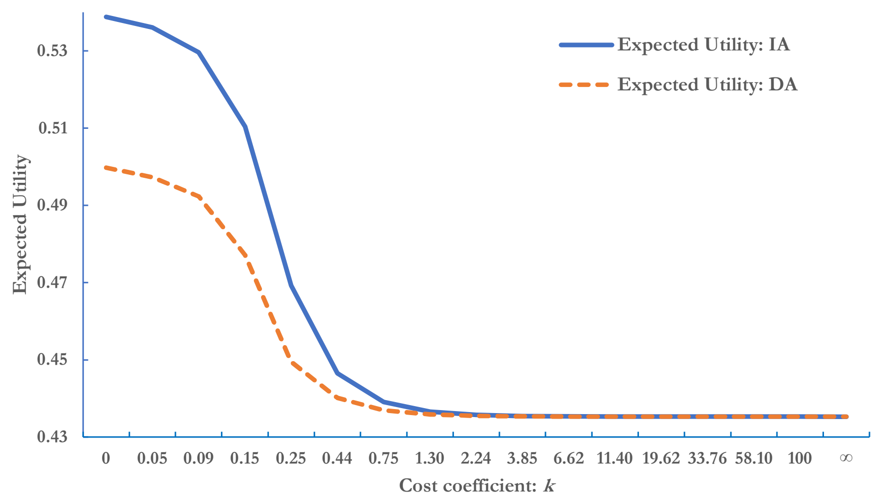

Figure 3: Equilibrium Payoffs with Information Acquisition on Own Preferences

Notes: This figure shows the ex ante payoffs (net of information acquisition costs) in symmetric equilibrium when students endogenously acquire information on their own preferences. The technology of information acquisition is described in Figure 1 of Section 3.3 and is further specified by $a(\alpha) = \frac{\sqrt{\alpha}}{k}$ (for ordinal information) and $b(\beta) = \frac{\sqrt{\beta}}{5k}$ (for cardinal information). That is, to have a probability p_o of learning one's own ordinal preferences, one needs to invest $(k \cdot p_o)^2$; given that ordinal preferences are known, to have a probability p_c of learning one's own cardinal preferences, the investment has to be $(10k^2 \cdot p_c)^2$. The cost function is $c(\alpha, \beta) = \alpha + \beta + 10k\alpha\beta$. k has 17 possible values, $\{0, 0.05, 0.09, 0.15, \ldots, 100, \infty\}$, and between 0.05 and 100, k increases on a logarithmic scale. Ex ante payoff in symmetric equilibrium is constant across students, as they are homogenous ex ante.

When $k = 0$ (i.e., free information), cardinal preferences are private information; we have the same result as in the literature that IA delivers higher welfare than DA. However, when k

[6] In other words, to have a probability $p_o \in [0, 1)$ of learning one's own ordinal preferences, one needs to invest $(k \cdot p_o)^2$; given that ordinal preferences are known, to have a probability $p_c \in [0, 1)$ of learning one's own cardinal preferences, the investment has to be $(10k^2 \cdot p_c)^2$.

increases, the welfare advantage of IA decreases and essentially disappears when $k \geq 1.30$.[7] This is because students invest less in information acquisition and thus more frequently fail to acquire information. The two mechanisms converge to the same equilibrium outcome $k \to \infty$ (i.e., impossible to acquire information). The welfare performance of DA also decreases when the cost becomes higher, because fewer students successfully acquire ordinal information.

The possibility of acquiring information on others' preferences only affects IA's welfare performance, but not DA's. As shown in below in part (iv) of Proposition 4, the information on others' preferences has an ambiguous effect on student welfare even when it is free. Therefore, the ex ante welfare of IA can be worse in some cases.

4 Information Provision

While students always have incentives to acquire information on their own preferences and sometimes on others' preferences, information is not always successfully acquired due to the costs. In this section, we examine the impact of information provision by education authorities.

In our model, we assume that the provision of information decreases the cost of information acquisition to zero, while the lack of it increases such cost to infinity. For simplicity, we focus on a special setting where everyone has the same ordinal (but different cardinal) preferences, similar to the setting in Abdulkadiroğlu et al. (2011) and Troyan (2012). This setting is unfortunately not a special case of the model in sections 3.3 and 3.4, because student preferences are correlated. However, it can be shown that the main results, Propositions 1 and 2, still hold true in the setting of this section.

We start with a prior F and thus $G(P|F)$ such that after a P is drawn, it becomes everyone's ordinal preference. Again, every school to be acceptable: $v_{i,s} > 0$ for all i and s. We use F_{v_s} to denote the marginal distribution of the cardinal preference for school s.

We next represent the education authority's decision regarding how much information to release by sending a vector of signals to every i: $\bar{\omega}_i = (\bar{\omega}_{1,i}, \bar{\omega}_{2,i}, \bar{\omega}_{3,i}) \in \bar{\mathcal{P}} \times \bar{\mathcal{V}} \times \bar{\mathcal{V}}^{(|I|-1)}$, where $\bar{\omega}_{1,i}$ and $\bar{\omega}_{2,i}$ are the signals of i's ordinal and cardinal preferences respectively, and $\bar{\omega}_{3,i}$ is the signal of others' cardinal preferences. All signals are such that $\bar{\omega}_{1,i} \in \left\{P^{\phi}, P_i\right\}$, $\bar{\omega}_{2,i} \in \left\{V^{\phi}, V_i\right\}$, and $\bar{\omega}_{3,i} = \left\{V_{-i}^{\phi}, V_{-i}\right\}$, i.e., they are either perfectly informative or completely uninformative.

We study the *ex ante* welfare in equilibrium under each of the following information structures:

(i) Uninformed (**UI**): $\bar{\omega}_i = \left(P^{\phi}, V^{\phi}, V_{-i}^{\phi}\right)$, $\forall i$;

(ii) Ordinally Informed (**OI**): $\bar{\omega}_i = \left(P_i, V^{\phi}, V_{-i}^{\phi}\right)$, $\forall i$;

[7] In fact, when $k = 58.10$ or 100, DA delivers slightly higher welfare than IA, although this may be due to simulation errors.

(iii) Cardinally Informed (**CI**): $\bar{\omega}_i = \left(P_i, V_i, V_{-i}^{\phi}\right), \forall i$;

(iv) Perfectly Informed (**PI**): $\bar{\omega}_i = (P_i, V_i, V_{-i}), \forall i$.

It should be noted that the identical ordinal preference is common knowledge under OI, CI, or PI. However, under UI, no one knows the realization of ordinal preference, but everyone knows that the ordinal preference will be the same across students.

These four information structures reflect possible outcomes of different school choice policies. When the education authority makes it difficult for students to acquire information on schools, we are likely to be in the UI scenario. When it makes some information easy to access, students may find it costless to learn their ordinal preferences, and thus we are likely in the OI scenario. If all information on own preferences is readily available, we are likely to be in the CI scenario.

We are also interested in the PI scenario, which relates to the gaming part of school choice under a non-strategy-proof mechanism. From Proposition 2, individual students have incentives to acquire information on others' preferences under IA. The literature has shown that this additional strategic behavior may create additional inequalities in access to public education. More precisely, if one does not understand the game and does not invest enough to acquire information on others' preferences, she may have a disadvantage when playing the school choice game. As a policy intervention, education authority can choose to make this information easier to obtain by publishing students' strategies and allowing students to revise their applications upon observing others' strategies as in Amsterdam (De Haan et al. 2015) and Wake County, NC (Dur, Hammond and Morrill 2015).

Note that a symmetric Bayesian Nash equilibrium, possibly in mixed strategies, always exists under any of the four information structures by the standard fixed point arguments. We summarize the results on *ex ante* welfare under DA and IA in the following two propositions.

Proposition 3 (Ex ante welfare under DA)**.** *Under DA, the ex ante welfare of every student under any of the four information structures (UI, OI, CI, and PI) equals* $\sum_{s\in S} \frac{q_s}{|I|} \int v_{i,s} dF_{v_s}(v_{i,s})$ *in any symmetric equilibrium.*

This implies that there is no gain in *ex ante* student welfare when students receive more information under DA.

Finally, we state our last proposition.

Proposition 4 (Ex ante welfare under IA)**.** *Under IA, we obtain the following ex ante student welfare comparisons in terms of Pareto dominance in a symmetric equilibrium:*

(i) When uninformed or ordinally informed, the student welfare is $\sum_{s\in S} \frac{q_s}{|I|} \int v_{i,s} dF_{v_s}(v_{i,s})$*;*

(ii) Welfare for cardinally informed students weakly dominates that for uninformed or ordinally informed students: CI $\geq$ *OI* $=$ *UI;*

(iii) Welfare for perfectly informed students weakly dominates that for uninformed or ordinally informed students: $PI \geq OI = UI$;

(iv) Welfare ranking for perfectly versus cardinally informed students is ambiguous.

The above proposition suggests that it is always beneficial to provide more information on one's own cardinal preferences, but the effect of providing information on others' preferences is ambiguous. To prove part (iv), we use two examples in Appendix A (sections A.5.4 and A.5.5). The intuition is as follows: when perfectly informed, it is possible that multiple high-type students at a school play mixed strategies in equilibrium instead of always top-ranking that school, as they compete for the same school seats knowing the presence of other high-type students. Consequently, always top-ranking that school becomes sub-optimal; the school may end up being assigned to a low-type student, leading to a welfare loss. By contrast, when cardinally informed in a symmetric Bayesian Nash equilibrium, high-type students may choose to always top rank the school.

5 Concluding Remarks

This paper provides insights for designing better school choice programs by studying endogenous information acquisition and the effects of information provision.

We distinguish between two types of information acquisition. One is to learn one's own preferences over schools, and the other is to discover others' preferences. Acquiring information on own preferences is necessary in school choice, given the complex nature of education production and the usual lack of information on schools. In contrast, learning about others' preferences is more related to competing with other students.

The two popular mechanisms, DA and IA, provide heterogeneous degrees of incentives for students to acquire information on preferences. Only IA incentivizes students to learn their own cardinal and others' preferences, while students under DA have no incentive to acquire information beyond their own ordinal preferences. We demonstrate that information acquisition costs affect the efficiency of each mechanism and the welfare ranking between the two. This implies that it is important to endogenize information acquisition in welfare analyses of school choice.

In the case where everyone has the same ordinal preferences, we show the welfare effects of various policies of information provision. The results reveal that information provision is irrelevant in DA, while providing more information on own cardinal preferences is always welfare-improving in IA. However, more information about others' preferences can sometimes be welfare-decreasing in IA.

Our model can be potentially extended in several dimensions. The results can be generalized to the setting in which students have interdependent values over schools. In this case, acquiring information on own values can be achieved by learning more about the schools as well as learning from others' preferences. The information acquisition on others' values in the model should be interpreted as information gathering for strategic purposes beyond learning one's own preferences. With interdependent values, students decipher signals on others' preferences in two ways, useful information on one's own values and that on others' values. Our results then describe under each mechanism which deciphering is necessary.

Our model considers the sequential acquisition of information, but, in reality, students may acquire information on one's own and others' preferences simultaneously. Given the lack of strategy-proofness and the role of cardinal utility under IA, we expect our results to hold.

Finally, our model assumes every student is rational; however, this assumption is not born out in laboratory or field studies (Chen and Sönmez 2006, Abdulkadiroğlu, Pathak, Roth and Sönmez 2006, He 2018). Further studies might explore a theoretical model with students of heterogenous sophistication levels, as in Pathak and Sönmez (2008). Given these considerations, the laboratory experiment in our companion paper (Chen and He 2018) may help us better understand how the theoretical predictions correspond to actual participant decisions in a school choice context.

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Appendix A Proofs

Before proving the propositions, let us summarize the properties of the two mechanisms. As the results can be easily verified by going through the mechanisms, we omit the formal proof.[8]

Lemma 2. *DA and IA (with single tie breaking) have the following properties:*

(i) Monotonicity: If the only difference between L_i and L_i' is that the positions of s and t are swapped such that tL_is, $sL_i't$, and $\#\{s'' \in S|s'L_is''\} = \#\{s'' \in S|s'L_i's''\}$ for all $s' \in S\backslash\{s,t\}$, then:

$$a_s(L_i', L_{-i}) \geq a_s(L_i, L_{-i}), \forall L_{-i};$$

the inequality is strict when $L_j = L_i$, $\forall j \neq i$.

(ii) Guaranteed share in first choice: If school s is top ranked in L_i by i, $a_s(L_i, L_{-i}) \geq q_s/|I|$, for all L_{-i}.

(iii) Guaranteed assignment: $\sum_{s\in S} a_s(L_i, L_{-i}) = 1$ for all L_{-i}.

A.1 Proof of Lemma 1.

The proof applies to either DA or IA. Note that given any $(\alpha_{-i}, \beta_{-i})$ of other students, $\sigma^*(\omega)$ exists. This can be proven by the usual fixed point argument. Note that $\sigma^*(\omega)$ does not depend on one's own investments in information acquisition, although it does depend on the signal that one has received (ω).

Given ω, i's payoff function can be written as:

$$\int\int\int u_i(V, \sigma, \sigma^*(\omega_{-i}))\, dF(V|\omega)\, dF(V_{-i}|\omega_{-i})\, dH(\omega_{-i}|\alpha_{-i}, \beta_{-i}),$$

which is continuous in σ. Therefore, the value function $\Pi(\omega, \alpha_{-i}, \beta_{-i})$ is continuous in $(\alpha_{-i}, \beta_{-i})$ by the maximum theorem.

For student i, the optimal information acquisition is solved by the first-order conditions (second-order conditions are satisfied by the assumptions on the functions $a()$, $b()$, and $c()$):

$$\begin{aligned}
&a'(\alpha^*)\int\left[\begin{array}{l} b(\beta^*(P))\int\Pi\left((P,V), \alpha^*_{-i}, \beta^*_{-i}\right)F(V|P) \\ +(1-b(\beta^*(P)))\,\Pi\left(\left(P, V^\phi\right), \alpha^*_{-i}, \beta^*_{-i}\right) - c(\alpha^*, \beta^*(P)) \end{array}\right] dG(P|F) \\
&-a'(\alpha^*)\left[\Pi\left(P^\phi, \alpha^*_{-i}, \beta^*_{-i}\right) - c(\alpha^*, 0)\right] \\
&-a(\alpha^*)\int c_\alpha(\alpha^*, \beta^*(P))\, dG(P|F) - (1-a(\alpha^*))\, c_\alpha(\alpha^*, 0) = 0
\end{aligned}$$

$$b'(\beta^*(P))\left[\int\Pi\left(V, \alpha^*_{-i}, \beta^*_{-i}\right)dF(V|P) - \Pi\left(P, \alpha^*_{-i}, \beta^*_{-i}\right)\right] - c_{\beta^*}(\alpha^*, \beta^*(P)) = 0, \forall P \in \mathcal{P}.$$

Given the non-negative value of information and the properties of $a()$, $b()$, and $c()$, one can verify that there must exist α^* and $\beta^*(P)$ for all $P \in \mathcal{P}$ such that the first-order conditions are satisfied.

[8] Similar results on IA and their proofs are available in He (2018).

A.2 Proof of Proposition 1.

A.2.1 Proof of $\alpha^* > 0$

Given the existence of a symmetric equilibrium, let us suppose instead that $\alpha^* = 0$. It implies that $\beta^*(P) = 0$ for all $P \in \mathcal{P}$ and that the value function can be simplified as:

$$\begin{aligned}\Pi(\omega, \alpha^*, \beta^*) &= \Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right) \\ &= \max_\sigma \left\{\int\int u_i\left(V, \sigma, \sigma^*\left(\omega_{-i}\right)\right) dF(V)\, dF\left(V_{-i}\right)\right\}.\end{aligned}$$

Since $\alpha^* = 0$ and $\beta^* = \mathbf{0}$ (a $|\mathcal{P}|$-dimensional vector of zeros) is a best response for i, $\forall \alpha > 0$,

$$\begin{aligned}&\Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right) \\ &\geq \left\{a(\alpha)\int \Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right) dG(P|F) + (1 - a(\alpha))\,\Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right) - c(\alpha, 0)\right\};\end{aligned}$$

or

$$c(\alpha, 0) \leq a(\alpha)\left[\int \Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right) dG(P|F) - \Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right)\right], \forall \alpha > 0,$$

which can be satisfied if and only if $\Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right) = \Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right)$ for all $P \in \mathcal{P}$, given that $\int \Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right) dG(P|F) \geq \Pi\left(\left(P^\phi, V^\phi\right), 0, \mathbf{0}\right)$ and $c_\alpha(0, 0) < a'(0) = \infty$.

In a given symmetric equilibrium σ^*, the finiteness of the strategy space implies that a finite set of lists $\left(L^{(1)}, ..., L^{(N)}\right)$ are played with positive probabilities $\left(p^{(1)}, ..., p^{(N)}\right)$ ($N \in \mathbb{N}$). Suppose that s_1 is bottom ranked in $L^{(1)}$ and s_2 is the second to the bottom. Moreover, there exists an ordinal preference P^* such that $s_1 P^* s P^* s_2$ for all $s \neq s_1, s_2$. We also define $L^{(1)\prime}$ which only switches the ranking of the bottom two choices in $L^{(1)}$, s_1 and s_2.

Since $\Pi\left(\left(P^*, V^\phi\right), 0, \mathbf{0}\right) = \Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right)$, it implies that $L^{(1)}$ is also a best response to σ^* even if i has learned $P_i = P^*$. We then compare i's payoffs from submitting $L^{(1)}$ and $L^{(1)\prime}$.

By the monotonicity of the mechanism (Lemma 2), $a_{s_1}\left(L^{(1)\prime}, L_{-i}\right) \geq a_{s_1}\left(L^{(1)}, L_{-i}\right)$ and $a_{s_1}\left(L^{(1)\prime}, L_{-i}\right) \leq a_{s_1}\left(L^{(1)}, L_{-i}\right)$ for all L_{-i}. Moreover, $a_{s^*}\left(P^*, L_{-i}\right) > a_{s^*}\left(P, L_{-i}\right)$ when everyone else submits $L^{(1)}$ in L_{-i}.

Besides, under either of the two mechanisms, given a list, lower-ranked choices do not affect the admission probabilities at higher-ranked choices. Together with the guaranteed assignment (Lemma 2), it implies that $a_{s_1}\left(L^{(1)}, L_{-i}\right) + a_{s_2}\left(L^{(1)}, L_{-i}\right) = a_{s_1}\left(L^{(1)\prime}, L_{-i}\right) + a_{s_2}\left(L^{(1)\prime}, L_{-i}\right)$.

σ^* leads to a probability distribution over a finite number of possible profiles of others' actions (L_{-i}). With a positive probability, everyone else plays $L^{(1)}$. In this event, therefore, by submitting $L^{(1)\prime}$, i strictly increases the probability of being accepted by s_1 and decrease the probability of the least preferred school s_2, comparing with that of submitting $L^{(1)}$. Furthermore, in any other possible profile of L_{-i}, the probability of being assigned to s^* is also always weakly higher when submitting $L^{(1)\prime}$. Hence, $L^{(1)}$ is not a best response to σ^* when $P_i = P^*$, and thus $\Pi\left(\left(P^*, V^\phi\right), 0, \mathbf{0}\right) \neq \Pi\left(\left(P, V^\phi\right), 0, \mathbf{0}\right)$.

This contradiction proves that $\alpha^* = 0$ is not an equilibrium. Since an equilibrium always exists, it must be that $\alpha^* > 0$.

A.2.2 Proof of $\beta^*(P) = 0$ under DA

Suppose $\beta^*(P) > 0$ for some $P \in \mathcal{P}$ under DA or any strategy-proof ordinal mechanism. It implies that:

$$\beta^*(P) \int \Pi\left((P,V), \alpha^*_{-i}, \beta^*_{-i}\right) dF(V|P) + (1-\beta^*(P)) \Pi\left(\left(P, V^\phi\right), \alpha^*_{-i}, \beta^*_{-i}\right) - c(\alpha^*, \beta^*(P))$$
$$> \Pi\left(\left(P, V^\phi\right), \alpha^*_{-i}, \beta^*_{-i}\right),$$

or,

$$\beta^*(P) \left[\int \Pi\left((P,V), \alpha^*_{-i}, \beta^*_{-i}\right) dF(V|P) - \Pi\left(\left(P, V^\phi\right), \alpha^*_{-i}, \beta^*_{-i}\right)\right] > c(\alpha^*, \beta^*(P)). \quad (1)$$

However, strategy-proofness implies that:

$$\int \Pi\left((P,V), \alpha^*_{-i}, \beta^*_{-i}\right) dF(V|P) = \Pi\left(\left(P, V^\phi\right), \alpha^*_{-i}, \beta^*_{-i}\right),$$

and thus Equation (1) cannot be satisfied. Therefore $\beta^*(P) = 0$ for all $P \in \mathcal{P}$.

A.2.3 Proof of $\beta^*(P) > 0$ for some P under IA

We construct an example where $\beta^*(P) > 0$ for some P given the distribution F under IA. For notational convenience and in this proof only, we assume the upper bound of utility $\overline{v} = 1$ and the lower bound $\underline{v} = 0$, although we bear in mind that all schools are more preferable than outside option. Suppose that F implies a distribution of ordinal preferences $G(P|F)$ such that for s_1 and s_2:

$$G(P|F) = \begin{cases} (1-\varepsilon) & \text{if } P = \bar{P}, \text{ s.t. } s_1 \bar{P} s_2 \bar{P} s_3 ... \bar{P} s_{|S|}; \\ \frac{\varepsilon}{|\mathcal{P}|-1} & \text{if } P \neq \bar{P}. \end{cases}$$

The distribution of cardinal preferences is:

$$F\left(V|\bar{P}\right) = \begin{cases} 1-\eta & \text{if } (v_{s_1}, v_{s_2}) = (1, \xi) \text{ and } v_s < \xi^2, \forall s \in S \backslash \{s_1, s_2\}; \\ \eta & \text{if } (v_{s_1}, v_{s_2}) = (1, 1-\xi) \text{ and } v_s < \xi^2, \forall s \in S \backslash \{s_1, s_2\}; \\ 0 & \text{otherwise.} \end{cases}$$

(ε, η, ξ) are all small positive numbers in $(0,1)$. Otherwise, there is no additional restriction on $F(V|P)$ for $P \neq \bar{P}$ nor on $v_s, \forall s \in S\backslash\{s_1, s_2\}$.

Suppose that $\beta^*(P) = 0$ for all $P \in \mathcal{P}$. Section A.2.1 implies that $\alpha^* > 0$. If $\omega_i = \left(\bar{P}, V^\phi\right)$ (i.e., ordinal preferences are known but not cardinal ones), the expected payoff of being assigned to s_2 is:

$$E\left(v_{i,s_2}|\bar{P}\right) = (1-\eta)\,\xi + \eta\,(1-\xi)\,.$$

And (η, ξ) are small enough such that $E\left(v_{i,s_2}|\bar{P}\right) < q_{s_1}/\,|I|$. Therefore, obtaining s_2 with certainty is less preferable than obtaining $q_{s_1}/\,|I|$ of s_1. In equilibrium, with a small enough (ε, η, ξ), it must be that:

$$\sigma^*\left(\left(\bar{P}, V^{\phi}\right), \alpha^*, \mathbf{0}\right) = \sigma^*\left(\left(P^{\phi}, V^{\phi}\right), \alpha^*, \mathbf{0}\right) = \bar{P}.$$

Therefore, from i's perspective, any other player, j, plays $\bar{P}$ with probability:

$$(1 - a(\alpha^*)) + a(\alpha^*)(1-\varepsilon) > 1 - \varepsilon.$$

It then suffices to show that student i has incentive to deviate from such equilibrium strategies. Suppose that i has learned her ordinal preferences and $P_i = \bar{P}$. If furthermore she succeeds in acquiring information on V_i, there is a positive probability that $(v_{s_1}, v_{s_2}) = (1, 1-\xi)$. In this case, if she plays L_i s.t., $s_2 L_i s_1 L_i s_3 ... L_i s_{|S|}$ (or other payoff-equivalent strategies), her expected payoff is at least:

$$(1-\xi)\,(1-\varepsilon)^{(|I|-1)}\,,$$

While playing $P_i (= \bar{P})$ leads to an expected payoff less than:

$$(1-\varepsilon)^{(|I|-1)}\left[\frac{q_{s_1}}{|I|} + \left(1 - \frac{q_{s_1}}{|I|}\right)\xi\right] + \left(1 - (1-\varepsilon)^{(|I|-1)}\right).$$

This upper bound is obtained under the assumption that one is always assigned to s_1 when not everyone submits $\bar{P}$. When (ε, ξ) are close to zero, it is strictly profitable to submit L_i instead of $\bar{P}$:

$$\int \Pi\left(\left(\bar{P}, V\right), \alpha^*_{-i}, \mathbf{0}\right) dF\left(V|\bar{P}\right) > \Pi\left(\left(\bar{P}, V^{\phi}\right), \alpha^*_{-i}, \mathbf{0}\right),$$

because in other realizations of V, i cannot do worse than submitting $\bar{P}$. The marginal payoff of increasing $\beta\left(\bar{P}\right)$ from zero by Δ is then:

$$\Delta\left(b'(0)\left[\int \Pi\left(\left(\bar{P}, V\right), \alpha^*_{-i}, \mathbf{0}\right) dF\left(V|\bar{P}\right) - \Pi\left(\left(\bar{P}, V^{\phi}\right), \alpha^*_{-i}, \mathbf{0}\right)\right] - c_\beta\left(\alpha^*, 0\right)\right),$$

which is strictly positive given $c_\beta(\alpha^*, 0) < b'(0) = +\infty$. This proves that under IA $\beta^*(P) > 0$ for some $P \in \mathcal{P}$ given F.

A.3 Proof of Proposition 2.

For the first part, by the definition of strategy-proofness, information on others' types does not change one's best response. Therefore, $\delta^*(V) = 0$ for all V under any strategy-proof mechanism.

To prove the second part, we construct an example of $F(V)$ to show $\delta^*(V) > 0$ for some V

under IA. For notational convenience and in this proof only, we assume the upper bound of utility $\overline{v} = 1$ and the lower bound $\underline{v} = 0$, although we bear in mind that all schools are more preferable than outside option. The distribution of cardinal preferences is:

$$F(V) = \begin{cases} \frac{1}{2} - \varepsilon & \text{if } V = V^{(1)} \text{ s.t. } (v_{s_1}, v_{s_2}) = (1, 0), v_s \in (0, \xi) \ \forall s \notin \{s_1, s_2\}; \\ \frac{1}{2} - \varepsilon & \text{if } V = V^{(2)} \text{ s.t. } (v_{s_1}, v_{s_2}) = (0, 1), v_s \in (0, \xi) \ \forall s \notin \{s_1, s_2\}; \\ \varepsilon & \text{if } V = V^{(3)} \text{ s.t. } (v_{s_1}, v_{s_2}) = (1, 1 - \eta), v_s \in (0, \xi) \ \forall s \notin \{s_1, s_2\}; \end{cases}$$

where (ε, ξ, η) are small positive values. Besides, $F\left(V \in [0,1]^{|S|} \setminus \{V^{(1)}, V^{(2)}, V^{(3)}\}\right) = \varepsilon$.

Suppose that for student i, $V_i = V^{(3)}$. If $\delta^*(V) = 0$ for all V, the best response for i in equilibrium is to top rank either s_1 or s_2.

Given $F(V)$, there is a positive probability, $\left(\frac{1}{2} - \varepsilon\right)^{|I|-1}$, that every other student has $V^{(1)}$ and top ranks s_1. In this case, the payoff for i top-ranking s_1 is less than $q_{s_1}/|I| + \xi$, while top-ranking s_2 leads to $(1 - \eta)$.

There is also a positive probability, $\left(\frac{1}{2} - \varepsilon\right)^{|I|-1}$, that every other student has $V^{(2)}$ and top ranks s_2. In this case, the payoff for i top-ranking s_1 is 1, while the one when top-ranking s_2 is at most $(1 - \eta) q_{s_2}/|I| + \xi$.

Since $\int \Phi\left(V, V_{-i}, \delta^*_{-i}\right) dF(V_{-i}) \geq \Phi\left(V, V^{\phi}_{-i}, \delta^*_{-i}\right)$ and the above shows they are different for some realization of (V_i, V_{-i}), thus,

$$\int \Phi\left(V, V_{-i}, \delta^*_{-i}\right) dF(V_{-i}) - \Phi\left(V, V^{\phi}_{-i}, \delta^*_{-i}\right) > 0.$$

The marginal payoff of acquiring information (increasing $\delta(V_i)$ from zero to Δ) is:

$$\Delta\left(d'(0)\left[\int \Phi\left(V, V_{-i}, \delta^*_{-i}\right) dF(V_{-i}) - \Phi\left(V, V^{\phi}_{-i}, \delta^*_{-i}\right)\right] - e'(0)\right),$$

which is positive for a small (ε, ξ, η) because $e'(0) < d'(0) = \infty$. This proves that $\delta^*(V) > 0$ for some V with a positive measure given F.

A.4 Proof of Proposition 3.

Under UI, the only information i has is that her preferences follow the distribution $F(V)$. Denote W^E_i as the expected (possibly weak) ordinal preferences of i such that $sW^E_i t$ if and only if $\int v_{i,s} dF_{v_s}(v_{i,s}) \geq \int v_{i,t} dF_{v_t}(v_{i,t})$. Given W^E_i, $\left(P^{E,1}_i, ..., P^{E,M}_i\right) \in \mathcal{P}$ are all the strict ordinal preferences that can be generated by randomly breaking ties in W^E_i if there is any. Therefore, $M \geq 1$.

When others play L_{-i}, the expected payoff of i playing L_i is:

$$\int \sum_{s \in S} a_s(L_i, L_{-i}) v_{i,s} dF(V) = \sum_{s \in S} a_s(L_i, L_{-i}) \int v_{i,s} dF_{v_s}(v_{i,s}).$$

Since DA with single tie breaking is essentially the random serial dictatorship, it is therefore a dominant strategy that i submits any $P_i^{E,m}$ $m \in \{1, ..., M\}$. Moreover, a strategy that is not in $\left(P_i^{E,1}, ..., P_i^{E,M}\right)$ can never be played in any equilibrium, because there is a positive-measure set of realizations of the lottery that such a strategy leads to a strictly positive loss.

We claim that in equilibrium for any L_{-i}^* such that $L_j^* \in \left(P_i^{E,1}, ..., P_i^{E,M}\right)$, $j \neq i$, the payoff to i is:

$$\sum_{s\in S} a_s \left(P_i^{E,m}, L_{-i}^*\right) \int v_{i,s} dF_{v_s}\left(v_{i,s}\right) = \sum_{s\in S} \frac{q_s}{|I|} \int v_{i,s} dF_{v_s}\left(v_{i,s}\right), \forall m. \tag{2}$$

Note that for any L_{-i}^*, $\sum_{s\in S} a_s \left(P_i^{E,m}, L_{-i}\right) \int v_{i,s} dF_{v_s}\left(v_{i,s}\right)$ does not vary across m given that any $P_i^{E,m}$ is a dominant strategy.

Since everyone has the same expected utility for being assigned to every school, the maximum utilitarian sum of expected utility is:

$$\sum_{s\in S} q_s \int v_{i,s} dF_{v_s}\left(v_{i,s}\right) \tag{3}$$

If Equation (2) is not satisfied and there exists i such that for some $\hat{L}_{-i}^*$:

$$\sum_{s\in S} a_s \left(P_i^{E,m}, \hat{L}_{-i}^*\right) \int v_{i,s} dF_{v_s}\left(v_{i,s}\right) > \sum_{s\in S} \frac{q_s}{|I|} \int v_{i,s} dF_{v_s}\left(v_{i,s}\right), \forall m. \tag{4}$$

The maximum utilitarian social welfare in (3) implies that there exists $j \in I \backslash \{i\}$ and $m \in \{1, ..., M\}$ such that:

$$\sum_{s\in S} a_s \left(P_j^{E,m}, \hat{L}_{-j}^*\right) \int v_{j,s} dF_{v_s}\left(v_{j,s}\right) < \sum_{s\in S} \frac{q_s}{|I|} \int v_{j,s} dF_{v_s}\left(v_{j,s}\right), \tag{5}$$

where $P_j^{E,m}$ is j's strategy in $\hat{L}_{-i}^*$ and $P_j^{E,m} = P_i^{E,m}$. We can always find such $P_i^{E,m}$ and $P_j^{E,m}$ because condition (4) is satisfied for all m. However, the uniform random lottery implies that:

$$a_s \left(P_i^{E,m}, \left(L_{-(i,j)}^*, P_j^{E,m}\right)\right) = a_s \left(P_j^{E,m}, \left(L_{-(i,j)}^*, P_i^{E,m}\right)\right) \ \forall s \text{ if } P_i^{E,m} = P_j^{E,m},$$

and thus:

$$\sum_{s\in S} a_s \left(P_j^{E,m}, \left(L_{-(i,j)}^*, P_i^{E,m}\right)\right) \int v_{j,s} dF_{v_s}\left(v_{j,s}\right) = \sum_{s\in S} a_s \left(P_i^{E,m}, \left(L_{-(i,j)}^*, P_j^{E,m}\right)\right) \int v_{i,s} dF_{v_s}\left(v_{i,s}\right),$$

which contradicts the inequalities (4) and (5). This proves (2) is always satisfied.

Under OI, CI, or PI, the unique equilibrium is for everyone to report her true ordinal preferences, and thus the expected payoff (*ex ante*) is:

$$\begin{aligned}
&\int\int\sum_{s\in S}a_s\left(P,L_{-i}\left(P\right)\right)v_{i,s}dF\left(V|P\right)dG\left(P|F\right)\\
&=\int\int\sum_{s\in S}\frac{q_s}{|I|}v_{i,s}dF_{v_s}\left(v_{i,s}|P\right)dG\left(P|F\right)\\
&=\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}\right),
\end{aligned}$$

where $L_{-i}\left(P\right)$ is such that $L_j=P, \forall j\in I\backslash\left\{i\right\}$.

A.5 Proof of Proposition 4.

A.5.1 Welfare under UI and OI

We first show UI = OI in symmetric equilibrium in terms of *ex ante* student welfare.

Under UI, the game can be transformed into one similar to that under PI but everyone has the same cardinal preferences that are represented in terms of the expected utilities $\left[\int v_{i,s}dF_{v_s}\left(v_{i,s}\right)\right]_{s\in S}$. In a symmetric equilibrium, everyone thus must play exactly the same strategy, either pure or mixed, which further implies that everyone is assigned to each school with the same probability and has the same *ex ante* welfare:

$$\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}\right).$$

Under OI, everyone knows that everyone has the same ordinal preferences P. The game again can be considered as one under PI where everyone has the same cardinal preferences, $\left[\int v_{i,s}dF_{v_s}\left(v_{i,s}|P\right)\right]_{s\in S}$. Similar to the argument above, the payoff conditional on P is:

$$\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}|P\right),$$

which leads to an *ex ante* payoff:

$$\int\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}|P\right)dG\left(P|F\right)=\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}\right).$$

A.5.2 Proof of CI $\geq$ UI = OI under IA

We then show CI $\geq$ OI = UI.

Under CI, everyone's cardinal preferences V_i are her private information, although her ordinal preferences P, which is common across i, are common knowledge. Suppose that $\sigma^{BN}\left(V\right)$:

$[0,1]^{|S|} \to \Delta(\mathcal{P})$ is a symmetric Bayesian Nash equilibrium. We show that:

$$\begin{aligned}
&\int\int\left(\int A\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_{-i}|P)\cdot V_i\right) dF(V_i|P)\, dG(P|F) \\
&\geq \sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s} dF_{v_s}(v_{i,s}).
\end{aligned}$$

The following uses the same idea as in the proof of Proposition 2 in (Troyan 2012). Note that $\int a_s\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_{-i}|P)$ is i's probability of being assigned to s in equilibrium when the realization of cardinal preferences is V_i. Furthermore, the *ex ante* assignment probability, i.e., the probability before the realization of P and V_i, is

$$\int\int\int a_s\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_{-i}|P)\, dF(V_i|P)\, dG(P|F),$$

which must be the same across students by symmetry. Therefore, we must have:

$$|I|\int\int\int a_s\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_{-i}|P)\, dF(V_i|P)\, dG(P|F) = q_s, \forall s\in S, \tag{6}$$

as in equilibrium all seats at all $s\in S$ must be assigned.

Suppose i plays an alternative strategy σ_i such that $\sigma_i = \int\int \sigma^{BN}(V_i)\, dF(V_i|P)\, dG(P|F) = \int \sigma^{BN}(V_i)\, dF(V_i)$. That is, i plays the "average" strategy of the equilibrium strategy regardless of her preferences. Her payoff given any realization of P is:

$$\begin{aligned}
&\int\left(\int A\left(\sigma_i, \sigma^{BN}(V_{-i})\right) dF(V_{-i}|P)\cdot V_i\right) dF(V_i|P) \\
&= \int\left(\int\left(\int\int A\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_i|P)\, dG(P|F)\right) dF(V_{-i}|P)\cdot V_i\right) dF(V_i|P) \\
&= \int\left(\textstyle\sum_{s\in S}\left(\int\int\int a_s\left(\sigma^{BN}(V_i), \sigma^{BN}(V_{-i})\right) dF(V_i|P)\, dG(P|F)\, dF(V_{-i}|P)\right) v_{i,s}\right) dF(V_i|P) \\
&= \int\left(\sum_{s\in S}\frac{q_s}{|I|} v_{i,s}\right) dF(V_i|P).
\end{aligned}$$

The last equation is due to (6). Since σ_i may not be optimal for i upon observing her preferences

V_i, we thus have for *ex ante* welfare:

$$\begin{aligned}
&\int\int\left(\int A\left(\sigma^{BN}\left(V_i\right),\sigma^{BN}\left(V_{-i}\right)\right)dF\left(V_{-i}|P\right)\cdot V_i\right)dF\left(V_i|P\right)dG\left(P|F\right)\\
&\geq\int\int\left(\int A\left(\sigma_i,\sigma^{BN}\left(V_{-i}\right)\right)dF\left(V_{-i}|P\right)\cdot V_i\right)dF\left(V_i|P\right)dG\left(P|F\right)\\
&=\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}\right),
\end{aligned}$$

which proves CI $\geq$ OI = UI in terms of Pareto dominance of *ex ante* student welfare.

A.5.3 Proof of PI $\geq$ OI = UI under IA

Under PI, everyone's cardinal preferences V_i are common knowledge. Given a symmetric equilibrium, by the same argument as above, we must have PI Pareto dominates OI and UI.

Suppose that $\sigma^{NE}\left(V_i,V_{-i}\right):[0,1]^{|S|\times|I|}\to\Delta\left(\mathcal{P}\right)$ is a symmetric Nash equilibrium. We show that:

$$\begin{aligned}
&\int\int\int\left(A\left(\sigma^{NE}\left(V_i,V_{-i}\right),\left[\sigma^{NE}\left(V_j,V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)\cdot V_i\right)dF\left(V_{-i}|P\right)dF\left(V_i|P\right)dG\left(P|F\right)\\
&\geq\sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s}dF_{v_s}\left(v_{i,s}\right).
\end{aligned}$$

Note that $a_s\left(\sigma^{NE}\left(V_i,V_{-i}\right),\left[\sigma^{NE}\left(V_j,V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)$ is i's probability of being assigned to s in equilibrium when the realization of cardinal preferences is $\left(V_i,V_{-i}\right)$. Furthermore, the *ex ante* assignment probability, i.e., the probability before the realization of P and $\left(V_i,V_{-i}\right)$, is

$$\int\int\int a_s\left(\sigma^{NE}\left(V_i,V_{-i}\right),\left[\sigma^{NE}\left(V_j,V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)dF\left(V_{-i}|P\right)dF\left(V_i|P\right)dG\left(P|F\right),$$

which must be the same across students by symmetry. Therefore, we must have, $\forall s\in S$:

$$|I|\int\int\int a_s\left(\sigma^{NE}\left(V_i,V_{-i}\right),\left[\sigma^{NE}\left(V_j,V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)dF\left(V_{-i}|P\right)dF\left(V_i|P\right)dG\left(P|F\right)=q_s,\tag{7}$$

as in equilibrium all seats at all $s\in S$ must be assigned.

Suppose i plays an alternative strategy σ_i such that

$$\sigma_i=\int\int\int\sigma^{NE}\left(V_i,V_{-i}\right)dF\left(V_{-i}|P\right)dF\left(V_i|P\right)dG\left(P|F\right).$$

That is, i plays the "average" strategy of the equilibrium strategy regardless of her and others' preferences. Her payoff given a realization of (V_i, V_{-i}) is:

$$
\begin{aligned}
& A\left(\sigma_i, \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)\cdot V_i \\
&= \left(\int\int\int A\left(\sigma^{NE}\left(V_i, V_{-i}\right), \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right) dF\left(V_{-i}|P\right) dF\left(V_i|P\right) dG\left(P|F\right)\right)\cdot V_i \\
&= \sum\nolimits_{s\in S}\left(\int\int\int a_s\left(\sigma^{NE}\left(V_i, V_{-i}\right), \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right) dF\left(V_i|P\right) dG\left(P|F\right) dF\left(V_{-i}|P\right)\right) v_{i,s} \\
&= \sum_{s\in S}\frac{q_s}{|I|} v_{i,s}.
\end{aligned}
$$

The last equation is due to (7). Therefore, her payoff given a realization of P is:

$$
\begin{aligned}
& \int\int\left(A\left(\sigma_i, \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)\cdot V_i\right) dF\left(V_{-i}|P\right) dF\left(V_i|P\right) \\
&= \int\left(\sum_{s\in S}\frac{q_s}{|I|} v_{i,s}\right) dF\left(V_i|P\right).
\end{aligned}
$$

Since σ_i may not be optimal for i upon observing her and others' preferences (V_i, V_{-i}), we thus have:

$$
\begin{aligned}
& \int\int\int\left(A\left(\sigma^{NE}\left(V_i, V_{-i}\right), \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)\cdot V_i\right) dF\left(V_{-i}|P\right) dF\left(V_i|P\right) dG\left(P|F\right) \\
&\geq \int\int\int\left(A\left(\sigma_i, \left[\sigma^{NE}\left(V_j, V_{-j}\right)\right]_{j\in I\setminus\{i\}}\right)\cdot V_i\right) dF\left(V_{-i}|P\right) dF\left(V_i|P\right) dG\left(P|F\right) \\
&= \sum_{s\in S}\frac{q_s}{|I|}\int v_{i,s} dF_{v_s}\left(v_{i,s}\right),
\end{aligned}
$$

which thus proves that $PI > OI = UI$ in terms of Pareto dominance.

We use two examples to show part (iii) in Proposition 4: Section A.5.4 shows that PI can dominate CI in symmetric equilibrium while the example in Section A.5.5 shows the opposite.

A.5.4 Example: PI dominates CI in symmetric equilibrium under IA

There are 3 schools (a, b, c) and 3 students whose cardinal preferences are i.i.d. draws from the following distribution:

$$
\begin{aligned}
\Pr\left(\left(v_a, v_b, v_c\right) = (1, 0.1, 0)\right) = 1/2 \\
\Pr\left(\left(v_a, v_b, v_c\right) = (1, 0.5, 0)\right) = 1/2
\end{aligned}
$$

Each school has one seat. For any realization of preference profile, we can find a symmetric Nash equilibrium as in Table A1.

Table A1: Symmetric Nash Equilibrium for Each Realization of the Game under PI

Realization of Preferences	Probability Realized	Strategy given realized type (1, 0.1, 0)	Strategy given realized type (1, 0.5, 0)	Payoff given realized type (1, 0.1, 0)	Payoff given realized type (1, 0.5, 0)
(1, 0.1, 0) (1, 0.1, 0) (1, 0.1, 0)	1/8	(a, b, c)	-	11/30	-
(1, 0.5, 0) (1, 0.1, 0) (1, 0.1, 0)	1/4	(a, b, c)	(b, a, c)	1/2	1/2
(1, 0.5, 0) (1, 0.5, 0) (1, 0.1, 0)	1/4	(a, b, c)	(a, b, c)	11/30	1/2
(1, 0.5, 0) (1, 0.5, 0) (1, 0.5, 0)	1/8	-	(a, b, c)	-	1/2

The above symmetric equilibrium leads to an *ex ante* student welfare:

$$\frac{1}{2}\left(\frac{1}{4}\frac{11}{30}+\frac{1}{2}\frac{1}{2}+\frac{1}{4}\frac{11}{30}\right)+\frac{1}{2}\left(\frac{1}{4}\frac{1}{2}+\frac{1}{2}\frac{1}{2}+\frac{1}{4}\frac{1}{2}\right)=\frac{14}{30}.$$

When everyone's preference is private information, we can verify that the unique symmetric Bayesian Nash equilibrium is:

$$\sigma^{BN}\left((1,0.1,0)\right)=\sigma^{BN}\left((1,0.5,0)\right)=\left(a,b,c\right).$$

That is, everyone submits her true preference ranking. This leads to an *ex ante* welfare of:

$$\frac{1}{2}\frac{11}{30}+\frac{1}{2}\frac{15}{30}=\frac{13}{30}$$

which is lower than the above symmetric equilibrium under PI.

Also note that always playing (a,b,c) is also a symmetric Nash equilibrium under PI in all realizations of preference profile, which leads to the same ex ante student welfare as σ^{BN}.

A.5.5 Example: PI is dominated by CI in symmetric equilibrium under IA

There are 3 schools (a,b,c) and 3 students whose cardinal preferences are i.i.d. draws from the following distribution:

$$\begin{array}{c}\Pr\left((v_a,v_b,v_c)=(1,0.1,0)\right)=3/4\\ \Pr\left((v_a,v_b,v_c)=(1,0.9,0)\right)=1/4\end{array}.$$

Each school has one seat. For any realization of preference profile, we can find a symmetric Nash equilibrium as in Table A2. The *ex ante* welfare under PI with the above symmetric equilibrium profile is:

$$\frac{3}{4}\left(\frac{9}{16}\frac{11}{30}+\frac{6}{16}\frac{1}{2}+\frac{1}{16}\frac{3073}{3610}\right)+\frac{1}{4}\left(\frac{1}{16}\frac{19}{30}+\frac{6}{16}\frac{99}{190}+\frac{9}{16}\frac{9}{10}\right)=\frac{22\,549}{43\,320}\approx 0.52052.$$

Table A2: Symmetric Nash Equilibrium for Each Realization of the Game under PI

Realization of Preference	Probability Realized	Strategy given realized type (1, 0.1, 0)	Strategy given realized type (1, 0.9, 0)	Payoff given realized type (1, 0.1, 0)	Payoff given realized type (1, 0.9, 0)
(1, 0.1, 0) (1, 0.1, 0) (1, 0.1, 0)	27/64	(a, b, c)	-	11/30	-
(1, 0.9, 0) (1, 0.1, 0) (1, 0.1, 0)	27/64	(a, b, c)	(b, a, c)	1/2	9/10
(1, 0.9, 0) (1, 0.9, 0) (1, 0.1, 0)	9/64	(a, b, c)	(a, b, c) w/ prob 3/19 (b, a, c) w/ prob 16/19	3073/3610	99/190
(1, 0.9, 0) (1, 0.9, 0) (1, 0.9, 0)	1/64	-	(a, b, c) w/ prob 11/19 (b, a, c) w/ prob 8/19	-	19/30

Under CI, i.e., when one's own preferences are private information and the distribution of preferences is common knowledge, there is a symmetric Bayesian Nash equilibrium:

$$\sigma^{BN}\left((1,0.9,0)\right)=(b,a,c)\,;\sigma^{BN}\left((1,0.1,0)\right)=(a,b,c)\,.$$

For a type-$(1,0.1,0)$ student, it is a dominant strategy to play (a,b,c). Conditional on her type, her equilibrium payoff is:

$$\frac{9}{16}\left(\frac{1}{3}\left(1+\frac{1}{10}+0\right)\right)+\frac{6}{16}\frac{1}{2}+\frac{1}{16}=\frac{219}{480}.$$

For a type-$(1,0.9,0)$ student, given others follow σ^{BN}, playing (b,a,c) results in a payoff of:

$$\frac{9}{16}\frac{9}{10}+\frac{6}{16}\left(\frac{1}{2}\left(\frac{9}{10}+0\right)\right)+\frac{1}{16}\left(\frac{1}{3}\left(\frac{9}{10}+1+0\right)\right)=\frac{343}{480}.$$

If a type-$(1, 0.9, 0)$ student deviates to (a, b, c), she obtains:

$$\frac{9}{16}\left(\frac{1}{3}\left(\frac{9}{10}+1+0\right)\right)+\frac{6}{16}\left(\frac{1}{2}(1+0)\right)+\frac{1}{16}(1)=\frac{291}{480}.$$

It is therefore not a profitable deviation. Furthermore, she has no incentive to deviate to other rankings such as (c, a, b) or (c, b, a).

The *ex ante* payoff to every student in this equilibrium under CI is:

$$\frac{219}{480}\frac{3}{4}+\frac{343}{480}\frac{1}{4}=\frac{25}{48}\approx 0.52083,$$

which is higher than that under PI.

In this example, the reason that PI leads to lower welfare is because it sometimes leads to type-$(1, 0.9, 0)$ students to play mixed strategies in equilibrium. Therefore, sometimes school B is assigned to a type-(1,0.1,0) student, which never happens under CI in symmetric Bayesian Nash equilibrium.