



Sealed bid auctions with ambiguity: Theory and experiments

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Abstract

This study presents a laboratory experiment of the first and second price sealed bid auctions with independent private values, where the distribution of bidder valuations may be *unknown*. In our experimental setting, in first price auctions, bids are lower with the presence of ambiguity. This result is consistent with ambiguity loving in a model that allows for different ambiguity attitudes. We also find that the first price auction generates significantly higher revenue than the second price auction with and without ambiguity. © 2007 Elsevier Inc. All rights reserved.

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1. Introduction

The theoretical and experimental auction literature often assumes that bidders know the distribution of other bidders' valuations.¹ Consequently, nearly all of the results derive from such assumptions. However, in many real-world auctions, it is inappropriate to assume that bidders know the distribution from which opponent valuations are drawn. This uncertainty may matter even more in online auctions than it does in standard “physical presence” auctions. Online auction technology introduces several interesting features not available to traditional auctions.

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¹ For a survey of the theoretical literature, see Klemperer [19]. For a survey of the experimental literature, see Kagel [16].

For example, bidders can be geographically dispersed and bidding can be asynchronous. These conveniences make it easier to obtain a relatively large group of bidders for an object. Thus, it is important to re-examine the implications of some key assumptions in auction theory and experiments. In this study, we focus on the assumption that bidders know the distribution of other bidder valuations.

The uncertainty about the probability distribution (of bidder valuations, for example) created by missing information is often called *ambiguity*. Ambiguity can affect decision making in important ways, as illustrated by the Ellsberg [8] paradox. Ellsberg's two-color problem uses two urns, one containing 50 red and 50 black balls called the known urn (or the risky urn), and one containing 100 balls in an unknown combination of red and black called the unknown urn (or the ambiguous urn). These two urns represent two distinct types of uncertainty. The first type of uncertainty, present in both urns, is uncertainty as to which outcome will occur: red or black, and is termed risk. The second type of uncertainty, present only in the unknown urn, is uncertainty about the probability of each outcome itself and is termed ambiguity. In Ellsberg experiments, many people bet on red from the known (vs. unknown) urn *and* on black from the known urn. However, they are indifferent between the two colors when betting on only one urn. This pattern of behavior is inconsistent with any model which uses probabilities, and is called *ambiguity aversion*. The opposite of ambiguity aversion is called *ambiguity loving*.

Apart from online auctions, ambiguity is prevalent in many other real-world situations, for example, the success rate of some new drugs or clinical treatments [7], the insurance of certain classes of highly ambiguous risks, such as environmental hazards [24] and terrorist attacks, the usefulness of new features of consumer products [18], the outcomes of R&D, incomplete contracting due to unforeseen contingencies, the audit selection procedures of the IRS [1], and initial public offerings (IPOs) of small privately held firms.

In this paper, we investigate the impact of ambiguity on bidding behavior and revenue in the first and second price sealed bid auctions in the laboratory. Our experiment compares treatments with an unknown distribution of bidder valuations to those with a known distribution of bidder valuations. Our study extends the large amount of research on auctions to a more realistic setting with the presence of ambiguity. Our main finding is that, in first price auctions, bids are *lower* with the presence of ambiguity. This result is consistent with ambiguity loving in a model that allows for different ambiguity attitudes. We also find that the first price auction generates significantly higher revenue than the second price auction with and without the presence of ambiguity.

Many researchers have studied ambiguity empirically. These studies can be broadly classified into three categories. The first kind of empirical ambiguity research is Ellsberg's original thought experiment and its replications. The second kind determines the psychological causes of ambiguity. The third kind studies ambiguity in applied settings. While many studies of the first kind find various degrees of ambiguity aversion, Curley and Yates [5], and Hogarth and Einhorn [15], among others, find ambiguity loving when subjects face an unknown urn and a known urn with a low probability of winning. Some studies of ambiguity in experimental markets find mixed results. For example, Sarin and Weber [28] study ambiguity in an experimental asset market using a double oral auction and a multi-unit Vickrey auction. This study finds that the market price for the unambiguous bet is considerably larger than the market price of the ambiguous bet.² The main lesson from past empirical studies is that ambiguity affects behavior, which is consistent with our findings.

² Note that in the Sarin and Weber experiments, ambiguity is operationalized as à la Ellsberg.

The paper is organized as follows. Section 2 introduces a theoretical model of sealed bid auctions with risk and ambiguity. Section 3 presents the experimental design. Section 4 presents the main results. Section 5 concludes the paper.

2. A model of bidding with ambiguity

This section develops a theoretical auction model incorporating risk and ambiguity. While we do not believe that this equilibrium model captures all aspects of behavior in the experiment, it provides a useful benchmark for our data analysis.

There are several different approaches to formally model ambiguity. Among them, maxmin expected utility³ (MMEU) and Choquet expected utility⁴ (CEU) models are the most prominent in applications. In this paper we use the α -MEU model, which is a natural and tractable generalization of the MMEU model. The α -MEU model allows for both ambiguity averse and ambiguity loving behavior.

Three theoretical studies address the role of ambiguity in auctions. Salo and Weber [27] analyze the first price sealed bid auction using the CEU model with a convex capacity. In particular, they consider the case where bidders have a constant relative risk aversion (CRRA) utility function and the Choquet capacity has a power representation. In this case, they show that the equilibrium bidding function is linear. In another study, Lo [20] analyzes sealed bid auctions using the MMEU framework. Specifically, he derives the equilibrium bidding function for linear utility functions, and compares the first and second price auctions. Using the MMEU framework, Ozdenoren [23] extends and generalizes the results in Lo. He derives conditions under which risk neutral bidders increase their bids in the first price auction as they become more ambiguity averse. He then uses this result to compare the first and second price auctions.

Our model differs from the above models in two important ways. First we use the α -MEU framework to allow for both ambiguity averse and ambiguity loving behavior. This framework is a generalization of both the maxmin and maxmax expected utility models. Second, we consider bidders with general concave utility functions. As a result, previous theory cannot be directly applied to our framework.

Throughout this section, we assume that there are two bidders $i = 1, 2$. In addition, we assume that there is one indivisible good for sale. In this model, we look at first and second price auctions with independent private values with zero reserve price. Bidders submit their bids simultaneously. For simplicity, we assume that the set of possible valuations of the bidders is $[0, 1]$, with V_i denoting bidder i 's valuation. Only the bidder knows his own valuation.

Our main departure from previous theoretical and experimental auction literature is the assumption that bidders do not know the distribution of valuations. We look at the case where bidder valuations are known to be independent draws from either $F^1(\cdot)$ or $F^2(\cdot)$, with positive and a.e.—continuous densities $f^1(\cdot)$ and $f^2(\cdot)$, respectively. In our experiment, we assume that F^2 first order stochastically dominates F^1 . Hence, we call F^1 the low value distribution and F^2 the high value distribution. We define δ to be the random variable corresponding to the probability that valuation is drawn from F^1 . For each bidder, the probability, δ , of the event that his opponent's valuation is drawn from the distribution F^1 is unknown.

³ In the MMEU model, decision makers have a set of priors and choose an action that maximizes the minimum expected utility over the set of priors.

⁴ In the CEU model, decision maker's beliefs are represented by a non-additive probability measure (capacity).

In the standard subjective expected utility (SEU) model, each bidder has a subjective prior about the value of δ . However, if a bidder’s information about δ is too vague to be represented by a single prior, it can be represented by a set of priors. In a seminal paper, Gilboa and Schmeidler [12] provide an axiomatization of the MMEU model using a set of priors. Expected utility is a special case of MMEU, where the set of beliefs contains only a single probability measure. In this model, a bidder’s prior on the event that his opponent’s valuation is drawn from the distribution F^1 is given by a set of probability measures. The bidder’s utility is given by the minimum expected utility over this set of priors. Intuitively, a set of priors reflects both ambiguity in the environment and the difficulty bidders face in forming a well-defined single prior. The min operator, on the other hand, reflects aversion to such ambiguity. To illustrate how MMEU explains Ellsberg type behavior, suppose a decision maker has a linear utility function and the set of priors is $\{(x, 1 - x) : 0.4 \leq x \leq 0.6\}$, where x is the probability of drawing a red ball and $1 - x$ is the probability of drawing a black ball from the unknown urn. The probability of drawing either color from the known urn is 0.5. In this case, betting \$1 on either color from the ambiguous urn will give an expected utility of 0.4, whereas betting \$1 on either color from the known urn will give an expected utility of 0.5.

In general, decision makers may also have preferences that represent ambiguity loving behavior [14]. Such behavior can be captured using the maxmax expected utility model, where the min operator is replaced by the max operator. We do not want to restrict bidders’ ambiguity attitude a priori, and therefore we use the α -MEU model that allows for both ambiguity averse and ambiguity loving behavior. The α -MEU model, axiomatized by Ghirardato et al. [11], is a generalization of both the maxmin and the maxmax expected utility models. In this model, bidders compute the utility of an act using α times the minimum plus $1 - \alpha$ times the maximum expected utility over the set of priors. When α equals 1, this model reduces to MMEU. When α equals 0, it reduces to maxmax EU. Note that the class of preferences this model represents is more general, since α can take all intermediate values.

Formally, let Δ be a closed and convex subset⁵ of the set of distribution functions over $[0, 1]$, representing bidder’s beliefs about the value of δ . Let $\underline{\delta} = \min_{G \in \Delta} \int \delta dG(\delta)$ and $\bar{\delta} = \max_{G \in \Delta} \int \delta dG(\delta)$. Note that the set Δ is subjective and the set $[\underline{\delta}, \bar{\delta}]$ can in general be a strict subset of $[0, 1]$. To see this, consider the case where the set Δ has a single element, F . In this case, $\underline{\delta} = \bar{\delta} =$ expected value of F . We assume that Δ is independent of bidder valuations and is common knowledge to all bidders.

In the first price auction, the bidder with the highest bid receives the object and pays his bid to the seller. Ties are broken with equal probability using a fair coin.⁶ A bid can be any number in $(0, \infty)$. The payoff for bidder i is given by

$$\pi_i(V_i, b_i, b_j) = \begin{cases} V_i - b_i & \text{if } b_i > b_j, \\ (V_i - b_i) / 2 & \text{if } b_i = b_j, \\ 0 & \text{if } b_i < b_j. \end{cases} \tag{1}$$

The bidding strategy of bidder i is given by $s_i : [0, 1] \rightarrow [0, \infty)$, mapping own valuation into a bid. We assume that, in equilibrium, bidder i knows both his own valuation, V_i , and bidder j ’s

⁵ The restrictions on Δ follow Gilboa and Schmeidler [12].

⁶ We assume that there is no ambiguity about the fair coin and the bidder maximizes expected utility when there is no ambiguity. Thus, a bidder’s *ex post* payoff in case of a tie is given by $(V_i - b_i)/2$.

strategy, s_j , but not j 's valuation. Bidder i best replies to bidder j 's strategy given his valuation and his beliefs Δ .

In order to capture bidders' risk attitude, we use a concave utility function, $u(\cdot)$, with $u(0) = 0$, $u' > 0$, and $u'' < 0$. Assuming that bidding strategies are strictly increasing in own valuation,⁷ given the other bidder's strategy s_j and bidder i 's own valuation V_i , bidder i chooses his bid by maximizing

$$U_i(b_i; V_i, s_j) \equiv u(V_i - b_i) F_\alpha \left[s_j^{-1}(b_i) \right], \tag{2}$$

where s_j^{-1} is the inverse of s_j , which, in equilibrium, is bidder j 's value, and $F_\alpha = \left(\alpha \underline{\delta} + (1 - \alpha) \bar{\delta} \right) F^1 + \left[1 - \left(\alpha \underline{\delta} + (1 - \alpha) \bar{\delta} \right) \right] F^2$ is the bidder's belief about his opponent's valuation. In other words, an α -MEU bidder will behave as if he believes that his opponent's valuation is drawn from F^1 with probability $\alpha \underline{\delta} + (1 - \alpha) \bar{\delta}$ and from F^2 with probability $1 - \left(\alpha \underline{\delta} + (1 - \alpha) \bar{\delta} \right)$. The derivation of Eq. (2) is in the Appendix.

Strategies s_1 and s_2 are *equilibrium strategies* if

$$U_i(s_i(V_i); V_i, s_j) \geq U_i(b_i; V_i, s_j) \tag{3}$$

for all $V_i \in [0, 1]$, $b_i \in [0, \infty)$, $i = 1, 2$, and $j = 3 - i$. In the following proposition, we characterize the symmetric equilibrium strategy.

Proposition 1. *The symmetric equilibrium bidding strategy, s , is characterized by*

$$\frac{\partial s}{\partial V} = \frac{F'_\alpha(V) u[V - s(V)]}{F_\alpha(V) u'[V - s(V)]}. \tag{4}$$

Proof. See Appendix.

This Proposition characterizes the symmetric equilibrium bidding strategy for an α -MEU bidder. Eq. (4) in Proposition 1 is analogous to the equilibrium characterization for the no-ambiguity case by Riley and Samuelson [25] and Milgrom and Weber [22].

We use a particular specification for F^1 and F^2 to further investigate the properties of the bidding function. We use this specification later in the experiments. To construct the low value distribution F^1 , we first choose the interval $\left[0, \frac{1}{2}\right]$ with probability $\frac{3}{4}$ and the interval $\left(\frac{1}{2}, 1\right]$ with probability $\frac{1}{4}$. Subsequently, we choose the valuation from the chosen interval uniformly. Similarly, to construct the high value distribution F^2 , we first choose the interval $\left[0, \frac{1}{2}\right]$ with probability $\frac{1}{4}$ and the interval $\left(\frac{1}{2}, 1\right]$ with probability $\frac{3}{4}$. Again, we then choose the valuation from the chosen interval uniformly. More precisely, the two distribution functions are specified as follows:

$$F^1(x) = \begin{cases} \frac{3}{2}x & \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{3}{4} + \left(x - \frac{1}{2}\right) \frac{1}{2} & \text{if } \frac{1}{2} < x \leq 1, \end{cases} \tag{5}$$

⁷ This assumption will later be verified as shown in Eq. (4).

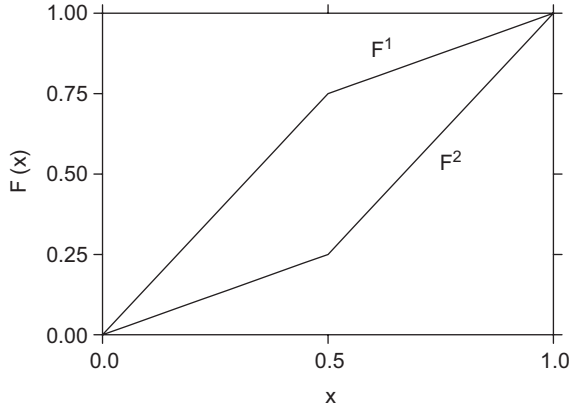


Fig. 1. Cumulative distribution functions F^1 and F^2 .

$$F^2(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{1}{4} + \left(x - \frac{1}{2}\right) \frac{3}{2} & \text{if } \frac{1}{2} < x \leq 1. \end{cases} \tag{6}$$

Fig. 1 presents graphs of the cumulative distribution functions F^1 and F^2 . Note that neither F^1 nor F^2 is uniform. A non-uniform distribution in first price auctions allows separation of equilibrium bidding functions from linear rules of thumb.

Recall that $F_\alpha = (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta}) F^1 + [1 - (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta})] F^2$. Thus, F_α can be expressed as

$$\begin{aligned} F_\alpha(x) &= \begin{cases} \theta x & \text{if } 0 \leq x \leq \frac{1}{2}, \\ \frac{1}{2}\theta + \left(x - \frac{1}{2}\right) (2 - \theta) & \text{if } \frac{1}{2} < x \leq 1 \end{cases} \\ &= \begin{cases} \theta x & \text{if } 0 \leq x \leq \frac{1}{2}, \\ (\theta - 1) + (2 - \theta)x & \text{if } \frac{1}{2} < x \leq 1, \end{cases} \end{aligned} \tag{7}$$

where

$$\begin{aligned} \theta &\equiv (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta}) \frac{3}{2} + [1 - (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta})] \frac{1}{2} \\ &= (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta}) + \frac{1}{2}. \end{aligned} \tag{8}$$

Eq. (8) implies that the higher α is, the lower θ will be. Recall from Eq. (2) that the higher the parameter α is, the more weight the decision maker puts on the min functional. Thus, α measures a bidder’s ambiguity attitude, where higher values of α reflect more ambiguity aversion.⁸ The interval, $[\underline{\delta}, \bar{\delta}]$, measures the amount of ambiguity in the environment. Fixing the amount of ambiguity in the environment, $[\underline{\delta}, \bar{\delta}]$, the parameter, θ , also measures a bidder’s ambiguity attitude.

⁸ In fact, Siniscalchi [29] shows that, once the set of priors Δ (or, equivalently, the interval $[\underline{\delta}, \bar{\delta}]$) is fixed, the index α can be interpreted as an ambiguity aversion parameter.

In the analysis, as we cannot separately identify α , $\underline{\delta}$ and $\bar{\delta}$, we will make an identifying assumption regarding $[\underline{\delta}, \bar{\delta}]$, which will allow us to use θ as a measure of ambiguity.

In order to identify when a bidder is ambiguity averse (or loving), we first need to know when the bidder is ambiguity neutral. This is characterized by the following proposition.

Proposition 2. *When $\alpha = \frac{1}{2}$, the decision maker is an expected utility maximizer with beliefs given by $\frac{1}{2} [\underline{\delta}F^1 + (1 - \underline{\delta})F^2] + \frac{1}{2} [\bar{\delta}F^1 + (1 - \bar{\delta})F^2]$*

Proof. This follows from Proposition 3 of Ghirardato et al. [10]. \square

This proposition gives us a natural benchmark for the case of ambiguity neutrality, which allows us to formally define ambiguity aversion and ambiguity loving.

Definition 1. When $\alpha = \frac{1}{2}$, the decision maker is ambiguity neutral; when $\alpha > \frac{1}{2}$, the decision maker is ambiguity averse; when $\alpha < \frac{1}{2}$, the decision maker is ambiguity loving.

Using the above parameterizations of F^1 and F^2 , we can extend the characterization of the bidding function provided in Proposition 1. In what follows, we include θ as an explicit argument of the bidding function.

Corollary 1. *With the parameterized distribution functions F^1 and F^2 , the equilibrium bidding strategy is characterized by*

$$\frac{\partial s}{\partial V}(V, \theta) = \begin{cases} (g[V - s(V, \theta)]) / V & \text{if } V \leq \frac{1}{2}, \\ g[V - s(V, \theta)]h(V, \theta) & \text{if } \frac{1}{2} < V \leq 1, \end{cases} \tag{9}$$

where

$$g(z) \equiv \frac{u(z)}{u'(z)} \quad \text{and} \quad h(V, \theta) \equiv \frac{2 - \theta}{\theta - 1 + (2 - \theta)V}. \tag{10}$$

Proof. Substituting Eq. (7) into Eq. (4), we obtain the result. \square

This more detailed characterization allows us to consider the impact of ambiguity on the bidding function. This issue is addressed by the following Proposition:

Proposition 3. *If $V \leq \frac{1}{2}$, $s(V, \theta)$ does not depend on θ . If $V > \frac{1}{2}$, $s(V, \theta)$ is strictly decreasing in θ .*

Proof. See Appendix.

This proposition shows that, in the range where $V > \frac{1}{2}$, an increase in ambiguity aversion (a decrease in θ) leads to higher bids, while an increase in ambiguity loving (an increase in θ) leads to lower bids. The intuition is the following. When a bidder is more ambiguity averse, she is more pessimistic, which implies that she thinks that her opponent’s valuation is more likely to be high. Therefore, she bids more.

In contrast, in second price auction, the bidder who has the highest bid receives the object and pays the second highest bid to the seller. Ties are broken by a random device. In this auction,

bidding one's true valuation is a weakly dominant strategy, even with ambiguity aversion (see, e.g., [20]). This leads to our next proposition.

Proposition 4. *In the second price sealed bid auction, regardless of the bidder risk and ambiguity attitudes, bidding one's true valuation is a weakly dominant strategy.*

All theoretical results presented in this section serve as a guidance for our experimental design and data analysis.

3. Experimental design

The experimental design reflects both theoretical and technical considerations. We design our experiment to determine the effect of ambiguity on bidder behavior and to re-evaluate the performance of two auction mechanisms in the presence of ambiguity.

3.1. Economic environments

To study the effect of ambiguity on bidder behavior and its consequences on the performance of two auction mechanisms, we chose a 2×2 design. In the information dimension, we include treatments with and without the presence of ambiguity, while in the mechanism dimension, we use the first and second price sealed bid auctions. The choice of the 2×2 design is based on the following considerations:

- (1) Known vs. unknown distributions: We use the treatment with known distributions to identify bidders' risk attitude. Since behavior in the treatment with unknown distribution involves both the bidders' risk attitude and their ambiguity attitude, comparing behavior in this treatment to the known treatment isolates the effect of ambiguity.
- (2) First price vs. second price auctions: As the theoretical predictions for the second price auction do not change with increased ambiguity while those for the first price auction do, we use the first price auction to measure participant ambiguity attitude, and the second price auction as a benchmark for detecting systematic behavioral changes with the presence of ambiguity which are unaccounted for by theory.

Table 1 summarizes the relevant features of the experimental sessions, including information conditions, auction mechanisms, treatment abbreviations, exchange rate and the total number of subjects in each treatment. The exchange rate is set so that participant earnings in equilibrium are comparable to the average earnings of past experiments conducted in the Research Center for Group Dynamics Laboratory. For each treatment, we conducted five independent sessions using networked computers at the Research Center for Group Dynamics Laboratory at the University of Michigan. This design gives us a total of 20 independent sessions and 160 subjects,⁹ recruited from an email list of Michigan undergraduate and graduate students.¹⁰

One crucial decision in the design was how to implement ambiguity. In many psychology experiments designed to test the Ellsberg paradox, subjects were told nothing about the distribution of the unknown urn. We adopted a similar design in a pilot experiment conducted in April 2001,

⁹ Despite our explicit announcement in the advertisement that subjects could not participate in the auction experiment more than once and our screening before each session, one subject participated three times.

¹⁰ Graduate students in Economics were excluded from the list.

Table 1
Features of experimental sessions

Information conditions	Auction mechanisms	Treatment abbreviation	Exchange rate	No. subjects per session	Total no. subjects
Known Distribution	First price	$K1$	20	8	40
	Second price	$K2$	20	8	40
Unknown Distribution	First price	$U1$	20	8	40
	Second price	$U2$	20	8	40

but found no basis to infer what prior (or set of priors) the subjects used. Thus, for analytical tractability, we narrow ambiguity to a single parameter in this experiment. More specifically, bidder valuations are known to be independent draws from either the low value distribution $F^1(\cdot)$ or the high value distribution $F^2(\cdot)$. We use the F^1 and F^2 specifications from Section 2, with two modifications. First, we re-scale the support to the interval $[0, 100]$. Second, we discretize the support to the set $\{1, 2, \dots, 100\}$. For each bidder, the probability, δ , of the event that his opponent's valuation is drawn from the distribution F^1 is unknown. Therefore, we generate ambiguity regarding the valuation distribution through δ .

In the experiment, each bidder's valuation in each round is a random draw from the set $\{1, 2, \dots, 100\}$. We choose δ_0 , the true value of δ , to be 0.70 for two reasons. First, we want the compound distribution to be non-uniform, which precludes $\delta_0 = 0.5$. We choose not to use a uniform distribution, since it might be a focal point in the absence of knowledge about the true distribution. Furthermore, with a uniform distribution, one cannot separate equilibrium bidding strategies from linear rules of thumb in the first price auction [2]. Second, since most previous experiments demonstrate ambiguity aversion, we want to create an environment which leaves room for ambiguity averse bidders to learn. This consideration precludes $\delta_0 < 0.5$. In treatments with a known distribution, $\delta_0 = 0.70$ implies that $\bar{\delta} = \underline{\delta} = 0.7$. It then follows from Eq. (8) that $\theta = \theta_0 = 1.2$.

3.2. Experimental procedure

At the beginning of each session, subjects randomly drew a PC terminal number. Then each subject was seated in front of the corresponding terminal, and given printed instructions. After the instructions were read aloud, subjects completed a set of Review Questions, to test their understanding of the instructions. Afterwards, the experimenter checked answers and answered questions. The instruction period varied between 15 and 30 min. Each round consisted of the following stages:

- (1) For treatments with an unknown distribution only, each bidder estimated the chance that the valuation of the *other* bidder in the group was drawn from the high value distribution, i.e., an estimate of $1 - \delta$. The bidder also indicated his confidence in his estimate: not confident at all, slightly confident, moderately confident, fairly confident, and very confident.¹¹

¹¹ This confidence rating method to elicit ambiguity attitude was proposed and evaluated by psychologists Curley et al. [7]. Among three different methods to elicit subject ambiguity attitude in decision making, they found this one to be the best.

- (2) Next, each bidder was informed of his own valuation. Then each bidder simultaneously and independently submitted a bid, which could be any integer between 1 and 100, inclusive.
- (3) Bids were then collected in each group and the object was allocated according to the rules of the auction.
- (4) Afterwards, each bidder received the following feedback on his screen: his valuation, his bid, the winning bid, whether he received the object, and his payoff.

The subjects did not receive the entire vector of valuations and the corresponding bids to slow down the learning of δ and thus to preserve ambiguity for the initial rounds.

In each treatment, each session lasted 30 rounds with no practice rounds. At the end of 30 rounds, all participants completed a questionnaire to elicit demographic information. The demographic results are reported in Chen et al. [3].

Compared to Salo and Weber [26] laboratory study of ambiguity in first price sealed bid auctions, our design has the following characteristics. First, we study both first and second price auctions, while Salo and Weber study only first price auctions. Second, we use a non-uniform distribution of valuations, while Salo and Weber use the uniform distribution. Third, while Salo and Weber also examine unknown number of competitors and dichotomous auctions, we do not. Last, we used 160 subjects, while Salo and Weber used 48 subjects. The larger number of observations enables us to obtain more precise estimates in our statistical analysis.

The experiments were conducted from October 2001 to January 2002. Each session lasted from 40 min to an hour. The exchange rate was 20 points to \$1. The average earning was \$16.20. Experimental Instructions are posted on the first author's website (<http://www.si.umich.edu/~yanchen/>). The data are available from the authors upon request.

4. Results

We present experimental results in this section. Fig. 2 presents the cross plot of bids against values in all four treatments. In all subsequent analysis, we normalize the valuations and bids to be on the interval $[0, 1]$, consistent with the notation in our theoretical model. The first row presents data for the first price auction, while the second row is for the second price auction. For each row, the left graph is for the known treatment, while the right graph is for the unknown treatment. An immediate observation is that, in the first price auction, most bids are below the value (i.e., below the diagonal), while in the second price auction, bids are often above the values. We now proceed to analyze the difference between treatments with and without ambiguity.

We first estimate bidders' ambiguity attitude in the first price auction by using two different approaches. The non-parametric approach compares bids in the no-ambiguity treatments and those in the ambiguity treatments, and infers bidders' ambiguity attitude based on Proposition 3. This approach imposes minimal assumptions on bidder behavior. The structural approach is based on the equilibrium bidding function to be derived in Corollary 2 and explicitly estimates the ambiguity parameter. Compared to the non-parametric analysis, the structural approach requires more assumptions on the bidders' utility function. We then examine the effect of ambiguity on bids, revenue, earnings and efficiency.

4.1. Non-parametric estimation of ambiguity attitude in first price auctions

To estimate bidders' ambiguity attitude, we first compare the bids in the no-ambiguity treatment to those in the ambiguity treatment. As we have a full factorial design, keeping everything else

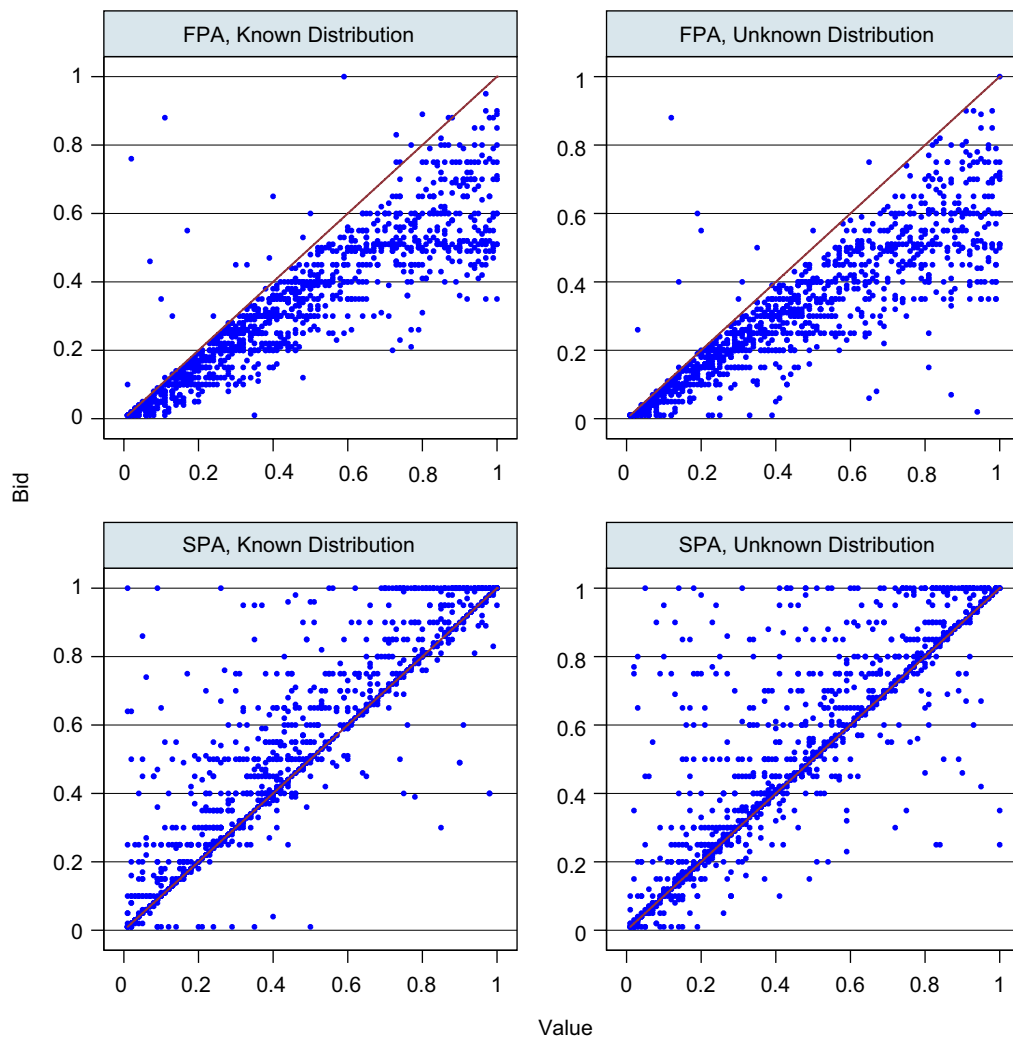


Fig. 2. Raw bids in all treatments.

constant, any systematic variations in bids in the ambiguity treatments compared to the no-ambiguity treatments can only be attributed to the variation in the amount of ambiguity.

Bids being lower in the ambiguity treatment compared to the no-ambiguity treatment is consistent with ambiguity loving under a weak assumption. Recall that both the amount of ambiguity in the environment and bidders' ambiguity attitude are summarized in the parameter θ . Proposition 3 implies that higher θ leads to lower bids. In the no-ambiguity treatments, $\theta = 1.2$ as $\delta = 0.7$ is known. Therefore, by comparing bids in the ambiguity treatments and those in the no-ambiguity treatments, we can determine whether θ in the ambiguity treatments is greater (or less) than 1.2. If bids in the ambiguity treatments are lower, we can infer that $\theta > 1.2$, and vice versa. To infer bidder's ambiguity attitudes (i.e., α) from θ , we need to assume that the center of the interval $[\underline{\delta}, \bar{\delta}]$ is at or below 0.7. This assumption puts a weak restriction on the amount of weight on the low value

distribution relative to the high value distribution. However, it does not rule out the possibility of putting more than 0.7 weight on the low value distribution (e.g., $[0.4, 1.0]$ is centered at 0.7 and thus is allowed by our assumption). A natural place where the interval might be centered is 0.5, as suggested by the “principle of insufficient reason,” which Luce and Raiffa [21, p. 284] attribute to Jacob Bernoulli. This case, too, is covered by this assumption. Under this assumption, if $\theta > 1.2$, then $\alpha < \frac{1}{2}$, implying ambiguity loving.¹² If $\theta < 1.2$, then bidder ambiguity attitude cannot be determined precisely.¹³ We now compare the mean bids in the no-ambiguity treatment ($K1$) with those in the ambiguity treatment ($U1$), using the Wilcoxon ranksum test. We also compare the median bids and get similar results.

Table 2 reports p -values for the Wilcoxon ranksum tests. The null hypothesis is that the distribution of bids is the same in treatments with and without ambiguity. The alternative hypothesis is that bids are higher in the no-ambiguity treatment. In Round 1, all bids are independent, and therefore we use each individual bid as an independent observation. From Round 2 on, we use a session mean as an independent observation. As we expect the amount of ambiguity to decrease over time, we partition the data into early rounds (Round 1, Rounds 1–3, Rounds 1–5) and later rounds. For each time interval, we compare bids over all values, as well as those in two subranges, $[0, 0.5]$ and $(0.5, 1]$.

Result 1 (Ambiguity attitude). *In first price auctions, bids are lower in the ambiguity treatment compared to the no-ambiguity treatment, which is consistent with ambiguity loving.*

Support: Table 2 reports p -values for one-sided Wilcoxon ranksum tests, comparing distributions of (mean) bids for treatments with and without ambiguity. There is a statistically significant difference in Round 1 for the value range of $(0.5, 1]$, for Rounds 1–3 and 1–5 for all V .

Result 1 presents a significant finding that bids are lower with the presence of ambiguity. From Proposition 3 and the analysis at the beginning of this subsection, this result is consistent with the hypothesis that bidders are ambiguity loving. This is the first main result of this paper. Result 1 is surprising, given that a large volume of empirical studies replicating the Ellsberg urn experiment and variations confirm ambiguity aversion. How do we reconcile our result with the “robust” ambiguity aversion finding in psychology?

Note that the interpretation of ambiguity loving in auction settings is not exactly the same as ambiguity loving in individual choice experiments, such as the Ellsberg experiment. In our auction setting, when the true underlying distribution is unknown, a bidder might be *ex ante* pessimistic in thinking that his own valuations are more likely to be drawn from the low value distribution. In a symmetric environment, since the opponent is just like himself, the same bidder might conclude that his opponent’s values are also more likely to be drawn from the low value distribution. This naive application of pessimistic reasoning implies ambiguity loving behavior.¹⁴ By contrast, in an Ellsberg urn experiment, ambiguity loving implies a preference for the unknown urn when choosing between known and unknown urns, or optimism when missing information. Since the literature on the psychological causes of ambiguity aversion focuses almost exclusively

¹² To see this, note that if $\theta > 1.2$, then $\alpha\bar{\delta} + (1 - \alpha)\bar{\delta} > 0.7$. Under our assumption, $(\bar{\delta} + \bar{\delta})/2 \leq 0.7$. So when $\alpha = \frac{1}{2}$, $\alpha\bar{\delta} + (1 - \alpha)\bar{\delta} \leq 0.7$. Moreover, $\alpha\bar{\delta} + (1 - \alpha)\bar{\delta}$ is decreasing in α . Together, these facts imply that $\alpha < \frac{1}{2}$.

¹³ To see this, suppose $[\underline{\delta}, \bar{\delta}] = [0.3, 0.5]$, and suppose $\alpha = 0$, which corresponds to ambiguity loving. Then $\theta = 1 < 1.2$, and hence such a bidder would increase his bid in the ambiguity treatment, even though he is ambiguity loving.

¹⁴ We thank an anonymous referee for suggesting this explanation.

Table 2
Comparison of bids with and without ambiguity

Round	All V		$0 \leq V \leq 0.5$		$0.5 < V \leq 1$	
1	0.504	(40,40)	0.204	(26,19)	0.022	(14,21)
2–30	0.133	(5,5)	0.133	(5,5)	0.183	(5,5)
1–3	0.006	(5,5)	0.062	(5,5)	0.540	(5,5)
4–30	0.310	(5,5)	0.183	(5,5)	0.133	(5,5)
1–5	0.012	(5,5)	0.038	(5,5)	0.310	(5,5)
6–30	0.242	(5,5)	0.310	(5,5)	0.133	(5,5)
1–30	0.133	(5,5)	0.183	(5,5)	0.183	(5,5)

Notes:

- (1) The table lists one-sided p -values for the Wilcoxon ranksum tests that bidders bid more under the known distribution than under an unknown distribution of valuations.
- (2) To assure independence of individual observations, first-period tests only use all the observations individually, while all the other tests use session means. Number of independent observations under the known and unknown distribution is listed in parentheses for each test.

on individual choice experiments (see, e.g., [6,9]), this naive application of pessimistic reasoning should be verified in future experiments on ambiguity using a variety of different contexts.

4.2. Structural estimation of the ambiguity parameter in first price auctions

In the previous subsection, we determined that bidders are ambiguity loving from comparison of bids in the two treatments. To get an idea of the magnitude of the ambiguity parameter, θ , we now use the structural approach to directly estimate θ in the α -MEU framework. As is common in the structural approach, we need additional assumptions to make the model tractable. Our first assumption is that an ambiguity neutral bidder will use the uniform prior in the ambiguity treatment, i.e., $\underline{\delta} + \bar{\delta} = 1$. As a result, Eq. (8) implies that $\theta < 1$ corresponds to ambiguity aversion, $\theta = 1$ corresponds to ambiguity neutrality, and $\theta > 1$ corresponds to ambiguity loving. Our second assumption is that bidders have constant relative risk averse (CRRA) utility functions of the form $u(x) = x^\beta$, where $\beta > 0$. While there has been no consensus on the right model for bidder behavior in first price auctions (see [16] and Cox (forthcoming) for surveys of this research), we choose to use CRRA due to its analytical tractability. Because of these assumptions, results on the magnitude of θ should be taken with caution. We now compute the equilibrium bidding strategies for an α -MEU bidder with a CRRA utility function, using Proposition 1.

Corollary 2. *With the parameterized distribution functions F^1 and F^2 , the equilibrium bidding strategy for a bidder with a CRRA utility function is characterized by*

$$s(V, \theta) = \begin{cases} \frac{V}{1 + \beta} & \text{if } 0 \leq V \leq \frac{1}{2}, \\ \frac{V(\theta - 2) + \beta(\theta - 1)}{(\theta - 2)(1 + \beta)} + \frac{\beta}{1 + \beta} \frac{\theta - 1}{2 - \theta} \left(\frac{\theta}{2}\right)^{\frac{1}{\beta}} [\theta - 1 + (2 - \theta)V]^{-\frac{1}{\beta}} & \text{if } \frac{1}{2} < V \leq 1. \end{cases} \tag{11}$$

Proof. See Appendix.

We use Corollary 2 to estimate the risk parameter, β , and ambiguity parameter, θ . In the treatment with a known distribution (K1), ambiguity does not play a role, as bidders know the value of δ . We use this treatment to estimate the bidders' risk attitude.

We make a simplifying assumption that, within the same treatment, the risk parameter is common and known across individuals. Allowing heterogeneous risk parameters across individuals would clearly fit the data better. However, one has to resort to the computational approach, which requires making *ad hoc* assumptions about the distribution of risk parameters in the population as well as about independence across individuals and rounds within the same session. Since our main goal is to separate the effects of risk from ambiguity, we assume symmetric bidders to get closed form solutions without distributional assumptions. Moreover, we believe that the main conclusions would remain unchanged even with heterogeneity in risk preferences. Thus, we estimate the following econometric model:

$$b_{it} = s(V_{it}, \beta, \theta_0) + \xi_{it}, \quad (12)$$

where $s(\cdot)$ is the bidding function characterized in Corollary 2; b_{it} is the bid submitted by bidder i at round t ; V_{it} is the private valuation of bidder i at round t ; β is the risk parameter; $\theta_0 = 1.2$; and ξ_{it} is the error term assumed to be orthogonal to the valuation, i.e., $E(\xi_{it}|V_{it}) = 0$. The method of nonlinear least squares is used for parameter estimations. In all estimations, standard errors and confidence intervals are computed by bootstrapping and are adjusted for clustering at the session level, implying that ξ_{it} is allowed to be heteroscedastic, and correlated across both individuals and rounds, but is independent across sessions. We use the bootstrap procedure to avoid distributional assumptions on ξ_{it} or relying on asymptotic distribution theory.

Table 3 reports the estimates of β for treatment K1. For each treatment, we first conduct a baseline estimation of β with the restriction that $\theta = 1.2$. We then repeat the same estimation separately for different subranges of valuations to evaluate the sensitivity of the estimate of β , since the bidding function has a different functional form for each subrange. Finally, we run a control estimation which jointly estimates β and θ . In the control estimation of both treatments, $\theta = 1.2$ lies within the 95% confidence interval, thus justifying the $\theta = 1.2$ restriction in the known distribution treatment. The estimated bidder risk parameter is $\beta = 0.3622$. This estimated risk parameter is consistent with recent estimates in private-value auction experiments, such as 0.33 [4], [0.35, 0.71] [2] and 0.48 [13].

In subsequent analysis, we use the estimated $\beta = 0.3622$ to isolate the effects of risk and ambiguity. As a robustness check, we repeat all the subsequent estimation procedures for $\underline{\beta} = 0.32$ and $\bar{\beta} = 0.42$, which are reasonable lower and upper bounds based on the estimates of β for different subranges of valuations and their respective confidence intervals reported in Table 3.

We now estimate θ using Corollary 2, with the modification of allowing θ to vary over time but not over bidders. More specifically, we let θ be a cubic polynomial of time to partially capture the effects of updating.

Fig. 3 presents estimated time paths of θ , together with their bootstrapped confidence intervals, with adjustment for clustering at the session level in the treatment with unknown distributions (U1). The top left graph uses the baseline estimates of the risk parameter β from the corresponding treatment with known distributions. The top right and bottom graph serve as robustness checks by using the corresponding lower and upper bounds of β , respectively. In all three graphs, the estimated ambiguity parameter θ is at least one, suggesting that bidders are ambiguity loving.

Table 3
Estimation of bidders' risk parameter (β)

Restriction on θ	Sample	Obs.	β coefficient	Std. error	95% confidence interval	
$\theta = 1.2$	All values	1200	0.3622	0.0242	0.3199	0.4160
N/A	$V_{it} \leq 0.5$	742	0.3573	0.0191	0.3169	0.3900
$\theta = 1.2$	$V_{it} > 0.5$	458	0.3633	0.0262	0.3185	0.4234
Unrestricted	All values	1200	0.3313 ($\theta = 1.288$)	0.0203 0.0549	0.2863 1.1809	0.3625 1.3914

Note: All standard errors and confidence intervals are bootstrapped with adjustment for clustering at session level.

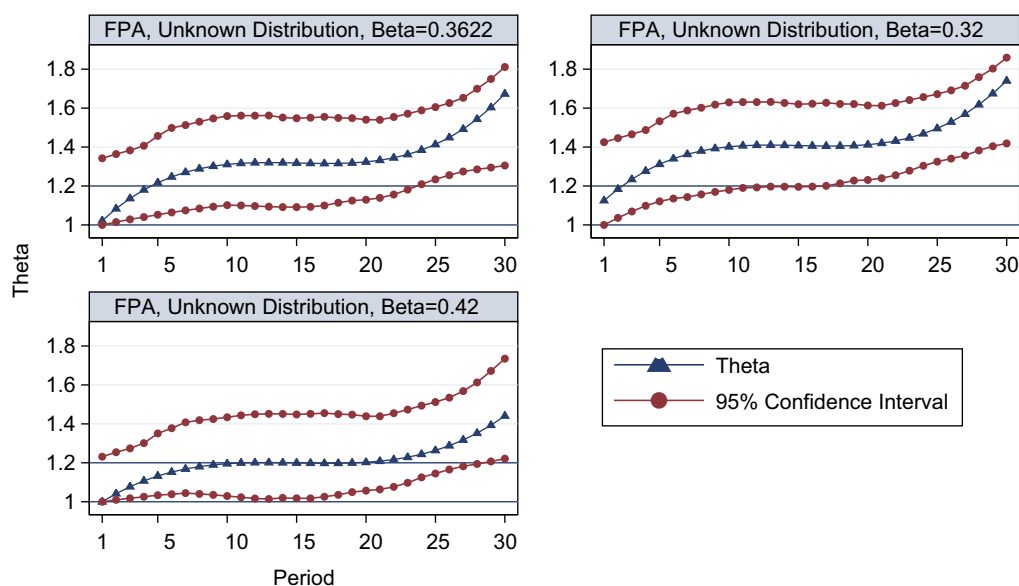


Fig. 3. Estimated ambiguity parameter θ in U1 treatments.

Result 2 (Estimation of the ambiguity parameter θ). In all rounds, but particularly in the early rounds (1–5), the estimated ambiguity parameter θ is at least one, with the lower boundaries of all confidence intervals being at least one. This rejects ambiguity aversion. Starting from round 2, both ambiguity aversion and ambiguity neutrality are rejected in favor of ambiguity loving.

Support: In all three graphs of Fig. 3, we see that the estimated θ is at least one. Furthermore, the lower boundaries of all confidence intervals are at least one.

Result 2 confirms Result 1 that our data are consistent with ambiguity loving in first price auctions. Apart from the two assumptions discussed earlier, the structural estimation restricts the ambiguity parameter θ to be the same across individuals in any given round. In an exercise not presented here due to space constraints, we also relax this assumption by modelling individual

learning using the SEU model. We find that the mean of the estimated prior distribution of δ is 0.8438, which suggests that bidders put more than 0.5 weight on the low value distribution.¹⁵

To summarize, we have used two different approaches to determine bidders' ambiguity attitude. The first approach compares the distribution of (mean) bids in treatments with and without ambiguity and finds that bids are lower in the treatment with ambiguity, which is consistent with ambiguity loving. The second approach estimates the ambiguity parameter to be at least one, rejecting ambiguity aversion. Combining both approaches, we conclude that ambiguity affects bidder behavior in the first price auction in our experimental setting, and our data are consistent with the hypothesis that bidders are ambiguity loving.

4.3. Second price auctions

For the second price auction, we use a structural approach based on Proposition 4, which states that bidding one's true valuation is a weakly dominant strategy with or without ambiguity. To test this hypothesis, we use an OLS regression with clustering at the session level. We use Bid as the dependent variable, and Value as the only independent variable. We do not include a constant because of the theoretical prediction. We conduct the estimation on treatments with known and unknown distributions for both the early (1–5 and 1–10) and later rounds (11–30). We combine both the known and unknown treatments in one regression to gain additional efficiency. Results are presented in Table 4.

Result 3 (*Effects of ambiguity in second price auctions*). *Ambiguity has no significant effect on bids in earlier rounds or later rounds. However, in rounds 1–10 of the known treatment and rounds 11–30 of both treatments, subjects bid significantly higher than their valuations.*

Support: Table 4 presents the OLS regression results for second price auctions. The coefficient estimates show how much subjects bid compared to their valuations. The standard errors are in parentheses. The asterisks next to the standard errors indicate the significance levels in two-sided Wald tests of the null hypothesis of bids being equal to values against the alternative hypothesis of bids not equal to values. The null hypothesis is rejected at the 5% significance level in rounds 1–10 of the known treatment and rounds 11–30 of both treatments. The last line of the table displays the Wald χ^2 statistics for the equality of coefficients between the known and unknown treatments for the early and later rounds, respectively. None of these statistics is significant at the 10% significance level.

The finding that ambiguity has no effects on bidding behavior in second price auctions confirms our theoretical prediction. The finding that participants overbid is consistent with previous experimental findings [17]. Interestingly, the extent of overbidding increases in later rounds, which not only confirms that participants do not seem to learn the dominant strategy, but also indicates that they depart further from the dominant strategy in later rounds.

4.4. Revenue, earnings and efficiency

In this subsection, we present aggregate results. Specifically, we examine the effects of the auction mechanisms (first vs. second price auctions) and information conditions (ambiguity vs. no-ambiguity treatments) on revenue, bidder earnings and overall auction efficiency.

¹⁵ The theoretical derivation and estimation results are available from the authors upon requests.

Table 4
Effects of ambiguity on bids in the second price auction

	Dependent variable: bid in second price auction		
	Rounds 1–5	Rounds 1–10	Rounds 11–30
Value (known case)	1.0341 (0.0139)**	1.0589 (0.0155)***	1.0882 (0.0324)***
Value (unknown case)	1.0269 (0.0154)	1.0453 (0.0143)***	1.0721 (0.0107)***
Observations	400	800	1,600
Test of known = unknown p -value of $\chi^2(1)$	0.7296	0.5172	0.6354

Notes:

- (1) Standard errors in parentheses are adjusted for clustering at the session level.
- (2) The asterisks next to the standard errors display significance in two-sided tests of the null hypothesis of the coefficient being unity against the alternative hypothesis of the coefficient not being equal to unity.
- (3) Significant at * 10% level; ** 5% level; *** 1% level.

Table 5
Average revenue and results of permutation tests (one-tailed)

Rounds 1–5	Session 1	Session 2	Session 3	Session 4	Session 5	H_1	p -value
$K1$	0.4665	0.4685	0.4235	0.5170	0.5485	$K1 > K2$	0.0040***
$U1$	0.3705	0.4795	0.4280	0.4420	0.3905	$U1 > U2$	0.0556*
$K2$	0.2815	0.2665	0.2600	0.3825	0.3795	$K1 > U1$	0.0397**
$U2$	0.2935	0.3870	0.4175	0.3130	0.3990	$K2 < U2$	0.0992*
Rounds 1–30	Session 1	Session 2	Session 3	Session 4	Session 5	H_1	p -value
$K1$	0.4459	0.3869	0.4443	0.4648	0.4559	$K1 > K2$	0.0079***
$U1$	0.3638	0.4419	0.4255	0.4277	0.4499	$U1 > U2$	0.0159**
$K2$	0.3335	0.3265	0.3423	0.3948	0.3506	$K1 > U1$	0.2341
$U2$	0.2953	0.3653	0.3628	0.3131	0.3588	$K2 > U2$	0.3730

Notes:

- (1) The null hypothesis is that the average revenue is equal in the two treatments.
- (2) Significant at * 10% level; ** 5% level; *** 1% level.

Result 4 (Revenue). *With or without ambiguity, the first price auction generates significantly higher revenue than the second price auction. In the early rounds of the first price auction, revenue is significantly higher without ambiguity.*

Support: Table 5 presents the average revenue in the early rounds (1–5) and over all 30 rounds for each session in each treatment. The last two columns report the alternative hypotheses and results of the one-tailed permutation tests for the effects of auction mechanisms and information conditions.

Result 4 is consistent with theory. The Revenue Equivalence Theorem states that, without ambiguity and with risk neutrality, first and second price auctions generate the same expected revenue. With risk aversion, bidders bid more in the first price auction but not in the second price

Table 6
Efficiency and results of permutation tests (one-tailed)

Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	H_1	p -value
$K1$	0.8667	0.9083	0.9167	0.9083	0.8750	$K1 > K2$	0.3373
$U1$	0.8833	0.8750	0.9000	0.8917	0.9083	$U1 > U2$	0.3214
$K2$	0.8583	0.9167	0.8917	0.8833	0.9000	$K1 > U1$	0.3810
$U2$	0.9333	0.7833	0.8250	0.9000	0.9417	$K2 > U2$	0.3413

Notes:

- (1) The null hypothesis is that efficiency is equal in the two treatments.
 (2) Significant at: * 10% level; ** 5% level; *** 1% level.

auction; therefore, we obtain the usual result that the first price auction generates more revenue than the second price auction. This results also holds when ambiguity is introduced.

In addition, we also observe that, in the early rounds of the first price auction, revenue is significantly lower when ambiguity is introduced, a consequence of ambiguity-loving bidders. In the second price auction, ambiguity does not affect revenue over all rounds, which is consistent with theory.

Closely related to auctioneer revenue are bidder earnings. We expect auction mechanisms and information conditions to have opposite effects on bidder earnings compared to auctioneer revenue. Indeed, we find that bidder earnings are significantly higher in the second price auction compared to the first price auction with or without ambiguity ($p < 0.01$ for one-tailed permutation tests).¹⁶

The last group level result we examine is efficiency. Following the tradition in the auction literature, we define efficiency as equal to 100% if the object goes to the bidder with the higher valuation. We therefore use the frequency with which the bidder with the higher valuation wins the object as our measure of efficiency.

Table 6 presents the average efficiency for each session in each treatment and the results of the one-sided permutation tests. Theoretically, both first and second price auctions should yield 100% efficiency under a zero reserve price. We find that average efficiency is fairly close to 90%, which is largely consistent with previous experiments.

5. Conclusions

In many real-world auctions, such as Internet auctions, bidder information regarding other bidders' valuations is vague. To explore the effect of this vagueness on bidder behavior, we study the first and second price sealed bid auctions with independent private values, where the distribution of bidder valuation is *not* known. We derive the symmetric equilibrium using the α -MEU framework. We then test our theoretical predictions to examine how ambiguity affects bidder behavior and to reassess the ranking of the first and second price sealed bid auctions.

Previous experimental studies on ambiguity mostly focus on Ellsberg individual choice experiments, while previous auction experiments mostly assume that the distribution of bidder valuations is common knowledge. Our study extends the experimental auction literature to a more realistic setting with ambiguity. It also extends studies of ambiguity to an important applied

¹⁶ Details are available from the authors upon request.

setting, to determine whether findings from individual choice experiments are robust in the auction context.

We show that ambiguity affects bidder behavior in the first price auction. Contrary to the results of many previous studies in Ellsberg urn experiments, in our experimental auction setting, in the first price auction, bids are lower with the presence of ambiguity. This result is consistent with ambiguity loving in a model which allows for different ambiguity attitudes. At the aggregate level, we also find that the first price auction generates significantly higher revenue than the second price auction with and without ambiguity.

These findings have important implications for auction design in settings with ambiguity. Our results suggest that from the revenue perspective, the designer ought to choose the first price auction. Another practical implication is that a reduction in ambiguity can lead to an increase in revenue.

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Appendix A. Derivation of Eq. (2)

Conditional on $\delta \in [0, 1]$, the distribution of the opponent’s valuations is given by $\delta F^1 + (1 - \delta) F^2$. Then, in light of the α -MEU theory, bidder i ’s utility is a weighted average of the utility of a maxmin EU bidder (weight α) and a maxmax EU bidder (weight $1 - \alpha$), where the set of beliefs over δ is given by Δ . Then, conditional on the opponent strategy being s_j and using the shorthand notation π_i for $\pi_i [V_i, b_i, s_j(V_j)]$, the bidder i ’s payoff $U(b_i; V_i, s_j)$ is given by

$$\begin{aligned}
 U_i(b_i; V_i, s_j) &= \alpha \min_{G \in \Delta} \left\{ \int_0^1 \int_0^1 u(\pi_i) d \left[\delta F^1(V_j) + (1 - \delta) F^2(V_j) \right] dG(\delta) \right\} \\
 &\quad + (1 - \alpha) \max_{G \in \Delta} \left\{ \int_0^1 \int_0^1 u(\pi_i) d \left[\delta F^1(V_j) + (1 - \delta) F^2(V_j) \right] dG(\delta) \right\} \\
 &= \alpha \min_{G \in \Delta} \left\{ \left(\int_0^1 \delta dG(\delta) \right) \left[\int_0^1 u(\pi_i) dF^1(V_j) \right] \right. \\
 &\quad \left. + \left(\int_0^1 (1 - \delta) dG(\delta) \right) \left[\int_0^1 u(\pi_i) dF^2(V_j) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + (1 - \alpha) \left\{ \max_{G \in \Delta} \left(\int_0^1 \delta dG(\delta) \right) \left[\int_0^1 u(\pi_i) dF^1(V_j) \right] \right. \\
 & \left. + \left(\int_0^1 (1 - \delta) dG(\delta) \right) \left[\int_0^1 u(\pi_i) dF^2(V_j) \right] \right\} \\
 & = \alpha \left\{ \underline{\delta} \left[\int_0^1 u(\pi_i) dF^1(V_j) \right] + (1 - \underline{\delta}) \left[\int_0^1 u(\pi_i) dF^2(V_j) \right] \right\} \\
 & \quad + (1 - \alpha) \left\{ \bar{\delta} \left[\int_0^1 u(\pi_i) dF^1(V_j) \right] + (1 - \bar{\delta}) \left[\int_0^1 u(\pi_i) dF^2(V_j) \right] \right\} \\
 & = \int_0^1 u(\pi_i) dF_\alpha(V_j) \\
 & = u(V_i - b_i) F_\alpha[s_j^{-1}(b_i)],
 \end{aligned}$$

where $F_\alpha = (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta}) F^1 + [1 - (\alpha \underline{\delta} + (1 - \alpha) \bar{\delta})] F^2$.

Proof of Proposition 1. By (2), bidder i solves

$$s_i(V_i) \in \arg \max_{b_i \in [0, \infty)} u(V_i - b_i) F_\alpha[s_j^{-1}(b_i)].$$

We know that $s_i(0) = 0$ since bidding above zero leads to negative utility for $V_i = 0$. When $V_i > 0$, we have $0 < s_i(V) < V_i$ and the bidding function of bidder i is characterized by the following first order condition:

$$-u' [V_i - s_i(V_i)] F_\alpha \left\{ s_j^{-1} [s_i(V_i)] \right\} + \frac{u [V_i - s_i(V_i)] F'_\alpha \left\{ s_j^{-1} [s_i(V_i)] \right\}}{\frac{\partial}{\partial V_j} s_j \left(s_j^{-1} [s_i(V_i)] \right)} = 0.$$

In a symmetric equilibrium $s_i = s_j = s$, and hence it follows that if $V > 0$,

$$-u' [V - s(V)] F_\alpha(V) + \frac{u [V - s(V)] F'_\alpha(V)}{\frac{\partial s(V)}{\partial V}} = 0,$$

which can be rewritten as

$$\frac{\partial s(V)}{\partial V} = \frac{F'_\alpha(V) u [V - s(V)]}{F_\alpha(V) u' [V - s(V)]}. \quad \square$$

Proof of Proposition 3. First, for $0 < V \leq \frac{1}{2}$, Corollary 1 shows that

$$\frac{\partial s(V, \theta)}{\partial V} = \frac{g [V - s(V, \theta)]}{V}.$$

The solution, $s(V, \theta)$, of the above differential equation does not depend on θ and hence the functional form of $s(V, \theta)$ is independent of θ .

Now consider all V such that $\frac{1}{2} < V \leq 1$. Corollary 1 shows that for this range of values

$$\frac{\partial s}{\partial V}(V, \theta) = g[V - s(V, \theta)]h(V, \theta). \tag{13}$$

Suppose, by contradiction, that there exist $V_0 \in (\frac{1}{2}, 1]$ and $\theta_1, \theta_2 \in [0.5, 1.5], \theta_1 < \theta_2$, such that $s(V_0, \theta_1) < s(V_0, \theta_2)$. Define the set M as

$$M \equiv \{V \in (\frac{1}{2}, V_0) : s(V, \theta_1) = s(V, \theta_2)\} \cup \{\frac{1}{2}\}.$$

By continuity of $s(\cdot, \theta)$, M is a compact set, and hence $m \equiv \max(M)$ is well defined. Continuity also implies that $s(V, \theta_1) < s(V, \theta_2)$ for all $V \in (m, V_0]$. But because $g(\cdot)$ is strictly increasing, $h(V, \theta)$ is strictly decreasing in θ , and, by construction, $s(m, \theta_1) = s(m, \theta_2)$, it follows from (13) that

$$\begin{aligned} s(V_0, \theta_1) &= s(m, \theta_1) + \int_m^{V_0} g[V - s(V, \theta_1)]h(V, \theta_1) dV \\ &> s(m, \theta_2) + \int_m^{V_0} g[V - s(V, \theta_2)]h(V, \theta_2) dV \\ &= s(V_0, \theta_2), \end{aligned}$$

which is a contradiction. Therefore it must be the case that $s(V, \theta_1) \geq s(V, \theta_2)$ for all $V \in (\frac{1}{2}, 1]$ and $\theta_1, \theta_2 \in [0.5, 1.5], \theta_1 < \theta_2$.

Now suppose by contradiction that there exists $V_0 \in (\frac{1}{2}, 1]$ and $\theta_1, \theta_2 \in [0.5, 1.5], \theta_1 < \theta_2$, such that $s(V_0, \theta_1) = s(V_0, \theta_2)$. Since $h(V, \theta)$ is continuous, positive, and strictly decreasing in θ , there must exist $\varepsilon > 0$ and $\gamma > 0$ such that

$$\frac{h(V, \theta_2)}{h(V, \theta_1)} < \frac{1}{1 + \varepsilon} \quad \text{for all } V \in (V_0 - \gamma, V_0).$$

In addition, since $s(V, \theta)$ is continuous in V , $g(\cdot)$ is continuous, positive, and strictly increasing, $s(V, \theta_1) \geq s(V, \theta_2)$ for all $V \in (\frac{1}{2}, V_0]$, there must exist $\delta > 0$ such that

$$\frac{g[V - s(V, \theta_2)]}{g[V - s(V, \theta_1)]} < 1 + \varepsilon \quad \text{for all } V \in (V_0 - \delta, V_0).$$

But then it follows that

$$g[V - s(V, \theta_2)]h(V, \theta_2) < g[V - s(V, \theta_1)]h(V, \theta_1) \quad \text{for all } V \in (V_0 - \min(\delta, \gamma), V_0).$$

This result, combined with the fact that $s[V_0 - \min(\delta, \gamma), \theta_1] \geq s[V_0 - \min(\delta, \gamma), \theta_2]$, implies that

$$\begin{aligned} s(V_0, \theta_1) &= s[V_0 - \min(\delta, \gamma), \theta_1] + \int_{V_0 - \min(\delta, \gamma)}^{V_0} g[V - s(V, \theta_1)]h(V, \theta_1) dV \\ &> s[V_0 - \min(\delta, \gamma), \theta_2] + \int_{V_0 - \min(\delta, \gamma)}^{V_0} g[V - s(V, \theta_2)]h(V, \theta_2) dV \\ &= s(V_0, \theta_2), \end{aligned}$$

which is a contradiction. Therefore it must be the case that $s(V, \theta_1) > s(V, \theta_2)$ for all $V \in (0.5, 1]$ and $\theta_1, \theta_2 \in [0.5, 1.5], \theta_1 < \theta_2$, meaning that $s(V, \theta_1)$ is strictly decreasing in θ when $V \in (0.5, 1]$. \square

Proof of Corollary 2. Substituting Eq. (7) into Eq. (4) gives

$$\frac{\partial s(V, \theta)}{\partial V} = \begin{cases} \frac{1}{\beta} \frac{V - s(V)}{V} & \text{if } 0 \leq V \leq \frac{1}{2}, \\ \frac{1}{\beta} [V - s(V)] \frac{2 - \theta}{\theta - 1 + (2 - \theta)V} & \text{if } \frac{1}{2} < V \leq 1. \end{cases}$$

The solution to this differential equation is

$$s(V, \theta) = \begin{cases} c_1 V^{-\frac{1}{\beta}} + \frac{V}{1 + \beta} & \text{if } 0 \leq V \leq \frac{1}{2}, \\ \frac{V(\theta - 2) + \beta(\theta - 1)}{(\theta - 2)(1 + \beta)} + c_2 [\theta - 1 + (2 - \theta)V]^{-\frac{1}{\beta}} & \text{if } \frac{1}{2} < V \leq 1. \end{cases}$$

Since $s(0, \theta) = 0$, we have $c_1 = 0$. By continuity at $V = \frac{1}{2}$,

$$\frac{1}{2(1 + \beta)} = \frac{1}{2(1 + \beta)} + \frac{\beta}{1 + \beta} \frac{\theta - 1}{\theta - 2} + c_2 \left(\frac{\theta}{2}\right)^{-\frac{1}{\beta}}$$

implying

$$c_2 = \frac{\beta}{1 + \beta} \frac{\theta - 1}{2 - \theta} \left(\frac{\theta}{2}\right)^{\frac{1}{\beta}}.$$

So we can write the bidding function as follows:

$$s(V, \theta) = \begin{cases} \frac{V}{1 + \beta} & \text{if } 0 \leq V \leq \frac{1}{2}, \\ \frac{V(\theta - 2) + \beta(\theta - 1)}{(\theta - 2)(1 + \beta)} + \frac{\beta}{1 + \beta} \frac{\theta - 1}{2 - \theta} \left(\frac{\theta}{2}\right)^{\frac{1}{\beta}} [\theta - 1 + (2 - \theta)V]^{-\frac{1}{\beta}} & \text{if } \frac{1}{2} < V \leq 1. \end{cases}$$

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