Public Choice **97:** 617–643, 1998.
© 1998 Kluwer Academic Publishers. Printed in the Netherlands.

Veto games: Spatial committees under unanimity rule *

YAN CHEN[1] & PETER C. ORDESHOOK[2]
[1]*Department of Economics, The University of Michigan, Ann Arbor, MI 48109-1220, U.S.A.;*
[2]*Division of Humanities and Social Sciences, 228-77, California Institute of Technology, Pasadena, CA 91125, U.S.A.*

Accepted 13 August 1996

Abstract. There exists a large literature on two-person bargaining games and distribution games (or divide-the-dollar games) under simple majority rule, where in equilibrium a minimal winning coalition takes full advantage over everyone else. Here we extend the study to an n-person veto game where players take turns proposing policies in an n-dimensional policy space and everybody has a veto over changes in the status quo. Briefly, we find a Nash equilibrium where the initial proposer offers a policy in the intersection of the Pareto optimal set and the Pareto superior set that gives everyone their continuation values, and punishments are never implemented. Comparing the equilibrium outcomes under two different agendas – sequential recognition and random recognition – we find that there are advantages generated by the order of proposal under the sequential recognition rule. We also provide some conditions under which the players will prefer to rotate proposals rather than allow any specific policy to prevail indefinitely.

1. Introduction

Despite the apparent ascendancy of democratic ideas and ideals, there are serious impediments to their universal acceptance and application. Foremost among them, especially in ethnically divided societies, is the matter of majority tyranny in majoritarian institutions. When the definition and description of minorities has deep historical roots, when a majority is easily identifiable, and when members of all groups are conscious of their status (e.g., Slovaks in Czechoslovakia, Tatars in Russia, Russians in most of the successor states of the former Soviet empire, Hungarians in Romania, and Turks in Bulgaria), then minorities are unlikely to acquiesce to the implementation of any structure that allows majorities to dictate policy. Indeed, as Calhoun (1953) argued, “the numerical majority, perhaps, should usually be one of the elements of a constitutional democracy; but to make it the sole element $\cdots$ is one of the greatest and most fatal of political errors”.

* The authors wish to thank Elizabeth Gerber, Richard McKelvey and Scott Page for their comments on earlier drafts of the paper. This research was partially funded by a grant from the United States Institute of Peace to the California Institute of Technology.

A number of devices have been proposed to treat the problems associated with simple majoritarinism, including federalism and bicameralism. Such arrangements seek to protect minorities by raising the vote quota necessary to alter the status quo (cf., Riker, 1982; Hammond and Miller, 1987). Generally, though, these devices refrain from taking matters to their natural limit – unanimity rule – in which every individual or identifiable group possesses a veto over change. Although minorities may demand a veto before agreeing to any constitutionally defined union, others fear that "misuse" of the veto will render the state incapable of formulating useful policy. The implementation of democratic principles, then, appears to entail a choice between the tyranny and deadlock (Buchanan and Tullock, 1962).

To evaluate this concern, though, requires that we take cognizance of the fact that constitutional matters rarely if ever focus on static situations. Political processes are ongoing so that agreements reached today can sometimes be enforced by punishments applied tomorrow. A constitutional issue such as minority rights is rarely "decided in perpetuity" – even if not explicitly debated, those rights must be implicitly and continuously maintained. Similarly, although a veto may yield deadlock in one period, unanimity rule may be little more than a device for upgrading the strategic capabilities of minorities so that they are better equipped to protect their rights over the long term. Thus, unanimity rule merely sets the stage for bargaining among groups, where the consequences of bargaining is a continual stream of outcomes that may or may not be Pareto efficient and that may or may not satisfy various criteria of fairness and equity.

Existing models of bargaining establish, in fact, that a veto need not imply deadlock or inefficiency. For example, Rubinstein (1982) and Binmore and Herrero's (1988) analyses of 2-person bargaining, which model unanimity rule to the extent that mutually disadvantageous outcomes are averted only if both persons reach agreement, reveal that mutually beneficial outcomes do correspond to subgame perfect equilibria. Unfortunately, these models and their extension to a more explicitly political realm (cf., Baron and Ferejohn, 1982; McKelvey and Riezman, 1990) are not sufficiently general for our purpose. They assume, first, that the decision confronting people is the division of some fixed pie. Although redistributive matters are important, the usual context for constitutional failure – ethnic conflict – entails issues over which sidepayments are difficult or impossible to implement directly. Second, they suppose that the "disagreement point" – the outcome that prevails if no unanimous agreement is reached – is mutually destructive rather than a status quo that one side of the dispute or the other finds "unsatisfactory".

A number of questions about unanimity rule's operation, then, remain unanswered. First, is a dynamic conceptualization of political processes sufficient to avert the most common objections to unanimity rule's implementation? Second, what types of policies will result if changes in the welfare of one group, owing to the structure of the issues being debated, necessarily exhibit both positive and negative externalities for other groups in society? And third, what disadvantages accrue to a minority that cannot control the agenda whereby alternatives to the status quo are considered?

This essay presents a framework for addressing these questions by offering a model of an n-person committee (legislature) in which there is a proposer empowered to offer an alternative to the status quo and a status quo that can be changed only by unanimous consent, but that departs from earlier bargaining models in four ways. First, rather than assume that alternatives correspond to divisions of a fixed pie, we assume that the committee is concerned with policies in some Euclidean policy space and that preferences in this space are modeled by Euclidean distance. Second, we assume that the disagreement point is a status quo outcome that need not be "bad" from everyone's perspective. Third, although, as in the Rubinstein et al. framework, we assume that proposals are made and voted on sequentially in an infinite sequence, we consider two rules whereby people are empowered to make proposals: a sequential rule and a random recognition rule. Finally, we consider the possibility that the committee might choose to "rotate" proposals rather than establish some specific outcome in perpetuity.

Generally, our conclusions match the intuition that existing models of bargaining might generate about veto games. We find that unanimity rule need not afford an overwhelming advantage to whoever controls the agenda whereby alternatives to the status quo are considered. On the other hand, once a policy is Pareto optimal in a dynamic sense, then, regardless of its perceived fairness, it can be sustained as the status quo in perpetuity. We also characterize the necessary and sufficient conditions for dynamically Pareto optimal policy paths, and show when a rotation scheme is advantageous implementable policy.

2. The general framework

We begin with some essential notation. First, we let $N = \{1, 2, \cdots, n\}$ be the set of *players* or voters, and $X \subseteq R^n$ be the compact set of alternative *policies*. Next, we assume that each voter $i \in N$ has a von Neumann-Morgenstern *utility function* $u_i : X \cup \{\phi\} \to R$, where $u_i(x) = u_i(|x - a_i|)$ represents the utility $i \in N$ receives when the policy position is $x \in X$, and ϕ is a *null outcome* with $u_i(\phi) = 0$. Thus, i's utility decreases with the distance between the policy position and his ideal point, a_i, and achieves its maximum at a_i. Also, we assume that $u_i(\cdot)$ is quasi-concave.

There are now three subsets of X that warrant special attention, $X_{PI}(x_0)$, $X_{PS}(x_0)$ and X_{PO}. Denoting the *status quo policy outcome* by $x_0 \in X$, these three sets are defined thus:

$$X_{PI}(x_0) = \{x \in X : u_i(x) < u_i(x_0), \forall i \in N\}$$

is the *Pareto inferior set*;

$$X_{PS}(x_0) = \{x \in X : u_i(x) \geq u_i(x_0), \forall i \in N\}$$

is the *Pareto superior set*. Note that $X_{PI}(x_0)$ and $X_{PS}(x_0)$ are shortened as X_{PI} and X_{PS} respectively hereafter. The *Pareto optimal set* is defined in the usual way as

$$\begin{aligned} X_{PO} = \ & \{x \in X : \nexists y \in X \text{ s.t. } u_i(y) \geq u_i(x), \forall i \in N \text{ and } u_i(y) > u_i(x) \\ & \text{for at least one } i\}. \end{aligned}$$

The preceding three sets are static concepts. Since we want to study the infinite horizon veto game, we need their corresponding dynamic formulation. Hence, we define a *path*, $\theta(x^0, x^1, \cdots, x^t, \cdots)$, as a sequence of policy positions, starting from period zero to period infinity. We let $\delta_i \in [0, 1]$ denote the discount factor i uses to discount future streams of utility. Next, let θ and θ' denote $\theta(x^0, x^1, \cdots, x^t, \cdots)$ and $\theta(y^0, y^1, \cdots, y^t, \cdots)$ respectively, and we let $U_i(\theta) \equiv \sum_{t=0}^{\infty} \delta_i^t u_i(x^t)$ and $U_i(\theta') \equiv \sum_{t=0}^{\infty} \delta_i^t u_i(y^t)$ denote the infinite stream of utility player i gets from the paths θ and θ', respectively. If $U_i(\theta) > U_i(\theta')$ for all i, then θ is *dynamically Pareto superior* to θ'; If $U_i(\theta) < U_i(\theta')$ for all i, then θ is *dynamically Pareto inferior* to θ'; Then the *dynamic Pareto optimal set* is simply $X_{DPO} = \{\theta \in X : \nexists \theta' \in X \text{ s.t. } U_i(\theta') \geq U_i(\theta), \forall i \in N$ and $U_i(\theta') > U_i(\theta)$ for at least one $i\}$.

Turning now to the bargaining game, if we take the example of n = 3, then under a sequential recognition rule, the three voters have a predetermined order for making proposals – first voter 1, then 2, then 3, then 1 again, and so on – where x_i denotes player i's proposal. Thus, if voter 1 proposes x_1 and if neither 2 nor 3 veto, then x_1 becomes the new status quo, at which point player 2 has the opportunity to offer a new proposal. But, if either 2 or 3 veto, x_0 remains the status quo and 2 has the next move. In contrast, with a random recognition rule, nature chooses the voter who will make a proposal at every stage.

To analyse these two situations formally, we make use of the following additional notation. First, we let T be the set of *states* that can be achieved in a game – the nodes in the game's extensive form. Next, we define a *stochastic game*, $\Gamma^t = (S^t, \pi^t, \psi^t)$, where S^t is the set of *pure strategy n tuples* (we do not consider mixed strategies), where $\pi^t : S^t \rightarrow \mu(T)$ is a *transition function* specifying for each $s^t \in S^t$ a probability distribution $\pi^t(s^t)$ on T, and where $\psi^t : S^t \to X$ is an *outcome function* that specifies for each $s^t \in S^t$ an outcome $\psi^t(s^t) \in X$. Finally, we use $S = \Pi_{t\in T} S^t$ to denote the collection of pure strategy n tuples, where $S^t = \Pi_{i\in N} s_i^t$.

3. Results

In the next three subsections, we first characterize the necessary and sufficient conditions for dynamically Pareto optimal paths. We then study stationary Nash equilibria under the two recognition rules – sequential and random. Finally, we discuss the implementation of a rotation scheme as an application for unanimity rule.

3.1. *Dynamic Pareto optimal paths*

We begin with some general lemmas that help us subsequently characterize the properties of equilibria. First,

Lemma 1: For all $x^t \in \theta \in X_{DPO},\ x^t \in X_{PO}$.

Proof: (By contradiction.) Suppose $\theta(x^1, \cdots, x^s, \cdots, x^t, \cdots) \in X_{DPO}$, and $x^s \notin X_{PO},\ x^t \in X_{PO},\ \forall t \neq s$. Then, by the definition of X_{PO}, for any $x \in X_{PO}$, we have $u_i(x) \geq u_i(x^s)$, for all i and, $u_i(x) > u_i(x^s)$, for at least one i. Denote $\theta'(x^1, \cdots, x^{s-1}, x, x^{s+1}, \cdots, x^t, \cdots) \equiv \theta'$, then $U_i(\theta') \geq U_i(\theta)$, for all i. Therefore, $\theta \notin X_{DPO}$, which is a contradiction. Similarly, for any path with more than one point outside X_{PO}, by replacing them by points inside X_{PO}, we get a Pareto superior path. By induction, we can show that for all $x^t \in \theta \in X_{DPO},\ x^t \in X_{PO}$. Q.E.D.

Lemma 1 says that any dynamically Pareto optimal path consists only of points from the stationary Pareto optimal set. Intuitively, if a path has one point outside the stationary Pareto set, we can always substitute a point inside that set for it and make every player better off for that period and thus better off for the infinite sequence.

However, the converse of Lemma 1 is not true: Paths that consist of points inside the stationary Pareto optimal set are not necessarily dynamically Pareto optimal paths. For example, suppose in a two dimensional policy space that all players have quadratic utility functions of the form, $u_i(z) = -[(x_1 - a_i)^2 + (y_1 - b_i)^2]$, where $z = (x_1, y_1)$. Let the players' ideal points be $z_1 = (a_1, b_1) = (-2, 0), z_2 = (a_2, b_2) = (2, 0), z_3 = (a_3, b_3) = (0, 3.46)$. Let $\delta_1 = \delta_2 = .5, \delta_3 = .7$. We consider two dynamic paths. The first one, $\theta =$

$(z_1, z_2, z_1, z_2, \cdots,)$, is a rotation of player 1 and 2's ideal point. The second one, $\theta' = (z, z, z, \cdots)$, where $z = (-0.67, 1.88)$, is a constant path inside the static Pareto set. It is easy to verify that $U_1(\theta) = U_1(\theta'), = U_2(\theta) = U_2(\theta')$, but $U_3(\theta) < U_1(\theta')$. So although θ consists only of points inside the static Pareto set, it is dominated by θ', and is therefore not dynamically Pareto optimal.

To see then what types of paths are dynamically Pareto optimal, let a *constant path* be a path that consists of the same point for every period. We use $\theta(x)$ to denote constant path $\theta(x, x, \cdots, x, \cdots,)$, and we say that θ is *equivalent* to another path θ' for player i, if $U_i(\theta) = U_i(\theta')$.

Lemma 2: Any path that consists of only Pareto optimal points, $\theta(x^0, x^1, \cdots, x^t, \cdots)$, where $x^t \in X_{PO}$ is equivalent to a constant path $\theta_i(x_i)$ for each player i, where $i \in N$ and $x_i \in X_{PO}$.

Proof: First we want to show how a path, $\theta(x^1, x^2, \cdots, x^t, \cdots)$, where $x^t \in X_{PO}$, can be broken down to an equivalent constant path.

$$\frac{1}{1-\delta_i} u_i(x_i) = \sum_{t=0}^{\infty} \delta_i^t u_i(x^t),$$

therefore,

$$\begin{aligned} u_i(x_i) &= \frac{1}{\sum_{t=0}^{\infty} \delta_i^t} [u_i(x^0) + \delta_i u_i(x^1) + \delta_i^2 u_i(x^2) + \cdots] \\ &= \sum_{t=0}^{\infty} \frac{\delta_i^t}{\sum_{t=0}^{\infty} \delta_i^t} u_i(x^t) \\ &\equiv \sum_{t=0}^{\infty} \alpha_{it} u_i(x^t), \end{aligned}$$

where $\sum_{t=0}^{\infty} \alpha_{it} = 1$, and $\alpha_{it} \in (0, 1)$. This means that $u_i(x_i) \in co\{u_i(x^t)\}$, i.e., $u_i(x_i)$ is in the convex hull of $u_i(x^t)$'s. Next, we want to show that $x_i \in X_{PO}$. Consider the two extreme cases for player i: since $u_i(x_i) = \sum_{t=0}^{\infty} \alpha_{it} u_i(x^t)$, $\forall i \in N$, and $x^t \in X_{PO}$, $\forall t$, so for any player i, the best case occurs whenever $x_i = a_i$; the worst case occurs whenever $x_i = a_j$, where $a_j = \mathrm{argmax}_{\{a_{-i}\}} |a_{-i} - a_i|$. Therefore, on the line segment between a_i and a_j, there is at least one point that is the solution to the problem, which we denote by x_i. Since X_{PO} is convex, $x_i \in X_{PO}$. Q.E.D.

Note that generally there is more than one solution to the problem of finding a constant path that is equivalent to the non-constant path θ. We call the set of all constant optimal paths that are equivalent to θ and that consist of all Pareto optimal points the *trajectory of θ for i*, denoted by $TR_i(\theta) =$

$\{x \in X_{PO} : u_i(x) = (1-\delta_i)U_i(\theta)\}$. Note that $TR_i(\theta)$ is just a segment of i's indifference curve inside the stationary Pareto set. From the quasiconcavity of $u_i(\cdot)$, it follows that $TR_i(\cdot)$ is also quasiconcave. A closely related concept of the trajectory for player i is player i's *better-than set* of θ, $B_i(\theta) = \{x \in X_{PO} : u_i(x) \geq (1-\delta_i)U_i(\theta)\}$. The better-than set is the set of Pareto optimal points that are Pareto superior to the constant equivalent path of θ. The close relationship of $TR_i(\cdot)$ and $B_i(\cdot)$ can be seen from Observation 1.

Observation 1:

$$| \cap_{i\in N} TR_i(\theta)| = 1 \text{ iff } | \cap_{i\in N} B_i(\theta)| = 1.$$

Proof: We first prove by contradiction that $| \cap_{i\in N} TR_i(\theta)| = 1 \Rightarrow | \cap_{i\in N} B_i(\theta)| = 1$. There are two possible cases, (1) $\cap_{i\in N}B_i(\theta) = \phi$, (2) $\cap_{i\in N}B_i(\theta) = B$, where $|B| > 1$. Suppose (1) holds. Since $TR_i(\theta) \subset B_i(\theta)$, it follows that $\cap_{i\in N}B_i(\theta) = \phi \Rightarrow \cap_{i\in N}TR_i(\theta) = \phi$, which is a contradiction. Suppose (2) holds, then B is a closed convex set with nonempty interior. Let ∂B and B^0 denote the boundary and interior of B respectively. Since $| \cap_{i\in N} TR_i(\theta)| = 1$, let $\cap_{i\in N}TR_i(\theta) = \{x\}$. It follows that $x \in \partial B$, $\forall y \in B^0$, $u_i(y) > (1-\delta_i)U_i(\theta)$ for all i. Given that $x = \cap_{i\in N}TR_i(\theta)$, $u_i(x) = (1-\delta_i)U_i(\theta)$ for all i. Therefore, $u_i(x) < u_i(y)$ for all i, which contradicts $x \in X_{PO}$.

We now prove that $|\cap_{i\in N}TR_i(\theta)| = 1 \Leftarrow |\cap_{i\in N}B_i(\theta)| = 1$. Let $\cap_{i\in N}TR_i(\theta) = A$. Let $B = \cap_{i\in N}B_i(\theta)$. By definition, $A \subset B$. It suffices to show that $|B| = 1 \Rightarrow B \subset A$. Let $B = \{x\}$. It is straight forward to show that $u_i(x) = (1-\delta_i)U_i(\theta)$ for all i. Therefore, $x \in A$, which completes the proof.

Q.E.D.

We now want to characterize the sufficient conditions for a path to be dynamically Pareto optimal.

Proposition 1: (1) $\forall\theta$ s.t. $| \cap_{i\in N} B_i(\theta)| > 1, \theta \notin X_{DPO}$.
(2) $\forall\theta$ s.t. $| \cap_{i\in N} B_i(\theta)| \leq 1, \theta \in X_{DPO}$ if $\nexists\theta'$ s.t. (i) and (ii) hold, where
(i) $TR_i(\theta') \subseteq B_i(\theta), \forall_i$;
(ii) $TR_i(\theta') \nsubseteq TR_i(\theta)$ for at least one i.

Proof: Part (1) is obtained by taking any $x \in B$, then the constant path $\theta'(x, x, x, \cdots)$ has the property that $U_i(\theta') \geq U_i(\theta)$, for all i. Then θ' Pareto dominates θ. So $\theta \notin X_{DPO}$.

Part (2) follows from the definition of dynamic Pareto optimality. The condition is equivalent to saying that there does not exist an x', such that $u_i(x') = (1-\delta_i)U_i(\theta') \geq (1-\delta_i)U_i(\theta)$ for all i, and $u_i(x') = (1-\delta_i)U_i(\theta') > (1-\delta_i)U_i(\theta)$ for at least one i. Q.E.D.

Proposition 1 characterizes conditions for whether a path is dynamically Pareto optimal or not. Given a path, we first compute the trajectories for each player. If the intersection of their better-than set is neither empty nor a singleton, then it is not dynamically Pareto optimal, i.e., in X_{DPO}. When it is empty or consists of a singleton and there does not exist a path whose trajectories lie inside its better-than set, it is dynamically Pareto optimal. In the case of constant paths, paths that consist of a single point for all periods, the intersection of the trajectories is a singleton, hence Corollary 1.

Corollary 1: A constant path $\theta(x) \in X_{DPO}$, if $\nexists\theta'$ such that $TR_i(\theta') \subseteq B_i(\theta),\ \forall i$ and $TR_i(\theta') \subset B_i(\theta)$ for at least one i.

Proof: For a constant path, $\theta(x), \cap_{i\in N}TR_i(\theta) = \{x\}$, hence, from Observation 1, $\cap_{i\in N}B_i(\theta) = \{x\}$. Then we apply Part (2) of Proposition 1. Q.E.D.

An extreme example of a constant path that is dynamically Pareto optimal is $\theta(a_i)$, where a_i is player i's ideal point.

3.2. *Stationary Nash equilibrium and two recognition rules*

In this section, we study the simplest Nash equilibria in this infinite horizon veto game – stationary Nash equilibria. For simplicity, we relegate to Appendix A the formal description of the stochastic veto game used to model unanimity rule and the formal characterization of equilibrium strategies for this class of games. Omitting the superscript t and concentrating on the position of the constant path, which is subscripted by the proposer, we need to define here only the indicator function $g(\cdot)$ in order to denote the results of a player's actions

$$g_j(x_i) = \begin{cases} 1, \text{ if player j accepts } x_i \\ 0, \text{ if player j vetos } x_i, \end{cases}$$

$$\text{if } g(x_i) = \prod_j g_j(x_i) = \begin{cases} 1, \text{ then } x_i \text{ passes} \\ 0, \text{ then } x_i \text{ fails.} \end{cases}$$

The best-responses for proposers and voters are,
Proposer i:

$$s_i = x_i,$$

$$x_i \in \ \text{argmax}_{x\in X}\{g(x_i)[u_i(x_i) + \delta_i v_i(x_i)] + (1 - g(x_i))[u_i(x_0) + \delta_i v_i(x_0)]\};$$

Voter j:

$$s_j = g_j(x_i) \in \{0, 1\},$$

$$g_j(x_i) \in \text{argmax}_{\{0,1\}}\{g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)]\},$$

where $v_j(\cdot)$ denotes player j's continuation value.

In the stationary case, it follows from Lemma 1 that the dynamic Pareto optimal set coincides with the stationary Pareto set. Now consider two cases: the first is that the status quo is inside the Pareto optimal set, and the second case is that the status quo is outside the Pareto set. In the first case, when only stationary strategies are considered, the constant path $\theta(x_0)$ can be supported as a Nash equilibrium. Since any defection from the status quo to another constant path makes at least one player worse off, that player will veto the proposal. Formally,

Observation 2: The following is a Nash equilibrium to the veto game:

$$\begin{aligned} x_i &= x_0, \ \forall i \in N, x_0 \in X_{PO}; \\ g_j(x_i) &= \begin{cases} 1, \text{if } U_j(\theta(x_i)) \geq U_j(\theta(x_0)) \\ 0, \ \text{otherwise.} \end{cases} \end{aligned}$$

Proof: If Proposer i proposes $\theta'(x_i)$, where $x_i \neq x_0$, then $U_k(\theta) > U_k(\theta')$ for at least one player, k, who will in turn veto the proposal, and the status quo prevails. So the Proposer is not better off defecting. For voter j, if $U_j(\theta(x_i)) \geq U_j(\theta(x_0))$ and $g_j(x_i) = 0$, x_0 prevails and he is worse off. If $U_j(\theta(x_i)) < U_j(\theta(x_0))$ and $g_j(x_i) = 1$, he is not better off whether $g(x_i) = 1$ or 0. Therefore he is not better off defecting. Q.E.D.

This observation says that when the status quo is inside the Pareto set, it can be supported as a Nash equilibrium for it to remain there infinitely. Obviously, this is not a very interesting situation. (Note this is no longer true when nonstationary strategies are admitted later in Section 3.3.) Now consider the second case, where the status quo is outside the Pareto set, and consider the two recognition rules – sequential and random.

We begin by offering an additional lemma (see Appendix B for proof), that is useful in analysing the game under both settings.

Lemma 3: There exists a Nash equilibrium strategy to the veto game that satisfies the following properties:

$$x_i \in X_{SO}, \ \forall i \in N;$$

$$g_j(x_i) = 0, \ \text{if } x_i \notin X_{SO}, \ \text{where } X_{SO} = X_{PO} \cap X_{PS}.$$

This lemma identifies a Nash equilibrium strategy in which the proposer offers a policy in the intersection of the Pareto optimal and Pareto superior sets, and the voters veto any other policy. The intuition behind this lemma and its characterization of the proposer's offer is two-fold. First, it should be evident that the new policy must be an element of the Pareto superior set, otherwise whoever prefers the status quo to the new policy will veto the proposal. Second, given the voters' continuation values, by moving in the direction of the Pareto optimal set, the proposer can make some voters better off without making the others worse off. Therefore, he can move to a higher indifference curve by proposing a proposition inside the Pareto optimal set while simultaneously giving each voter their continuation values. So the proposer is never worse off by offering a policy in the intersection of the Pareto superior set and the Pareto optimal set.

I. Sequential recognition rule

Under a sequential recognition rule, the proposer offers in equilibrium a policy closest to his ideal point that gives every other voter their continuation values; voters accept any proposal that gives them their continuation values and veto any proposal that gives them utility less than these values.

Proposition 2: The following is a stationary Nash equilibrium to the veto game under the sequential proposing rule:

For Proposer i:

$$x_i \in \operatorname{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \sum_{l=0}^{k-1} \delta_j^1 u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \forall j \neq i,$$

$$\text{where } k = \begin{cases} j-i, & \text{if } j > i \\ n-i+j, & \text{if } j < i. \end{cases}$$

For voter j:

$$g_j(x_i) = \begin{cases} 1, \text{if } \frac{u_j(x_i)}{1-\delta_j} \geq v_j^*(x_i) \\ 0, \text{ otherwise,} \end{cases}$$

where

$$v_j^*(x_i) = \sum\nolimits_{l=0}^{k-1} \delta_j^l u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j).$$

Proof: See Appendix C.

This proposition gives equilibrium strategies for both the initial proposer and all other voters. We use x_j to denote voter j's optimal proposal when he becomes the proposer. Notice in particular that, like the results of most bargaining models, in equilibrium, the first proposition is accepted and lies in the Pareto optimal set. Thus, punishments, i.e., vetos, are never implemented. That every player is a blocking coalition, however, leads to a more “equitable” outcome. That is, the kind of equilibria that we often find under majority rule, in which a minimal winning coalition divides the pie among its members and leaves all others with nothing, cannot occur here.

II. Random recognition rule

Under the random recognition rule, we get a similar result as under the sequential recognition rule – a stationary Nash equilibrium where the proposer proposes a policy position closest to his own ideal point that still gives every voter their continuation values. Voters accept any proposal that gives them their continuation values, and veto any position that gives them less. More formally, if we let $v_{ii} \equiv \frac{u_i(x_i)}{1-\delta_i}$, then

Proposition 3: The following is a stationary Nash equilibrium under a random recognition rule:

For Proposer i:

$$x_i \in \operatorname{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \frac{nu_j(x_0) + \delta_j v_{jj}}{n-(n-1)\delta_j} \equiv v_j^*, \forall j \neq i.$$

For Voter j:

$$g_j(x_i) = \begin{cases} 1, \text{ if } \frac{u_j(x_i)}{1-\delta_j} \geq v_j^* \\ 0, \text{ otherwise} \end{cases}$$

Proof: See Appendix C.

Unlike outcomes under sequential recognition rule, then, and as expected, all voters are treated equally here, without the bias from the order to propose. In this sense, then, random recognition is more “equitable” than sequential recognition.

3.3. *Rotation schemes*

To this point we have considered only strategies in which each person’s continuation value is calculated under the assumption that any Pareto optimal policy remains in effect forever. Consider, though, the possibility that the voters agree beforehand to rotate policies among themselves – choosing first a policy that is “good” for voter 1, then one that is “good” for voter 2, and so on. The question is whether, with Euclidean preferences, such a scheme has any advantages for all voters over a fixed policy and whether it can be implemented under unanimity rule.

To address this question, we begin by representing a rotation scheme in a n-person veto game as a dynamic path, $\theta(x^1, x^2, \cdots, x^n, x^1, x^2, \cdots, x^n, \cdots)$. The trajectory of the path for player i is $TR_i(\theta) = \{x \in X_{PO} : u_i(x) = (1-\delta_i)U_i(\theta)\}$, which can be simplified to $TR_i(\theta) = \{x \in X_{PO} : u_i(x) = \sum_{j=1}^{n} \alpha_j u_i(x^j)\}$, where $\alpha_j = \frac{\delta_i^{j-1}}{1+\delta_i+\cdots+\delta_i^{n-1}}$. From Proposition 1, we know that a rotation path θ is dynamically Pareto optimal if the intersection of all trajectories of θ is either empty or consisted only of a singleton, and there is no other path whose trajectories lies inside the better-than set of θ. Thus,

Corollary 2: A rotation scheme $\theta(x^1, x^2, \cdots, x^n, x^1, x^2, \cdots, x^n, \cdots) \subset X_{DPO}$ if$| \cap_{i\in N} B_i(\theta)| \leq 1$, and $\nexists\theta'$ such that $TR_i(\theta') \subseteq B_i(\theta)\forall i$, and $TR_i(\theta') \nsubseteq TR_i(\theta)$ for at least one i, where $\alpha_j = \frac{\delta_i^{j-1}}{1+\delta_i+\cdots+\delta_i^{n-1}}$.

After we expand the possible paths to include the rotation schemes, a constant path within the Pareto set is not necessarily dynamically Pareto efficient. We compare the constant paths with rotation paths that also only consist of points inside the Pareto optimal set and see when a rotation path is Pareto dominated by a constant path, and when a constant path is Pareto inferior to a rotation path. From Proposition 1, we obtain the following corollary:

Corollary 3: A rotation scheme $\theta(x^1, x^2, \cdots, x^n, x^1, x^2, \cdots, x^n, \cdots)$ is Pareto superior to any constant path if $\cap_{i\in N} B_i(\theta) = \phi$; it is equivalent to the constant path $\theta(x)$ if $\cap_{i\in N} B_i(\theta) = \{x\}$; and it is Pareto inferior to a constant path if $| \cap_{i\in N} B_i(\theta)| > 1$.

This corollary says that if the intersection of the better-than set of a rotation path is empty then there is no constant path that dominates it; if the intersection is a singleton, then the rotation path is equivalent to the constant path at the singleton; if the intersection is neither empty nor a singleton, then a constant path consisting of any point inside the intersection is Pareto superior to the rotation path.

We illustrate Corollary 3 with the three person case, let $\theta(x, x, \cdots, x, \cdots)$ be the constant path, where $x \in X_{PO}$, and denote the rotation path by $\theta'(x^1, x^2, x^3, x^1, x^2, x^3, \cdots)$. Since $U_i(\theta) = \frac{1}{1-\delta_i} u_i(x)$ and $U_i(\theta') = \frac{1}{1-\delta_i^3} u_i(x^1) + \frac{\delta_i}{\delta_i^3} u_i(x^2) + \frac{\delta^2 i}{1-\delta_i^3} u_i(x^3)$, the comparison of the constant path and the rotation path can be reduced to looking at the sign of D_i, where $D_i = u_i(x^1) + \delta_i u_i(x^2) + \delta_i^2 u_i(x^3) - (1+\delta_i+\delta_i^2) u_i(x)$, for $i = 1, 2, 3$. The rotation path is Pareto superior to the constant path if $D_i \geq 0$ for all i and $D_i > 0$ for at least one i; it is Pareto inferior to the constant path if $D_i \leq 0$ for all i and $D_i < 0$ for at least one i; they are equivalent if $D_i = 0$ for all i.

For example, suppose in a two-dimensional policy space that all players have quadratic utility functions of the form, $u_i(z) = -[(x_1 - a_i)^2 + (y_1 - b_i)^2]$, where $z = (x_1, y_1)$. Let the players' ideal points be $A_1 = (a_1, b_1) = (-2, 0)$, $A_2 = (a_2, b_2) = (2, 0)$, and $A_3 = (a_3, b_3) = (0, 3.46)$. Let $\theta'(A_1, A_2, A_3)$ be the rotation of the three players' ideal points. The trajectories are

$$\begin{aligned} TR_1(\theta') &= (x_1 + 2)^2 + y_1^2 = 16\tfrac{\delta_1 + \delta_1^2}{1 + \delta_1 + \delta_1^2}, \\ TR_2(\theta') &= (x_2 - 2)^2 + y_2^2 = 16\tfrac{1 + \delta_2^2}{1 + \delta_2 + \delta_2^2}, \\ TR_3(\theta') &= x_3^2 + (y_3 - 3.46)^2 = 16\tfrac{1 + \delta_3}{1 + \delta_3 + \delta_3^2}. \end{aligned}$$

Then, when $\delta_1 = 0.01,\ \delta_2 = \delta_3 = 0.9,\ \ \cap_i B_i = \phi$, there is no constant path Pareto superior to the rotation path; when $\delta_1 = 0.04,\ \delta_2 = 0.70,\ \delta_3 = 0.49,\ \ \cap_i B_i = z = (-1.27, 0.42)$, then the constant path $\theta(z)$ is equivalent to rotation path θ'; when $\delta_1 = 0.74,\ \delta_2 = 0.34,\ \delta_3 = 0.73,\ \ \cap_i B_i = B$ with $|B| > 1$, any constant path $\theta(x) \in B$ is Pareto superior to the rotation path θ', such as when $x = (0, 1)$.

Insofar as implementing this scheme is concerned, its enforcement is assured if a very bad default alternative prevails whenever any player defects from the path by vetoing the next policy in the sequence. Let x_d be the default outcome. We can formalize this idea in the following proposition (see Appendix C for proof).

Proposition 4: When a veto causes the default outcome to be $x_0 \notin X_{PO} \cup X_{PS}$, a Pareto optimal rotation scheme can be implemented as a Nash equilibrium.

This proposition says that when defecting from the Pareto optimal rotation path leads to a "bad" default outcome that makes everybody worse off, the rotation scheme can be implemented in the sense that it is a Nash equilibrium for every proposer to offer a policy along the path, and it is a Nash equilibrium for every voter to agree to any proposal along the path and to veto any defection from the path.

4. Conclusions

For the most part, our results are not unexpected. The outcomes that prevail parallel in form that prevail under the assumption that voters must divide some fixed pie. First, equilibria are efficient in the sense that all outcomes and all dynamic paths are Pareto optimal. Second, unanimity is more "equitable" than simple majority rule in that a majority cannot wholly expropriate from a minority. Third, different recognition rules yield different equilibrium outcomes. A sequential recognition rule is more advantageous to players who propose early, whereas this advantage disappears under a random recognition rule. Finally, Euclidean preferences allow for the implementation of a rotation scheme that is enforced by a combination of unanimity rule and the threat of a mutually disadvantageous default alternative that prevails if any voter defects from his equilibrium strategy. More generally, our analysis establishes that much of our intuition about unanimity rule does not require any specific assumptions about transferable utility and the like for its validity. The results that we might infer from pre-existing bargaining models hold when we extend the analysis to Euclidean preferences and nontransferable utility.

It is of course true that there are a great many strategies (paths and punishments) in addition to the ones we consider, and that any definitive argument about the virtues of unanimity rule must consider them. At the very least, then, this essay establishes a framework for a more comprehensive analysis of this voting rule in a context – spatial preferences – that is more universally applicable than the divide-the-dollar scenario of earlier bargaining models.

References

Banks, J.S. and Gasmi, F. (1987). Endogenous agenda formation in three-person committees. *Social Choice and Welfare* 4: 133–152.

Baron, D.P. and Ferejohn, J.A. (1989). Bargaining in legislatures. *American Political Science Review* 83: 1181–1206.

Binmore, K.G. and Herrero, M.J. (1988). Security equilibrium. *Review of Economic Studies* 55: 33–48.

Buchanan, J.M. and Tullock, G. (1962). *The calculus of consent*. Ann Arbor: University of Michigan Press.

Calhoun, J.C. (1853). *A disquisition on government*. Indianapolis: Bobbs-Merrill.

Hammond, T.H. and Miller, G.J. (1987). The core of the Constitution. *American Political Science Review* 81: 1155–1174.

Laing, J.D. and Slotznick, B. (1991). When anyone can veto: A laboratory study of committees governed by unanimous rule. *Behavioral Science* 36: 179–195.

Matthews, S.A. (1989). Veto threats: Rhetorics in a bargaining game. *The Quarterly Journal of Economics* 104: 348–369.

McKelvey, R.D. and Riezman, R. (1990). Seniority in legislature. Social Science Working Paper 725. Caltech.

Miller, G.J. and Hammond, T.H. (1990). Committees and the core of the constitution. *Public Choice* 66: 201–227.

Ordeshook, P.C. and Schwartz, T. (1987). Agendas and the control of political outcomes. *American Political Science Review* 81: 179–199.

Riker, W.H. (1982). *Liberalism against populism*. San Francisco: W.H. Freeman.
Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica* 50: 97–109.
Sobel, M.J. (1971). Noncooperative stochastic games. *The Annals of Mathematical Statistics* 42: 1930–1935.

Appendix A

We define the stationary strategy sets $\sum$, $\sum = \prod_i \sum_i = \prod_i \prod_t \sum_i^t$, where $\sum_i^t = \mathrm{Prob}(s_i^t)$; and its element $\sigma(s) = \prod_t \sigma^t(s^t) = \prod_i \prod_t \sigma_i^t(s_i^t)$. We define the stationary Nash equilibrium in this game following McKelvey and Riezman (1990). For existence and characterization of stationary equilibrium, see Sobel (1971). That is, the stationary Nash equilibrium is characterized by a set of values $\{v_t\} \subseteq \mathcal{R}^n$ for each stage of the game, and a strategy profile $\sigma^* \in \sum$, such that

a) $\forall t$, σ^* is Nash equilibrium with payoff function $G^t : \sum^t \to R^n$ defined by

$$\begin{aligned} G^t(\sigma^t; v) &= u(\psi^t(\sigma^t)) + \sum_{y \in T} \pi^t(\sigma^t)(y)v^y \\ &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= \sum_{s^t \in S^t} \sigma^t(s^t)[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y]. \end{aligned}$$

b) $\forall t$, $v^t = G^t(\sigma^t; v)$.

Next, we define the stochastic game that we use to model unanimity rule. Let $T = M_0 \cup D \cup R \cup P \cup V$ be the set of states. An element of T will be denoted by t. We use y to denote the possible states the game moves to. We use M_0 to denote *Termination Game 0*, D to denote the *Discounting Game*, R to denote the *Recognition Game*, P to denote the *Proposal Game*, and V to denote the *Voting Game*.

Under the sequential recognition rule, the strategy sets, transition functions and outcome functions for the game elements are defined as follows:

$$\begin{array}{ll} \text{For } t \in M_0 : & S_i^t = \{0\}, \forall i \in N, \\ \text{(Termination Game 0)} & \pi^t(s^t)(0) = 1, \\ & \psi^t(s^t) = \phi, \forall s^t \in S^t. \end{array}$$

If $t \in M_0$, we are in the Termination Game 0, where the whole game terminates. Here each player's strategy set is $\{0\}$, the probability that the game stays at this stage is one, and the null outcome prevails.

$$\begin{array}{ll} \text{For } t \in D : & S_i^t = \{0\}, \forall i \in N, \\ \text{(Discounting Game)} & \pi^t(s^t)(y) = \begin{cases} \delta & \text{if } y \in R, \\ 1-\delta & \text{if } y \in M_0, \end{cases} \\ & \psi^t(s^t) = \phi, \forall s^t \in S^t. \end{array}$$

If $t \in D$, we are in the Discounting Game, where each player's strategy set is $\{0\}$. With probability $1 - \delta$ the game proceeds to the Termination Game 0, where the whole game terminates; with probability δ the game goes to the Recognition Game. This is equivalent to saying that the players discount the future payoffs at the rate δ (see McKelvey and Riezman, 1990). Next,

$$\begin{array}{ll} \text{For } t \in R: & S_i^t = \{0\}, \forall i \in N, \\ \text{(Recognition Game)} & \pi^t(s^t)(y) = 1, \ \text{if } y \in P \\ & \psi^t(s^t) = \phi, \forall s^t \in S^t. \end{array}$$

The Recognition Game is indexed by $t \in R$. We assume that there is an exogenously given order of recognition; therefore, the strategy set of each player is $\{0\}$. The game proceeds to the Proposal Game with probability 1, and the null outcome prevails.

$$\begin{array}{ll} \text{For } t \in P: & S_i^t = \begin{cases} \{x_i\} & \text{if } i = p, \\ \{0\} & \text{if } i \in N - \{p\}, \end{cases} \\ \text{(Proposal Game)} & \pi^t(s^t)(y) = 1, \ \text{if } y \in V, \\ & \psi^t(s^t) = \phi, \forall s^t \in S^t. \end{array}$$

In the Proposal Game, we use p to denote the Proposer. The strategy set for the Proposer is the set of policy positions $\{x_i\}$, while the strategy set for each voter is still $\{0\}$. The game proceeds to the Voting Game with probability one, and the null outcome prevails in this game.

$$\begin{array}{ll} \text{For } t \in V: & S_i^t = \{0, 1\}, \forall i \in N, \\ \text{(Voting Game)} & \pi^t(s^t)(y) = 1, y \in D, \\ & \psi^t(s^t) = \begin{cases} x_i^t & \text{if } s^t = 1, \\ & \\ x_0 & \text{if } s^t = 0, \end{cases} \quad \forall s^t \in S^t. \end{array}$$

In the Voting Game, each player can veto or accept (0 or 1) the new proposal. If the new proposal, x_i, is accepted by all players, it becomes the new status quo; if it is vetoed by one or more players, the old status quo, x_0, prevails. The game moves to D with probability 1.

Appendix B

Lemma 3: There exists a Nash equilibrium strategy that satisfies the following properties:

$$x_i \in X_{SO}, \ \forall i \in N;$$

$$g_j(x_i) = 0, \ \text{if } x_i \notin X_{SO}, \ \text{where } X_{SO} = X_{PO} \cap X_{PS}.$$

Proof of Lemma 3:

(1) For Proposer i, if he chooses $x_i \in X_{SO}$, the corresponding payoff is

$$G(x_i) = g(x_i)[u_i(x_i) + \delta_i v_i(x_i)] + (1 - g(x_i))[u_i(x_0) + \delta_i v_i(x_0)].$$

If he defects from this strategy, and proposes $x_i' \notin X_{SO}$, the voters, following their equilibrium strategies, will veto this proposal, i.e., $g(x_i') = 0$. The status quo prevails and the game moves to the next proposer. Therefore, his corresponding payoff is

$$G(x_i') = u_i(x_0) + \delta_i v_i(x_0).$$

Take the difference of the two payoffs, we get

$$G(x_i) - G(x_i') = g(x_i)\left[\frac{u_i(x_i)}{1-\delta_i} - u_i(x_0) - \delta_i v_i(x_0)\right].$$

Suppose $\frac{u_i(x_i)}{1-\delta_i} < u_i(x_0) + \delta_i v_i(x_0)$, then

$$x_i \notin \text{argmax}_{x \in X_{SO}} \frac{u_i(x_i)}{1-\delta_i},$$

This contradicts the assumption on the maximizing behavior of the players. Therefore, $\frac{u_i(x_i)}{1-\delta_i} - u_i(x_0) - \delta_i v_i(x_0) \geq 0$. And since $g(x_i) \geq 0$, we have $G(x_i) \geq G(x_i')$. So the proposer has no positive incentive to defect unilaterally from the equilibrium proposal.

(2) For the voters, the strategy specified in the lemma is $g_j(x_i) = 0, \forall j,\ \text{if } x_i \notin X_{SO}$.

Suppose voter k defects from the specified strategy, i.e., $g_k(x_i) = 1,\ \text{if } x_i \notin X_{SO}$. Since no other voter defects from the specified strategy, i.e., $g_j(x_i) = 0, \forall j \neq k,\ \text{if } x_i \notin X_{SO}$, then $g(x_i) = \prod_{j \neq i} g_j(x_i) = 0$. Therefore, the unilateral defection of any single voter can not change the outcome or his own payoff.

It follows that no voter will have a positive incentive to defect unilaterally from the specified strategy in Lemma 1, which is the Nash equilibrium strategy. Q.E.D.

The following lemma will be used to prove Propositions 2 and 3.

Lemma 4: The optimal x_i of the proposer's constrained maximization problem is obtained when all constraints are binding.

Proof of Lemma 4: Proposer i will make a proposal such that

$$x_i \in \text{argmax}_{x \in X_{SO}} g(x_i)[u_i(x_i) + \delta_i v_i(x_i)] + (1 - g(x_i))[u_i(x_0) + \delta_i v_i(x_0)]$$

Since $x_i \in X_{SO}$, we have

$$u_i(x_i) + \delta_i v_i(x_i) \geq u_i(x_0) + \delta_i v_i(x_0).$$

So Proposer i maximizes his objective function when $g(x_i) = 1$.

Next we specify when this condition is satisfied. For voter j, he chooses

$$g_j(x_i) \in \text{argmax}_{\{0,1\}} g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)].$$

When $g_j(x_i) = 1$, he gets $g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)]$. When $g_j(x_i) = 0$, he gets $u_j(x_0) + \delta_j v_j(x_0)$.

Therefore $g_j(x_i) = 1$, iff

$$g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)] \geq u_j(x_0) + \delta_j v_j(x_0),$$

$$\Leftrightarrow u_j(x_i) + \delta_j v_j(x_i) \geq u_j(x_0) + \delta_j v_j(x_0).$$

It follows that $g(x_i) = 1$ iff

$$u_j(x_i) + \delta_j v_j(x_i) \geq u_j(x_0) + \delta_j v_j(x_0), \forall_j \neq i.$$

Also, we know that

$$u_i(x_i) + \delta_i v_i(x_i) = \frac{u_i(x_i)}{1 - \delta_i}, \forall i \in N.$$

Then proposer i's maximization problem is simplified to

$$\max_{x_i \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1 - \delta_j} \geq u_j(x_0) + \delta_j v_j(x_0), \forall j \neq i.$$

Suppose

$$x_i \in \text{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1 - \delta_j} \geq u_j(x_0) + \delta_j v_j(x_0), \forall j \neq i,$$

and

$$\overline{x}_i \in \text{argmax}_{\overline{x} \in X_{SO}}[u_i(\overline{x}_i)]$$

$$\text{s.t. } \frac{u_j(\overline{x}_i)}{1 - \delta_j} \geq u_j(x_0) + \delta_j v_j(x_0), \forall j \neq i, m,$$

$$\frac{u_m(\overline{x}_i)}{1 - \delta_m} = u_m(x_0) + \delta_m v_m(x_0).$$

It follows that $u_m(\overline{x}_i) \leq u_m(x_i)$. Since both $x_i, \overline{x}_i \in X_{PO}$, and $v_j(x_i) = v_j(\overline{x}_i), \forall j \neq i, m$, from the definition of Pareto optimality, proposer i is at least as well off from the new proposal as from the old one, i.e., $u_i(\overline{x}_i) \geq u_i(x_i)$. Nex let

$$x_i' \in \text{argmax}_{x' \in X_{SO}}[u_i(x_i')]$$

$$\text{s.t. } \frac{u_j x_i')}{1 - \delta_j} \geq u_j(x_0) + \delta_j v_j(x_0), \forall j \neq i, m, l,$$

$$\frac{u_m(x_i')}{1 - \delta_m} = u_m(x_0) + \delta_m v_m(x_0)$$

$$\frac{u_l(x_i')}{1-\delta_l} = u_l(x_0) + \delta_l v_l(x_0).$$

Comparing the last two maximization problems, i is at least as well off from x_i' as from $\overline{x}$. By converting the unbinding constraints into binding constraints one by one, it follows that the policy position that maximizes the proposer's own utility is obtained by solving the constrained maximization problem when all constraints are binding. Q.E.D.

Appendix C

In the subsequent text, we employ the following notations: $v_{ii} \equiv \frac{u_i(x_i)}{1-\delta_i}$, $u_i \equiv u_i(x_0)$.

Proposition 2: The following is a stationary Nash equilibrium to the veto game under sequential proposing rule:

For $t \in P$ and $i = p$ (Proposer i):

$$x_i \in \text{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \sum_{l=0}^{k-1} \delta_j^1 u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \forall j \neq i,$$

$$\text{where } k = \begin{cases} j - i, & \text{if } j > i \\ n - i + j, & \text{if } j < i. \end{cases}$$

For $t \in V$ and $j \in N - \{p\}$ (Voter j):

$$g_j(x_i) = \begin{cases} 1, & \text{if } \frac{u_j(x_i)}{1-\delta_j} \geq v_j^*(x_i) \\ 0, & \text{otherwise,} \end{cases}$$

where

$$v_j^*(x_i) = \sum_{l=0}^{k-1} \delta_j^l u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j).$$

Proof of Proposition 2:

The main steps to prove Proposition 2 follow the definition of stationary Nash equilibrium in the previous section. We first specify the values associated with the equilibrium strategies, and then show that these values are self-generating. The third step is to show that the strategies specified in the proposition are Nash equilibria.

The values of the games are defined below. The interpretations of these values go back to the definitions of each game elements above.

For $t \in M_0$: $\quad v_i^t = v_i^0 = 0, \forall i \in N.$

(Termination Game 0)

For $t \in D$: $\quad v_i^t = \delta v_i^{(R)}, \forall i \in N.$

(Discounting Game)

For $t \in R$: $\quad v_i^t = v_i^{(R)}, \forall i \in N.$

(Recognition Game)

For $t \in P$: $\quad v_i^t(x_i) = \frac{u_i(x_i)}{1-\delta_i}, \text{ for } i = t,$

(Proposal Game) $\quad v_j^t(x_i) = \sum_{l=0}^{k-1} \delta_j^l u_j(x_i) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \text{ for } j \in N - \{t\},$

where

$x_i \in \text{argmax}_{x \in X_{SO}}[u_i(x_i)]$

s.t. $\frac{u_j(x_i)}{1-\delta_j} = \sum_{l=0}^{k-1} \delta_j^l(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \forall j \in N - \{t\}.$

For $t \in V$: $\quad v_i^t = \prod_i s_i^t v_i^t(x_i) + (1 - \prod_i s_i^t)[u(x_0) + \delta v^{(R)}], \forall i \in N.$

(Voting Game)

The next step is to verify that these values are self-generating, i.e., that they correspond to the payoffs under the equilibrium strategies. To do this, we plug the equilibrium strategies and other game elements into the definition of G, and show that they equal the corresponding values.

For $t \in M_0$: (Termination Game 0)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u(\psi) + \pi^t(s^t)(y) \cdot v^t = v^t. \end{aligned}$$

For $t \in D$: (Discounting Game)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum \pi^t(s^t)(y)v^y] \\ &= u(\phi) + \delta v^{(R)} + (1-\delta)v^{(0)} \\ &= \delta v^{(R)} = v^t. \end{aligned}$$

For $t \in R$: (Recognition Game)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u(\phi) + \pi^t(s^t)(y) \cdot v^t \\ &= v^{(R)} = v^t. \end{aligned}$$

For $t \in P$: (Proposal Game)
For $i = t$ (Proposer i):

$$\begin{aligned} G_i^t(\sigma^t, v_i^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u_i(\phi) + 1 \cdot \frac{u_i(x_i)}{1-\delta_i} \\ &= \frac{u_i(x_i)}{1-\delta_i} \\ &= v_i^t(x_i). \end{aligned}$$

For $j = N - \{t\}$ (Voter j):

$$\begin{aligned} G_j^t(\sigma^t; v_j^t(x_i)) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u_i(\phi) + 1 \cdot \frac{u_i(x_j)}{1-\delta_j} \\ &= \frac{u_j(x_i)}{1-\delta_j} \\ &= \sum_{l=0}^{k-1} \delta_j^l u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j) \\ &= v_j^t(x_i). \end{aligned}$$

For $T \in V$: (Voting Game)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \tau^t(s^t)(y)v^y] \\ &= \prod_i s_i^t[u(\phi) + 1 \cdot v_i^t(x_i)] + (1 - \prod_i s_i^t)[u(x_0) + 1 \cdot v^{(D)}] \\ &= \prod_i s_i^t v_i^t(x_i) + (1 - \prod_i s_i^t)[u(x_0) + \delta v^{(R)}] \\ &= v^t. \end{aligned}$$

Next, we verify that the strategies specified in Proposition 2 are Nash equilibrium strategies. We do this by showing that for each game element no player will benefit from a unilateral one-shot deviation.

For $t \in P$, we want to show that policy position x_i is the equilibrium strategy for Proposer i, where

$$x_i \in \operatorname{argmax}_{x \in X_{SO}} u_i(x_i)$$
$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \sum_{l=0}^{k-1} \delta_i^l u_j(x_i) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \forall j \neq i.$$

The corresponding payoff is

$$G_i^t(\sigma^t; v^t(x_i)) = \frac{u_i(x_i)}{1-\delta_i}.$$

If the proposer defects to any other pure strategy $x_i' \neq x_i, \forall i \in N$, there are two possible consequences:

$$\text{(i) } u_i(x_i') \leq u_i(x_i) :$$

he is not better off by defection, so he will not defect in this case.

$$\text{(ii) } u_i(x_i') > u_i(x_i):$$

in this case, if $\frac{u_j(x_i')}{1-\delta_j} = \sum_{l=0}^{k-1} \delta^l u_j(x_i) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j)$ still holds, $\forall j \neq i$, then

$$x_i \notin \operatorname{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \sum_{l=0}^{k-1} \delta_i^l u_j(x_0) + \frac{\delta_j^k}{1-\delta_j} u_j(x_j), \forall j \neq i,$$

but this contradicts the definition of x_i. Therefore the $n-1$ constraints cannot hold simultaneously: at least one of them has to be violated. Since all voters still use their equilibrium strategy, whoever gets a lower continuation value vetos the proposal. Consequently, $g(x_i') = \prod_j g_j(x_i') = 0$, and i's payoff is

$$\begin{aligned}
G_i^t(\sigma_i', \sigma_{-i}^t; v_i^t(x_i')) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\
&= u_j(\phi) + 1 \cdot [u_j(x_0) + \delta_i v_i(x_0)] \\
&= \sum_{l=0}^{k} \delta_i^l u_i(x_0) + \frac{\delta_i^{k+1}}{1-\delta_i} u_i(x_i) \\
&= \sum_{l=0}^{k} \delta_i^l [u_i(x_0) - u_i(x_i)] + \frac{u_i(x_i)}{1-\delta_i} \\
&\leq \frac{u_i(x_i)}{1-\delta_i}.
\end{aligned}$$

Therefore, $G_i^t(\sigma_i', \sigma_{-i}^t); v_i^t(x_i')) \leq G_i^t(\sigma^t; v^t(x_i))$. So the proposer has no positive incentive to defect unilaterally from his strategy specified in Proposition 2, which means that it is a Nash equilibrium for the Proposer.

For $t \in V$, we want to check if voters' strategies specified in the proposition are Nash equilibrium strategies. This can be done in two steps:

(1) Suppose x_i is such that

$$\frac{u_j(x_i)}{1-\delta_j} \geq \sum_{l=0}^{k-1} \delta_j^l u_j(x_0) + \delta_j^k u_j(x_j),$$

the corresponding equilibrium strategy is $s_j = g_j(x_i) = 1$, and the payoff is

$$G_j^t(s_j, v_j) = g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)].$$

If he detects from his equilibrium strategy for one period, i.e., $s_j' = g_j(x_i) = 0$, player j's corresponding payoff will be

$$G_j^t(s_j', v_j) = u_j(x_i) + \delta_j v_j(x_0).$$

Therefore,

$$\begin{aligned} G_j^t(s_j, v_j) - G_j^t(s_j', v_j) &= g(x_i)[u_j(x_i) + \delta_j v_j(x_i) - u_j(x_0) - \delta_j v_j(x_0)] \\ &= g(x_i)\left\{\frac{u_j(x_i)}{1-\delta_j} - \delta_j\left[\sum_{l=0}^{k-1}\delta_j^l u_j(x_0) + \delta_j^k u_j(x_j)\right]\right\} \\ &\geq 0, \end{aligned}$$

so voter j has no positive incentive to defect from his stated strategy in this situation.

(2) Suppose x_i is such that

$$\frac{u_j(x_i)}{1-\delta_j} < \sum_{l=0}^{k-1}\delta_j^l u_j(x_0) + \delta_j^k u_j(x_j),$$

the corresponding equilibrium strategy for player j is $s_j = g_j(x_i) = 0$, and the payoff is

$$G_j^t(s_j, v_j) = u_j(x_i) + \delta_j v_j(x_0).$$

If he defects from his equilibrium strategy for one period, i.e., $s_j' = g_j(x_i) = 1$, player j's corresponding payoff will be

$$G_j^t(s_j', v_j) = g(x_i)[u_j(x_i) + \delta_j v_j(x_i)] + (1 - g(x_i))[u_j(x_0) + \delta_j v_j(x_0)].$$

Therefore,

$$\begin{aligned} G_j^t(s_j, v_j) - G_j^t(s_j', v_j) &= g(x_i)[u_j(x_0) + \delta_j v_j(x_0) - u_j(x_i) - \delta_j v_j(x_i)] \\ &= g(x_i)\left\{\delta_j\left[\sum_{l=0}^{k-1}\delta_j^l u_j(x_0) + \delta_j^k u_j(x_j)\right] - \frac{u_j(x_i)}{1-\delta_j}\right\} \\ &\geq 0, \end{aligned}$$

so voter j has no positive incentive to defect from his stated strategy in this situation.

From (1) and (2), we know any voter j has no positive incentive to defect unilaterally from his strategies specified in Proposition 2, which in turn means that they are Nash equilibrium strategies for voter j. Q.E.D.

Under the random recognition rule, let $T = M_0 \cup D \cup R \cup P \cup V$ be the set of states.

The strategy sets, transition functions and outcome functions for the game elements are the same as those in the sequential recognition rule.

Proposition 3: The following is a stationary Nash equilibrium under a random recognition rule:

For Proposer i:

$$x_i \in \operatorname{argmax}_{x \in X_{SO}}[u_i(x_i)]$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \frac{nu_j(x_0) + \delta_j v_{jj}}{n - (n-1)\delta_j} \equiv v_j^*, \forall j \neq i.$$

For Voter j:

$$g_j(x_i) = \begin{cases} 1, & \text{if } \frac{u_j(x_i)}{1-\delta_j} \geq v_j^* \\ 0, & \text{otherwise.} \end{cases}$$

Proof of Proposition 3: The values of the games are defined below:

For $t \in M_0$: $v_i^t = 0, \forall i \in N.$

(Termination Game 0)

For $t \in D$: $v_i^t = \delta v_i^{(R)}, \forall i \in N.$

(Discounting Game)

For $t \in R$: $v_i^t = v_i^{(R)}, \forall i \in N.$

(Recognition Game)

For $t \in P$: $v_i^t(x_i) = \frac{u_i(x_i)}{1-\delta_i}$, for $i = t$,

(Proposal Game) $v_j^t(x_i) = \frac{nu_j(x_0)+\delta_j v_{jj}}{n-(n-1)\delta_j}$ for $j \in N - \{t\}$,

where

$x_i \in \text{argmax}_{x \in X_{SO}}[u_i(x_i)]$

s.t. $\frac{u_j(x_i)}{1-\delta_j} = \frac{nu_j(x_0)+\delta_j v_{jj}}{n-(n-1)\delta_j}, \forall j \in N - \{t\}.$

For $t \in V$: $v_i^t = \prod_i s_i^t v_i^t(x_i) + (1 - \prod_i s_i^t)[u(x_0) + \delta v^{(R)}], \forall i \in N.$

(Voting Game)

Then we verify that these values are self-generating, i.e., they equal the payoffs of the game.

For $t \in M_0$: (Termination Game 0)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u(\phi) + \pi^t(s^t)(y) \cdot v^t = v^t. \end{aligned}$$

For $t \in D$: (Discounting Game)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u(\psi) + \delta v^{(R)} + (1-\delta)v^{(0)} \\ &= \delta v^{(R)} = v^t. \end{aligned}$$

For $t \in R$: (Recognition Game)

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u(\phi) + \pi^t(s^t)(y) \cdot v^t \\ &= v^{(R)} = v^t. \end{aligned}$$

For $t \in P$: (Proposal Game)
For $i = t$ (Proposer i):

$$\begin{aligned} G^t(\sigma^t; v_i^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u_i(\phi) + 1 \cdot \tfrac{u_i(x_i)}{1-\delta_i} \\ &= \tfrac{u_i(x_i)}{1-\delta_i} \\ &= v_i^t(x_i). \end{aligned}$$

For $j = N - \{t\}$ (Voter j):

$$\begin{aligned} G_j^t(\sigma^t; v_j^t(x_i)) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= u_i(\phi) + 1 \cdot \tfrac{u_j(x_i)}{1-\delta_j} \\ &= \tfrac{u_i(x_j)}{1-\delta_j} \\ &= \tfrac{nu_j(x_0)+\delta_j v_{jj}}{n-(n-1)\delta_j}, \\ &= v_j^t(x_j). \end{aligned}$$

For $t \in V$: (Voting Game):

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[u(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v^y] \\ &= \prod_i s_i^t[u(\phi) + 1 \cdot v_i^t(x_i)] + (1 - \prod_i s_i^t)[u(x_0) + 1 \cdot v^{(D)}] \\ &= \prod_i s_i^t v_i^t(x_i) + (1 - \prod_i s_i^t)[u(x_0) + \delta v^{(R)}] \\ &= v^t. \end{aligned}$$

The third step is to show that the strategies specified in Proposition 3 are Nash equilibrium strategies, i.e., that for each game element no player will benefit from a unilateral one-shot defection.

For $t \in P$: we want to show that policy position x_i is an equilibrium strategy for Proposer i, where:

$$x_i \in \operatorname{argmax}_{x \in X_{SO}} u_i(x_i)$$

$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = \frac{nu_j(x_0) + \delta_j v_{jj}}{n - (n-1)\delta_j} \equiv v_j^*, \forall j \in N - \{i\}.$$

The corresponding payoff is

$$G_i^t(\sigma^t; v^t(x_i)) = \frac{u_i(x_i)}{1-\delta_i}.$$

If the proposer defects to any other pure strategy $x_i' \neq x_i$, and since $u_i(\cdot)$ is monotone, $\forall i \in N$, there are two possible consequences.

(i) $u_i(x_i') \leq u_i(x_i)$: he is not better off by defection, so he will not defect in this case.

(ii) $u_i(x_i') > u_i(x_i)$: in this case, if $\frac{u_j(x_i')}{1-\delta_j} = v_j^*$ still holds $\forall j \neq i$, then

$$x_i \notin \operatorname{argmax}_{x \in X_{SO}}[u_i(x_i)]$$
$$\text{s.t. } \frac{u_j(x_i)}{1-\delta_j} = v_j^*, \forall j \neq i,$$

but this contradicts the definition of x_i. Therefore the $n-1$ constraints cannot hold simultaneously: at least one of them has to be violated. Since all voters still use their equilibrium strategy, whoever gets a lower continuation value vetos the proposal. Consequently, $g(x_i') = \prod_j g_j(x_i') = 0$, and i's payoff is

$$\begin{aligned} G_i^t(\sigma_i', \sigma_{-i}^t; v_i^t(x_i')) &= E_{\sigma'}[u(\psi^t(s')) + \textstyle\sum_{y \in T} \pi^t(s')(y)v^y] \\ &= u_j(\phi) + 1 \cdot [u_j(x_0) + \delta_i v_i(x_0)] \\ &= u_j(x_0) + \delta_i v_i(x_0). \end{aligned}$$

Therefore,

$$\begin{aligned} G_i^t(\sigma_i', \sigma_{-i}, \cdot) - G_i^t(\sigma, \cdot) &= u_i(x_0) + \delta_i v_i(x_0) - \tfrac{u_i(x_i)}{1-\delta_i} \\ &= u_i(x_0) + \delta_i \tfrac{nu_i(x_0)+\delta_i v_{ii}}{n-(n-1)\delta_i} - v_{ii}. \end{aligned}$$

To show that x_i is a Nash equilibrium strategy, it suffices to show that the above expression is less than or equal to zero, i.e.,

$$\begin{aligned} u_j(x_0) + \delta_i \tfrac{nu_i(x_0)+\delta_i v_{ii}}{n-(n-1)\delta_i} &\leq v_{ii} \\ \Leftrightarrow (1 + \tfrac{n\delta}{n-(n-1)\delta})u_i(x_0)) &\leq (1 - \tfrac{\delta^2}{n-(n-1)\delta})v_{ii} \\ \Leftrightarrow (n+\delta)u_i(x_0) &\leq (n-(n-1)\delta - \delta^2)\tfrac{u_i(x_i)}{1-\delta_i} \\ \Leftrightarrow u_i(x_0) &\leq u_i(x_i), \end{aligned}$$

which holds obviously, since $x_i \in X_{SO}$. Therefore, $G_i^t(\sigma_i', \sigma_{-i}^t; v_i^t(x_i')) \leq G_i^t(\sigma^t; v^t(x_i))$. So, the proposer has no positive incentive to defect from the stated strategy, which is proven to be the Nash equilibrium strategy.

Last, we show that voter j's strategy in Proposition 3 is Nash equilibrium strategy. Since this part of the proof is similar to the corresponding part in the proof of Proposition 2, we will not repeat it here. Q.E.D.

Proposition 4: When veto by a player causes the default outcome to be $x_d \notin X_{PO} \cup X_{PS}$, a Pareto optimal rotation scheme $\theta(x^1, x^2, \cdots, x^n, x^1, x^2, \cdots, x^n, \cdots)$ can be supported as a Nash equilibrium.

Proof: We want to show that when $\theta(x^1, x^2, \cdots, x^n, \cdots) \in X_{DPO}$ and $x_0 \notin X_{PO} \cup X_{PS}$, in equilibrium, for proposer i, $x_i = x^i$; and for voter j, $g_j(x_i) = 1$ if $x_i = x^i$, $g_j(x_i) = 0$ if $x_i \neq x^i$.

For proposer i, if $x_i = x^i$, his payoff function $G_i(x_i) = U_i(\theta)$, given that everyone else follows their equilibrium strategies. If $x_i \neq x^i$, $G_i'(x_i) = U_i(\theta) + u_i(x_d) - u_i(x^i)$. Since $u_i(x_d) - u_i(x^i) < 0$, $G_i'(x_i) < G_i(x_i)$, i.e., he is worse off defecting from the optimal rotation path. Therefore, it is a Nash equilibrium for any proposer to offer the policy in the rotation path.

For voter j, when $x_i = x^i$, $g_j(x_i) = 1$ gives him $G_j(x_i) = U_j(\theta)$, while $g_j(x_i) = 0$ gives him $G_j'(x_i) = U_j(\theta) + u_i(x_d) - u_i(x^i) < G_j(x_i)$, therefore, it is a Nash equilibrium for him to accept when the proposal is along the equilibrium path. Consider the second case, when $x_i \neq x^i$, given that all other voters still follow their equilibrium strategies, $g(x_i) = 0$ regardless of voter j's decision. Therefore, he is not better off defecting from the specified strategy, which is a Nash equilibrium strategy. Q.E.D.