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An experimental study of serial and average cost pricing mechanisms

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Abstract

This paper reports the first experimental study of the serial and the average cost pricing mechanisms under five different treatments: a complete information treatment and four treatments designed to simulate distributed systems with extremely limited information, synchronous and asynchronous moves. Although the proportion of Nash equilibrium play under both mechanisms is statistically indistinguishable under complete information, the serial mechanism performs robustly better than the average cost pricing mechanism in distributed systems, both in terms of the proportion of equilibrium play and system efficiency.

Keywords: Serial mechanism; Asynchrony; Experiment

JEL classification: C91; D83

1. Introduction

In a wide variety of real world situations a group of agents share a common production process transforming input into output. Examples of shared resources include computing facilities, secretarial support and lab facilities within an organization. A cost-sharing mechanism distributes the service and allocates the corresponding costs to each agent. Two prominent cost-sharing mechanisms are

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the serial mechanism (Shenker, 1990; Moulin and Shenker, 1992) and the average cost pricing mechanism (see Tauman (1988) for a survey). We will use two examples to illustrate how the mechanisms work.[1]

We first look at a group of ranchers who might share the cost of constructing and maintaining an irrigation network. Aadland and Kolpin (1998) provide an empirical and axiomatic analysis of cost-sharing arrangements of irrigation ditches located in south-central Montana. In their sample, a typical ditch begins at the headgate and then continues on a sequential path through the lands of each rancher using the main ditch. Ranchers' private ditches branch off from the main ditch and transport water to their land. The costs associated with the main ditch are shared among the ranchers. Kolpin and Aadland (2001) find that the cost sharing rules employed on these ditches are variations of the average and serial cost sharing mechanisms. A rule is in the average class if all agents pay according to an identical fixed 'rate', which may be defined on a per capita basis, per irrigated acre basis, etc. A serial rule partitions the ditch into 'a sequence of segments such that all agents require the first segment to be operational in order to receive water, all but the first agent on the ditch additionally require the second segment to be operational, ... Each segment is then treated like a separate ditch whose costs are covered by having all agents requiring its use pay an identical fixed rate' (Kolpin and Aadland, 2001). An agent's total cost share is the sum of his obligations on each of these individual segments. This example provides a more traditional setting where the ranchers know the rules of the game as well as each other's demand fairly well.

A more recent example is provided by the Internet, which has becoming increasingly important in global telecommunications. In the context of several agents sharing a network link, the cost to be shared is congestion experienced. Each agent controls the rate at which she is transmitting data. If the sum of the transmission rates is greater than the total link capacity, then the link becomes congested and the agents' packets experience delays. Most current Internet routers use a FIFO packet scheduling algorithm, which results in each agent's average queue proportional to her transmission rate. This corresponds to the average cost pricing mechanism (Shenker, 1990). In contrast, the Fair Queueing packet scheduling algorithm, which corresponds to the serial mechanism, leads to congestion allocations such that an agent's average queue is independent of transmission rates higher than her own. The latter has been proposed as an alternative to the former, based on theoretical and simulation results (Stoica et al., 1998). The new generation of Cisco 7200, 3600 and 2600 routers have both the FIFO and Fair Queueing options.[2] This paper presents the first experimental study

[1] A formal definition of each mechanism is provided in Section 2.

[2] http://www.cisco.com/warp/public/121/7200_per-vc-CBWFQ.htm

of the performance of the two mechanisms in both the traditional complete information setting and the Internet setting.

Most of the theoretical literature has focused on the axiomatic characterization of these mechanisms (e.g. Moulin and Shenker, 1994; Friedman and Moulin, 1999) and their static properties in a complete information setting with synchronous actions. However, as Friedman and Shenker (1998) pointed out, in a *distributed system*[3] such as the Internet where agents have very limited a priori information about other agents and the payoff structure and where there is no synchronization of actions, traditional solution concepts that we use to characterize these mechanisms, such as Nash equilibrium or even the serially undominated set,[4] might not be achieved as a result of learning. They propose new solution concepts for distributed systems describing convergence for learning algorithms satisfying certain theoretical properties.

Although Friedman and Shenker (1998) used the Internet as the context for their new theory, limited information and asynchrony are more realistic assumptions than complete information and synchronous play in many real economic situations. To my best knowledge there have not been experimental studies that incorporate both limited information and asynchrony to study implementation. The average cost pricing mechanism has not been studied in the laboratory either. The only other experimental study of the serial mechanism is by Dorsey and Razzolini (1999). They investigate the performance of the serial mechanism with each human subject against three computerized players, where each human player knows his own cost share and payoff structure but has no information about the opponents' payoff structures. Their information condition is in between the complete information and limited information setting in this study. They do not consider the performance of the mechanism under limited information or asynchrony.

In this paper I design an experiment to evaluate the serial and the average cost pricing mechanism in an easy environment with complete information, and more challenging environments with extremely limited information and asynchronous moves. The goal of this paper is to compare the performance of the two mechanisms in various settings and to assess the plausibility of the new solution concepts.

The paper is organized as follows. Section 2 introduces the theoretical properties of the serial (sometimes shortened as SER) and average cost pricing (hereafter shortened as ACP) mechanisms. Section 3 presents the experimental

[3]Following Friedman and Shenker (1998), a system is called a *distributed system* 'because the users are geographically dispersed and are accessing the resource through the network'. The Internet is a prominent example.

[4]The serially undominated set is the set of outcomes of a strategic game that survives iterated elimination of strictly dominated actions.

design. Section 4 compares the performance of the mechanisms under complete information. Section 5 presents results in distributed systems with limited information and asychrony. Section 6 discusses the robustness of the experimental results with respect to changes in the environment. Section 7 concludes the paper.

2. The serial and ACP mechanisms—theoretical properties

Suppose a group of n agents share a one-input, one-output technology with decreasing returns. Each of the n agents announces his demand q_i of output. Each agent gets her demand q_i and pays a cost share, x_i. Note x_i is the total cost agent i pays. In the irrigation example, q_i corresponds to the total amount of maintenance of the main ditch demanded by agent i, while x_i is what agent i pays to get the maintenance done. In the example of Internet routers, q_i is agent i's data transmission rate, while x_i is the congestion, i.e. the average queue experienced by agent i. Therefore, x_i is the reduction in agent i's utility due to congestion. Let $q_1 \leq q_2 \leq \cdots \leq q_n$. The cost function is denoted by C, which is strictly convex. A cost-sharing mechanism must allocate the total cost $C(\Sigma_i \; q_i)$ among the n agents.

The serial mechanism, originally introduced by Shenker (1990), was analyzed by Moulin and Shenker (1992) in the context of cost and surplus sharing with complete information. The mechanism can be characterized by four properties: unique Nash equilibrium at all profiles,[5] anonymity (the name of the agents does not matter), monotonicity (an agent's cost share increases when she demands more output) and smoothness (an agent's cost share is a continuously differentiable function of the vector of demands). Among agents endowed with convex, continuous and monotonic preferences, the serial mechanism is the only cost sharing rule which is dominance-solvable and its unique Nash equilibrium is also robust to coalitional deviations when agents cannot transfer outputs.

Under the serial mechanism, agent 1 (with the lowest demand) pays $(1/n)$th of the cost of producing nq_1. Agent 2 pays agent 1's cost share plus $1/(n-1)$th of the incremental cost from nq_1 to $(n-1)q_2 + q_1$, i.e.

$$x_1^S(c, q_1) = C(nq_1)/n; \text{ and}$$

$$x_2^S(c, q_1, q_2) = \frac{C(nq_1)}{n} + \frac{C(q_1 + (n-1)\, q_2) - C(nq_1)}{n-1}.$$

And so on. Therefore, an agent's cost share under the serial mechanism is only affected by her own demand and those whose demands are lower than hers. That is, an agent's cost share is independent of demands higher than her own.

[5] Assume agents have convex, continuous and monotonic preferences.

Like the serial mechanism, the average cost pricing mechanism satisfies anonymity, monotonicity and smoothness. It is the only method that is robust to arbitrage, i.e. agents cannot benefit from merging or splitting their demands. In contrast to the serial mechanism, the normal form game induced by the average cost pricing mechanism is in general not dominance-solvable, nor does it have a unique equilibrium at all profiles when agents have convex, continuous and monotonic preferences.

When agent i demands q_i amount of output, the general formula for agent i's cost share under the average cost pricing mechanism is given by

$$x_i^A(c, q) = \left(q_i / \sum_k q_k \right) \cdot C\left(\sum_k q_k \right), \quad \text{for all } i = 1, \ldots, n.$$

Therefore, under ACP an agent's cost share is proportional to her demand. It is affected by her own demand, and the sum of all other agents' demands.

There is no systematic efficiency comparison between the two mechanisms. In general there exists no differentiable and monotonic cost sharing mechanism where all Nash equilibrium outcomes are first best Pareto optimal at all preference profiles. Moulin and Shenker (1992) provide a definition of second best efficiency[6] and show that the serial mechanism yields a second best efficient equilibrium while ACP does not.

A particularly interesting question is the performance of the two mechanisms in distributed systems where users are geographically dispersed and are accessing the resource through the network. Friedman and Shenker (1998) address the issue of learning and implementation in distributed systems. They argue that when agents have very limited a priori information about the other players and the payoff structure, standard solution concepts like Nash equilibrium or even the serially undominated set are not necessarily achieved as a result of learning in the network setting. Therefore, new solution concepts, such as the serially unoverwhelmed set and the Stackelberg undominated set are proposed. Loosely speaking, one action *overwhelms* another if all payoffs, over all sets of other players' actions, for the one are greater than all payoffs, over all sets of other players' actions, for the other.[7] Therefore, if action U overwhelms action D, then U dominates D, but the converse is not true

[6] 'For an arbitrary cost sharing mechanism ξ, say that $(q_1, \ldots, q_n)$ is a Nash equilibrium outcome at some utility profile. We ask if there is another vector of demands $(q_1', \ldots, q_n')$ such that at the corresponding allocation dictated by the mechanism ξ, no one is worse off and someone is better off than at the equilibrium allocation corresponding to $(q_1, \ldots, q_n)$. If no such vector of demands exists, we call our equilibrium second best efficient' Moulin and Shenker (1992, p.1025).

[7] See Friedman and Shenker (1998) for a precise definition.

		Player 2 L	Player 2 R
Player 1	U	$\pi_1(UL),\ \pi_2(UL)$	$\pi_1(UR),\ \pi_2(UR)$
	D	$\pi_1(DL),\ \pi_2(DL)$	$\pi_1(DR),\ \pi_2(DR)$

For example, in the above 2×2 game, action U dominates D if $\pi_1(UL) \geq \pi_1(DL)$ and $\pi_1(UR) \geq \pi_1(DR)$; action U overwhelms D if $\min\{\pi_1(UL), \pi_1(UR)\} \geq \max\{\pi_1(DL),\ \pi_1(DR)\}$. The *serially unoverwhelmed set* is the set remaining after iterated elimination of overwhelmed actions. One main result of Friedman and Shenker (1998) is that reasonable learners[8] converge to the serially unoverwhelmed set. In comparison, Milgrom and Roberts (1990) showed that adaptive learners converge to the serially undominated set. A game is *D-solvable* if iterated elimination of dominated strategies leads to a single eventual outcome. A game is *O-solvable* if iterated elimination of overwhelmed strategies leads to a single eventual outcome. Among the cost sharing mechanisms, the serial mechanism is O-solvable[9] while ACP is not.

3. Experimental design

The experimental design reflects both theoretical and technical considerations. The goal of the design is to compare the performance of the serial and ACP mechanisms in two different settings: a complete information setting that tests the prediction of dominance-solvability, and a more challenging network setting to compare the performance of the two mechanisms and to assess the plausibility of the new solution concepts. The economic environment and experimental procedures are discussed in the sections below.

3.1. The economic environment

In a simple environment to test the serial and ACP mechanism under various treatments, agents are endowed with linear preferences $\pi_i(x_i,\ q) = \alpha_i q_i - x_i$, where α_i is agent i's marginal utility for the output, and x_i is her cost share. The cost function is chosen to be quadratic, $C(q) = q^2$. In the network context with several agents sharing a network link, α_i is agent i's value for the amount of data transmitted per unit of time, and the cost to be allocated corresponds to the congestion experienced. Therefore, the cost should be interpreted as the reduction

[8]The key components of a reasonable learner are optimization, monotonicity and responsiveness. See Friedman and Shenker (1998).

[9]This is proved in Theorem 8 in Friedman and Shenker (1998).

in agent i's utility due to congestion. I chose linear utility and quadratic cost functions in order to get a unique interior Nash equilibrium. In Section 6 I present simulation results for more general utility and cost functions.

Consider a two-player game with $\alpha_1 \leq \alpha_2$. Then under the serial mechanism, the cost share for agent 1 is $x_1^S = C(2q_1)/2 = 2q_1^2$. Agent 2 picks up the remaining cost, $x_2^S = C(q_1 + q_2) - C(2q_1)/2 = (q_1 + q_2)^2 - 2q_1^2$. The unique, dominance-solvable Nash equilibrium is thus characterized by

$$q_1^S = \frac{\alpha_1}{4}, \text{ and } q_2^S = \frac{1}{4}(2\alpha_2 - \alpha_1).$$

The Stackelberg equilibrium for the serial game coincides with the above Nash equilibrium.[10]

For the ACP mechanism, the cost shares of the two agents are $x_1^A = q_1/(q_1 + q_2) \cdot C(q_1 + q_2) = q_1(q_1 + q_2)$, and $x_2^A = q_2/(q_1 + q_2) \cdot C(q_1 + q_2) = q_2(q_1 + q_2)$, respectively. Therefore, the unique dominance-solvable Nash equilibrium is characterized by

$$q_1^{A_n} = \frac{2\alpha_1 - \alpha_2}{3}, \text{ and } q_2^{A_n} = \frac{2\alpha_2 - \alpha_1}{3}.$$

Note that the Stackelberg equilibria under ACP usually differ from the Nash equilibrium. The Stackelberg equilibrium with player 2 as the leader is

$$q_1^{A_{s2}} = \frac{3\alpha_1}{4} - \frac{\alpha_2}{2}, \text{ and } q_2^{A_{s2}} = \alpha_2 - \frac{\alpha_1}{2}.$$

In the asynchronous treatment discussed in Section 3.2 player 2 will be the Stackelberg leader.

The mechanisms are implemented as normal form games with a discrete strategy space, $S_i = \{1, 2, \ldots, 11, 12\}$ for each i. Parameters are chosen to ensure: (1) the serial game is both D-solvable and O-solvable, while the ACP game is D-solvable but not O-solvable; (2) the Stackelberg equilibrium and Nash equilibrium under ACP are sufficiently far away from each other; (3) the normal form games with a discrete strategy space have a unique Nash equilibrium under ACP;[11] (4) most of the payoffs are positive in both normal form games. Note that since the stage game under SER and ACP are both dominance-solvable, the equilibrium

[10] This is a general property of the SER mechanism (Corollary 1 to Theorem 1 in Moulin and Shenker (1992)).

[11] I thank Scott Shenker for suggesting using non-integer $\{\alpha_i\}_i$ to avoid the multiple equilibria problem in ACP. In environments with linear preferences and quadratic cost functions even though there exists a unique Nash equilibrium in a continuous strategy space under the ACP mechanism, there are multiple equilibria when the strategy space is discrete and the preference parameters, $\{\alpha_i\}_i$, are integers. The proof is available from the author upon request.

Table 1
Parameters and equilibrium quantities

ID	Parameters	Equilibrium quantities			
		SER		ACP	
	α_i	Nash	Stackelberg (Leader: 1 or 2)	Nash	Stackelberg (Leader: 2)
1 (Blue)	16.1	4	4	4	2
2 (Red)	20.1	6	6	8	12

of the complete information repeated game is simply repeated play of the static equilibrium.

Table 1 reports the parameters and equilibrium quantities for each type of equilibrium for the two mechanisms. In the second column we let $\alpha_1 = 16.1$ and $\alpha_2 = 20.1$. Under the serial mechanism the Nash as well as the Stackelberg equilibrium quantities are (4, 6). Under ACP, the Nash equilibrium quantities are (4, 8), while the Stackelberg equilibrium quantities with player 2 as the leader are (2, 12). Note that we use Blue for player 1 and Red for player 2 in the instructions (see Appendix A).

3.2. Experimental procedures

I implement five different treatments. For a baseline treatment I conducted 12 sessions of the serial and ACP mechanisms under complete information with the round robin design (hereafter shortened as Round Robin). Each session has eight pairs of players. Each of the player 1s is matched with each of the player 2s only once. The entire session lasts for eight rounds. Under the Round Robin treatment, each player is given complete information about the payoff matrix and the structure of the game. They are also given information about quantities chosen and the corresponding payoffs of all players. This treatment is designed to compare the performance of the two mechanisms as one-shot games under complete information. The natural solution concept for this treatment is dominance-solvability.

To evaluate the possibility of applying these mechanisms to distributed systems such as the Internet, I designed four treatments with limited information and various degrees of asynchrony. Learning in distributed systems is characterized by two features. Firstly, players have extremely limited information—they often do not know the payoff functions, nor do they know how their payoffs depend on the actions of others, probably due to the lack of information about the detailed nature of the resources itself. Therefore, in the experimental set-up the only information players have is their own action and the resulting own payoffs. Secondly, there is no synchronization. The rate at which updating occurs can vary by many orders of magnitude. This feature is reflected by the following design: two treatments with synchronous play and updating, two with asynchronous play and updating. In the synchronous treatments (hereafter shortened as SYN) every player receives his own payoff feedback after each round. In the asynchronous treatments (hereafter shortened as ASYN) player 1 submits a demand and gets a payoff feedback after each round, but player 2 submits a demand which is matched with his opponents' demands for the next five rounds and gets a cumulative payoff feedback every five rounds. Therefore, in the asynchronous treatment player 2 acts five times slower than player 1 and becomes the de facto Stackelberg leader.[12]

With both synchronous and asynchronous play, I designed one treatment where players are randomly re-matched into pairs in each of the 150 rounds, and another treatment where players are matched into fixed pairs at the beginning of each session, and play the same partner for 150 rounds. The former captures the inherent randomness in many network settings, while the latter reflects situations with fixed sets of players, such as cost sharing in irrigation ditches (Aadland and Kolpin, 1998). In all four treatments the game lasts for 150 rounds (30 rounds for player 2 in ASYN) and the players always keep their own type. To summarize, I implement the following five different treatments.

1. Round Robin: complete information, round robin.
2. SYN_r: limited information, synchronous play, with random re-matching for each of the 150 rounds.
3. SYN_f: limited information, synchronous play, with repeated fixed pairs for 150 rounds.
4. $ASYN_r$: limited information, asynchronous play, with random re-matching for each of the 150 rounds.
5. $ASYN_f$: limited information, asynchronous play, with repeated pairs for 150 rounds.

Computerized experiments were conducted by the author at the EEPS Laboratory at the California Institute of Technology (hereafter shortened as CIT) in June and July, 1997, and the RCGD Laboratory at the University of Michigan in November, 1999 and January, 2000. Subjects were students and staff from CIT, Pasadena City College (hereafter shortened as PCC) and the University of

[12] Asynchrony as defined by Friedman and Shenker (1998) requires the ratio of expected reaction time of different players to be fixed. Therefore, there can be two different implementations of asynchrony in the experimental setting: a fixed ratio as described above, or more random speed differentials with fixed expected ratio. Both implementations of asynchrony are faithful to the theoretical model. I chose the former because of its simplicity.

Michigan (hereafter shortened as UM).[13] A total of 484 subjects participated in the experiment. No subject was used in more than one session.

Table 2 lists the features of each session, including session number and subject pool, number of subjects in each session, mechanisms implemented, and game length under each treatment. At the beginning of each session subjects randomly drew an identification number. Then each of them was seated in front of the corresponding terminal, with a folder containing the instructions and record sheets. After the instructions were read aloud, subjects were required to finish the Review Questions, which were designed to test their understanding of the instructions. Afterwards the experimenter checked answers and answered questions. In all sessions the instruction period was within 20 min. There was no practice round in any session. The Round Robin sessions consisted of eight rounds and typically lasted for 40 min. The SYN and ASYN sessions consisted of 150 rounds and typically lasted for 1.5 h. The average earnings of experimental subjects was $20.16, not including the $8 participation fee for PCC subjects.[14]

Instructions for the experiments are in Appendix A. Note that in both the SYN

Table 2
Features of experimental sessions

Treatments	Session no. (subject pool)	No. of subjects per session	Mechanisms	Game length
Round Robin	1, 2, 3 (CIT)	16, 16, 16	SER	
	4, 5, 6 (PCC)	16, 16, 16	SER	
	7, 8, 9 (CIT)	16, 16, 16	ACP	8
	10, 11, 12 (PCC)	16, 16, 16	ACP	
Synchronous	13 (CIT), 14 (PCC), 15 (UM)	16, 12, 12	SER	
(fixed pair)	16 (CIT), 17 (PCC), 18 (UM)	14, 12, 12	ACP	
Synchronous	19, 20, 21 (UM)	12, 12, 12	SER	
(random match)	22, 23, 24 (UM)	12, 12, 12	ACP	
Asynchronous	25 (CIT), 26 (PCC), 27 (UM)	10, 12, 12	SER	150
(fixed pair)	28 (CIT), 29 (PCC), 30 (UM)	12, 12, 12	ACP	
Asynchronous	31, 32, 33 (UM)	12, 12, 12	SER	
(random match)	34, 35, 36 (UM)	12, 12, 12	ACP	

[13]I checked the subject pool effects by using the data from the fixed-pairs and Round Robin treatments. In the fixed pairs treatments, one-tailed *t*-tests show that the difference in the proportion of equilibrium play are not significant at the 10% level ($z = 2.32$ for CIT vs. PCC, $z = 1.22$ for CIT vs. UM, and $z = 1.02$ for UM vs. PCC), and that the differences in efficiency are not significant between CIT and UM ($z = 1.88$), PCC and UM ($z = 0.12$). The only significant difference in efficiency is CIT > PCC, with $z = 2.63$ ($P < 0.05$). Under Round Robin, CIT subjects played Nash equilibrium strategy significantly more than those from PCC ($P < 0.01$, one-tailed permutation test) while efficiency difference is weakly significant ($P = 0.768$, one-tailed permutation test).

[14]The participation fee was used to compensate the PCC subjects for transportation costs. Since the experiment was conducted on the CIT and UM campuses, subjects from CIT and UM did not receive a participation fee.

and ASYN treatments players had extremely limited information—they were told that they were in a game, the game length and their strategy space. At the end of each round each player was informed of his own choice in the previous round and his own payoff corresponding to his previous round's choice of quantity. They had no information about the payoff matrix, nor whom they were playing with.

4. Performance of the mechanisms under complete information

In this section I compare the performance of the two mechanisms under the Round Robin treatment, using two criteria—the proportion of Nash equilibrium play and the system efficiency.[15] Under the Round Robin treatment, the theoretical prediction for both mechanisms is the dominance-solvable Nash equilibrium.

Table 3 tabulates the proportion of Nash equilibrium play in each round under the Round Robin treatment, as well as the proportion of equilibrium play in all rounds. The last row presents the *P*-values for one-tailed permutation tests under the null hypothesis that the proportion of Nash equilibrium play is the same under both mechanisms.

Result 1. (Equilibrium play under Round Robin) In the Round Robin treatment, at the 8th round an average of 88.6% of the subjects played the unique Nash equilibrium strategy under SER; while an average of 80.9% of the subjects played

Table 3
Proportion of subjects choosing Nash equilibrium strategies under the Round Robin treatment and results of permutation tests (H_0: SER = ACP; H_1: SER > ACP)

Session no.	Mechanism (subj. pool)	Round no.								All rounds
		1	2	3	4	5	6	7	8	
1	SER (CIT)	0.625	0.688	0.938	1.000	0.938	1.000	1.000	1.000	0.899
2	SER (CIT)	0.688	0.750	0.938	0.938	0.938	0.938	1.000	1.000	0.899
3	SER (CIT)	0.750	0.938	0.825	0.938	1.000	1.000	1.000	1.000	0.931
4	SER (PCC)	0.188	0.313	0.438	0.625	0.750	0.625	0.825	0.813	0.572
5	SER (PCC)	0.313	0.375	0.438	0.313	0.375	0.625	0.625	0.688	0.469
6	SER (PCC)	0.125	0.313	0.313	0.438	0.500	0.750	0.688	0.813	0.493
7	ACP (CIT)	0.438	0.438	0.688	0.688	0.813	0.813	0.825	0.825	0.691
8	ACP (CIT)	0.375	0.500	0.688	0.625	0.750	0.813	0.813	0.750	0.664
9	ACP (CIT)	0.688	1.000	1.000	0.938	0.938	1.000	0.825	0.813	0.900
10	ACP (PCC)	0.250	0.375	0.688	0.625	0.813	0.813	0.813	0.825	0.650
11	ACP (PCC)	0.313	0.438	0.375	0.563	0.625	0.813	0.688	0.825	0.580
12	ACP (PCC)	0.500	0.563	0.563	0.750	0.813	0.813	0.938	0.813	0.719
Perm. tests	*P*-value	0.461	0.484	0.549	0.481	0.667	0.656	0.323	0.083	0.456

[15] A complete set of the data is available from the author upon request.

the unique Nash equilibrium strategy under ACP. The proportion of the Nash equilibrium play under the two mechanisms is not significantly different.

Support. Table 3 presents the proportion of Nash equilibrium play for each round. Permutation tests under the null hypothesis that the proportion of Nash equilibrium play under SER is the same as that under ACP for round t, where $t = 1, 2, \ldots, 8$, show that none of the P-values is significant at the 5% level. The overall proportion of Nash equilibrium play under the two mechanisms is not significantly different either ($P = 0.456$, one-tailed).

Result 1 is not surprising since both games are dominance-solvable, and the presentation in the form of bimatrix games is fairly transparent. Under complete information we expect that adaptive learning converges to the unique Nash equilibrium.

Although there is no theoretical systematic efficiency comparison between the two mechanisms in general, it is informative to check the actual efficiency of the system in this particular experiment. Group efficiency is calculated by taking the ratio of the sum of the actual earnings of all subjects in a session and the Pareto-optimal earnings of the group. Note that in this experimental setting the Pareto optimal payoff is 970 at strategy two-tuple (1, 9) in both SER and ACP. As a benchmark, the efficiency of Nash (and Stackelberg) equilibrium under the serial mechanism is 87.63%. Under ACP the efficiency of Nash equilibrium is 83.71%, while the efficiency of the Stackelberg equilibrium with player 2 as leader is 79.71%.

Result 2. (Efficiency under Round Robin) The efficiency of the serial mechanism is significantly higher than that of the ACP mechanism under the Round Robin treatment.

Support. The last column of Table 4 shows the efficiency of each session under the Round Robin treatment. Permutation tests show that the efficiency of SER $>$ ACP at a significance level of 0.023 (one-tailed).

Therefore, under Round Robin although the amount of Nash equilibrium play is not significantly different between the two mechanisms, the serial mechanism generated significantly higher system efficiency than ACP.

5. Performance of the mechanisms in distributed systems

Although the proportion of Nash equilibrium play was not significantly different under the two mechanisms under complete information, the performance of the two mechanisms differed dramatically in distributed systems. In this section I will evaluate the two mechanisms under SYN and ASYN in terms of the proportion of

Table 4
Efficiency of each session under the Round Robin treatment

Session no.	Mechanism	Subj. pool	Efficiency
1	SER	CIT	0.8669
2	SER	CIT	0.8655
3	SER	CIT	0.8696
4	SER	PCC	0.8241
5	SER	PCC	0.8069
6	SER	PCC	0.8206
7	ACP	CIT	0.8066
8	ACP	CIT	0.7809
9	ACP	CIT	0.8326
10	ACP	PCC	0.7788
11	ACP	PCC	0.8070
12	ACP	PCC	0.8310

equilibrium play and efficiency, and the plausibility of new solution concepts proposed for distributed systems.

Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation[16] under each of the four different treatments in distributed systems.

Result 3. (Equilibrium play under SER and ACP) Under all four treatments the ranking of the proportion of equilibrium play is highly significant: SER$>$ACP.

Support. Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation. One-tailed permutation tests show that the proportion of equilibrium play under SER is greater than the proportion of equilibrium play under ACP, with $P = 0.05$ under SYN_r, $P = 0.05$ under $ASYN_r$, $P < 0.01$ under SYN_f, and $P < 0.01$ under $ASYN_f$.

Therefore, in contrast to Result 1 where the proportion of Nash equilibrium play is not significantly different under the Round Robin treatment, the proportion of Nash and Stackelberg equilibrium play do differ significantly in distributed systems. The SER mechanism induces significantly more equilibrium play than the ACP mechanism.

Result 4. (Efficiency under SER and ACP) Under all four treatments the ranking of group efficiency is highly significant: SER$>$ACP.

[16]Note under the random matching treatment each session is an independent observation, while under the fixed pair treatment each pair is an independent observation.

Table 5
Proportion of Nash/Stackelberg equilibrium play in distributed systems

No. of sessions	SER		ACP	
	SYN_r	$ASYN_r$	SYN_r	$ASYN_r$
1	0.529	0.544	0.166	0.051
2	0.669	0.447	0.149	0.045
3	0.468	0.596	0.174	0.066
No. of pairs	SYN_f	$ASYN_f$	SYN_f	$ASYN_f$
1	0.873	0.683	0.197	0.077
2	0.343	0.383	0.207	0.003
3	0.893	0.480	0.383	0.393
4	0.710	0.687	0.400	0.040
5	0.937	0.737	0.743	0.053
6	0.760	0.553	0.610	0.123
7	0.883	0.363	0.180	0.030
8	0.657	0.477	0.117	0.007
9	0.937	0.367	0.200	0.120
10	0.773	0.630	0.110	0.117
11	0.133	0.327	0.067	0.043
12	0.683	0.677	0.167	0.050
13	0.380	0.637	0.107	0.077
14	0.283	0.317	0.160	0.020
15	0.690	0.557	0.143	0.087
16	0.767	0.647	0.047	0.363
17	0.503	0.703	0.147	0.040
18	0.590		0.170	0.027
19	0.847		0.060	
20	0.737			

Support. Table 6 presents the efficiency of each independent observation under SER and ACP. One-tailed permutation tests show that the efficiency under SER is greater than the efficiency under ACP, with $P = 0.05$ under SYN_r, $P = 0.05$ under $ASYN_r$, $P < 0.01$ under SYN_f, and $P < 0.01$ under $ASYN_f$.

Although both games are dominance-solvable and the amount of equilibrium play is not statistically different under complete information, their performance does differ dramatically in distributed settings with limited information and asynchrony: the serial mechanism performs robustly better than the ACP mechanism both in terms of Nash and Stackelberg equilibrium play and system efficiency.

One of the characteristics of distributed systems is the asynchrony of actions. In the following result I examine the effects of asynchrony on the proportion of equilibrium play and efficiency.

Table 6
Efficiency in distributed systems

No. of sessions	SER		ACP	
	SYN_r	$ASYN_r$	SYN_r	$ASYN_r$
1	0.800	0.797	0.607	0.690
2	0.821	0.779	0.612	0.690
3	0.793	0.799	0.658	0.625
No. of pairs	SYN_f	$ASYN_f$	SYN_f	$ASYN_f$
1	0.853	0.829	0.709	0.688
2	0.524	0.797	0.670	0.761
3	0.851	0.789	0.844	0.772
4	0.788	0.841	0.778	0.653
5	0.858	0.817	0.787	0.708
6	0.859	0.776	0.765	0.729
7	0.845	0.710	0.640	0.682
8	0.812	0.769	0.600	0.729
9	0.847	0.733	0.587	0.700
10	0.831	0.789	0.562	0.572
11	0.779	0.767	0.520	0.709
12	0.821	0.785	0.600	0.760
13	0.742	0.790	0.567	0.697
14	0.683	0.768	0.648	0.693
15	0.827	0.795	0.474	0.777
16	0.832	0.848	0.489	0.552
17	0.784	0.797	0.515	0.689
18	0.803		0.711	0.712
19	0.817		0.301	
20	0.832			

Result 5. (Effects of asynchrony) The proportion of Nash equilibrium play under SYN is significantly higher than the proportion of Stackelberg equilibrium play under ASYN. Efficiency under SYN and ASYN is not significantly different.

Support. Table 5 presents the proportion of Nash and Stackelberg equilibrium play for each independent observation. One-tailed *t*-test (H_0: SYN = ASYN; H_1: SYN > ASYN) yields $z = 2.18$ ($P < 0.05$). Table 6 presents the efficiency of each independent observation under SYN and ASYN. One-tailed *t*-test yields $z = 1.26$ ($P > 0.10$).

Intuitively, under the asynchronous treatments the Stackelberg leaders moved five times slower than the followers. Therefore they did not have the same opportunity to learn the equilibrium strategies. It is interesting to note that even though we observe significantly more equilibrium play in the synchronous case, the presence of asynchrony does not reduce the system efficiency significantly.

Results in this section lend support for the following result:

Result 6. (O-solvable vs. D-solvable mechanisms) The SER mechanism, which is O-solvable, performs significantly and robustly better than the ACP mechanism, which is D-solvable but not O-solvable, in terms of efficiency and the proportion of equilibrium play.

Results in Sections 4 and 5 provide empirical support for Friedman and Shenker's (1998) argument that traditional solution concepts such as Nash equilibrium or dominance-solvability are not adequate for predicting what can happen in distributed systems. Analysis of experimental data shows that O-solvable games exhibited rapid and robust convergence to the unique Nash equilibrium regardless of the degree of asynchrony, while D-solvable games did not converge as well. In Chen and Khoroshilov (2000) we examine the learning dynamics induced by the two mechanisms by comparing the explanatory power of three learning models. In Section 6 I examine whether the experimental results in the last two sections are robust in more general environments.

6. Robustness of experimental results in more general environments

In this section I assess the extent to which the experimental results in Sections 4 and 5 depend on the linearity of the utility function and the quadratic cost function employed. I consider nine different environments. For simplicity I use polynomial utility and cost functions. The utility function is $\pi_i(x_i,\ q) = \alpha_i q_i^b - x_i$, where $\alpha_1 = 16.1$, $\alpha_2 = 20.1$ are agents' marginal utility for the output, $b = 0.5$, 1, and 2, and x_i is her cost share. The cost function is chosen to be $C(q) = q^c$, where $c = 0.5$, 1 and 2. Varying parameters b and c will give us nine combinations of concave, linear and convex utility and cost functions. Note that $b = 1$ and $c = 2$ is the original experimental design.

Table 7 presents the Nash equilibrium quantities and payoffs for the two types of players under each of the nine environments. Note that all 14 boundary Nash equilibrium, (12, 12), and one interior Nash equilibrium under SER, (2, 2), are dominant strategy Nash equilibria, whereas the other three interior Nash equilibria are dominance solvable.

For the complete information, Round Robin treatment, I expect Result 1 to hold in each of the nine environments, i.e. the proportion of Nash equilibrium play will be indistinguishable between SER and ACP, since both games have either a dominance-solvable or a dominant strategy equilibrium in each of the nine environments, and the presentation in the form of bimatrix games is fairly

Table 7
Nash equilibrium quantities (q) and payoffs (π) for utility function $\alpha_i q_i^b - x_i$ and cost function $C(q) = q^c$

Parameters		SER			ACP		
		$c = 0.5$	$c = 1$	$c = 2$	$c = 0.5$	$c = 1$	$c = 2$
	q	(12, 12)	(12, 12)	(2, 2)	(12, 12)	(12, 12)	(1, 3)
$b = 0.5$	π	(533, 672)	(438, 576)	(148, 204)	(533, 672)	(438, 576)	(148, 204)
	q	(12, 12)	(12, 12)	(4, 6)	(12, 12)	(12, 12)	(4, 8)
$b = 1$	π	(1908, 2388)	(1812, 2292)	(324, 526)	(1908, 2388)	(1812, 2292)	(164, 648)
	q	(12, 12)	(12, 12)	(12, 12)	(12, 12)	(12, 12)	(12, 12)
$b = 2$	π	(23160, 28920)	(23064, 28824)	(20304, 26064)	(23160, 28920)	(23064, 28824)	(20304, 26064)

transparent. Under complete information I expect that adaptive learning leads to convergence to the unique Nash equilibrium.

Note that in Table 7 the Nash equilibrium payoffs for the players are the same under the two mechanisms in eight out of nine environments, where the Nash equilibria are symmetric. This is because SER and ACP allocate the same cost share to each player when they demand the same quantity. Indeed, the only environment where the payoffs differ is the experimental environment ($b = 1$ and $c = 2$). Therefore, I expect that the efficiency will be indistinguishable between SER and ACP in each of these eight environments under complete information and Round Robin treatment. That is, Result 2 might not hold in these eight environments. This is not surprising, since in general there is no systematic efficiency comparison between the two mechanisms, as I discussed in Section 2. Therefore, any efficiency comparison between the two mechanisms will necessarily depend on the environment.

To assess how robust the experimental results are in distributed systems in different environments, I conduct Monte Carlo simulations for each of the nine environments. Since Chen and Khoroshilov (2000) study the learning dynamics induced by the SER and ACP mechanisms, I use the learning algorithm that performs the best on these data sets and the calibrated parameters in Chen and Khoroshilov (2000) to conduct simulations.

Chen and Khoroshilov (2000) study how human subjects learn under extremely limited information. They use experimental data on cost sharing games reported in this paper, and Van Huyck et al. (1996) data on coordination games to compare three payoff-based learning models: the payoff-assessment learning model (Sarin and Vahid, 1999), a modified experience-weighted attraction learning model (Camerer and Ho, 1999) and a simple reinforcement learning model. They show that the payoff-assessment learning model tracks the data the best in both the cost sharing games as well as the coordination games. Therefore, I use the payoff-assessment learning model and the parameters calibrated on the cost sharing games to conduct simulation in other environments. Admittedly, even though the payoff-assessment learning model performs the best in capturing how human subjects learn under limited information in one environment ($b = 1$ and $c = 2$), it is possible that it might not be the best learning model when the environment changes. However, this is the best approximation we have. At least, the simulation results can show us the relative performance of the two mechanisms in other environments *if* agents are myopic maximizers described by the payoff-assessment algorithm.

The *payoff-assessment* learning model assumes that a player is a myopic subjective maximizer. She chooses among alternate strategies only on the basis of the payoff she assesses she would obtain from them. These assessments do not explicitly take into account her subjective judgements regarding the likelihood of alternate states of the world. At each stage, the player chooses the strategy that she myopically assesses to give her the highest payoff and updates her assessment adaptively. Let $u_j(t)$ denote the subjective assessment of strategy s_j at time t. The initial assessments are denoted by $u_j(0)$. Payoff assessments are updated by taking a weighted average of her previous assessments and the objective payoff she actually obtains at time t. If strategy k is chosen at time t, then

$$u_j(t+1) = (1-r)u_j(t) + r\pi_k(t), \quad \forall j. \tag{1}$$

Suppose that at time t the decision-maker experiences zero-mean, symmetrically distributed shocks, $Z_j(t)$ to her assessment of the payoff she would receive from choosing strategy s_j, for all s_j. Denote the vector of shocks by $Z = (Z_1, \ldots, Z_{12})$, and their realizations at time t by $z(t) = (z_1(t), \ldots, z_{12}(t))$. The decision maker makes choices on the basis of her shock-distorted subjective assessments, denoted by $\tilde{u}(t) = u(t) + Z(t)$. At time t she chooses strategy s_j if

$$\tilde{u}_j(t) > \tilde{u}_l(t), \quad \forall s_l \neq s_j. \tag{2}$$

Note that mood shocks only affect her choices and not the manner in which assessments are updated. Sarin and Vahid (1999) prove that such a player converges to stochastically choose the strategy that first order stochastically dominates another among the strategies she converges to play with positive probability.

For parameter estimation, Chen and Khoroshilov (2000) conducted Monte Carlo simulations designed to replicate the characteristics of each of the experimental settings. They then compare the simulated paths with the actual paths of a subset of the experimental data to estimate the parameters which minimize the mean-squared deviation scores. I use these estimated parameters to conduct Monte Carlo simulations for each of the nine environments.

In each simulation, 10,000 pairs of players were created.[17] In each simulation the following steps were taken.

[17]This yields a statistical accuracy of 1%.

1. Initial values: since Kolmogorov–Smirnov tests of the initial choice distribution by experimental subjects cannot reject the null hypothesis of uniform distribution. I set $u_j(0) = 200$ for all players when $b = 0.5$ and 1, since in the experimental data the average first-round payoffs were around 200 which also result in a probability prediction around the centroid, $(1/12, \ldots, 1/12)$, for the first round. With concave ($b = 0.5$) and linear ($b = 1$) utility functions, the magnitude of payoffs are similar to the experimental setting. With convex utility function ($b = 2$), the payoffs are about two orders of magnitude larger than the payoff matrices in the experiment, therefore I set $u_j(0) = 3000$ for all players when $b = 2$.
2. Simulated players were matched into fixed pairs, or randomly rematched pairs for each period, depending on the treatment.
3. Shocks are drawn from a uniform distribution, $[-a, a]$, where a is estimated[18] in Chen and Khoroshilov (2000).
4. The simulated players' strategies were determined via Eq. (2).
5. Payoffs were determined using the SER or ACP payoff rule for each (b, c) parameter combination.
6. Assessments were updated according to Eq. (1), using discount factor, r, estimated[19] in Chen and Khoroshilov (2000). Updating occurs every period under SYN for both players, every period for player 1 in ASYN and every five periods for player 2 in ASYN.

Fig. 1 shows the simulated time series paths for player 1 in an environment with concave utility function ($b = 0.5$) under SERSYN_f (left column) and ACPSYN_f (right column), respectively. Simulated paths for player 2 exhibit similar patterns, therefore are not displayed. The first row presents the simulated paths under concave utility ($b = 0.5$) and concave cost function ($c = 0.5$). The second row presents the same information under a linear cost function ($c = 1$). The last row presents the same information under a convex cost function ($c = 2$). Each graph presents the mean (the black dots), standard deviation (the grey error bars) and stage game equilibria (the dashed lines) for each mechanism. Larger error bars indicate more variance in the choice of strategies and thus worse convergence to the mean. Figs. 2 and 3 present the simulated time series paths for player 1 under SERSYN_f (left column) and ACPSYN_f (right column) in environments with linear ($b = 1$) and convex ($b = 2$) utility functions, respectively. Simulation results for the random matching treatments display similar patterns. Therefore they are not displayed.

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[18]The best fit parameters are $a = 3$ for SER SYN, $a = 2$ for SER ASYN, $a = 50$ for ACP SYN, and $a = 45$ for ACP ASYN. The larger a in ACP reflects the relatively volatile paths of the ACP data.

[19]The best fit parameters are $r = 0.2$ for SERSYN, $r = 0.0$ for SERASYN, $r = 0.6$ for ACPSYN, and $r = 0.2$ for ACPASYN.

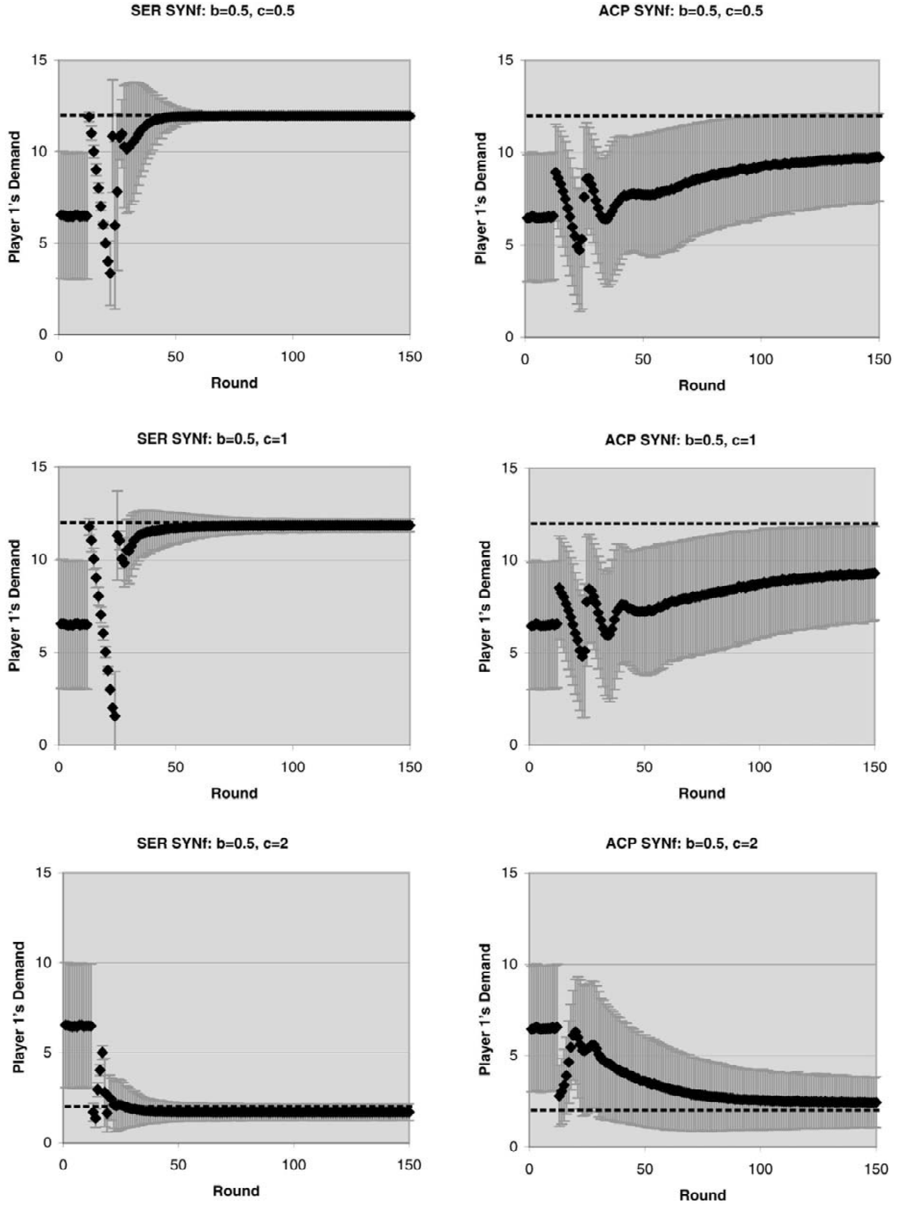

Fig. 1. Simulation results with concave utility function, concave, linear, and convex cost functions.

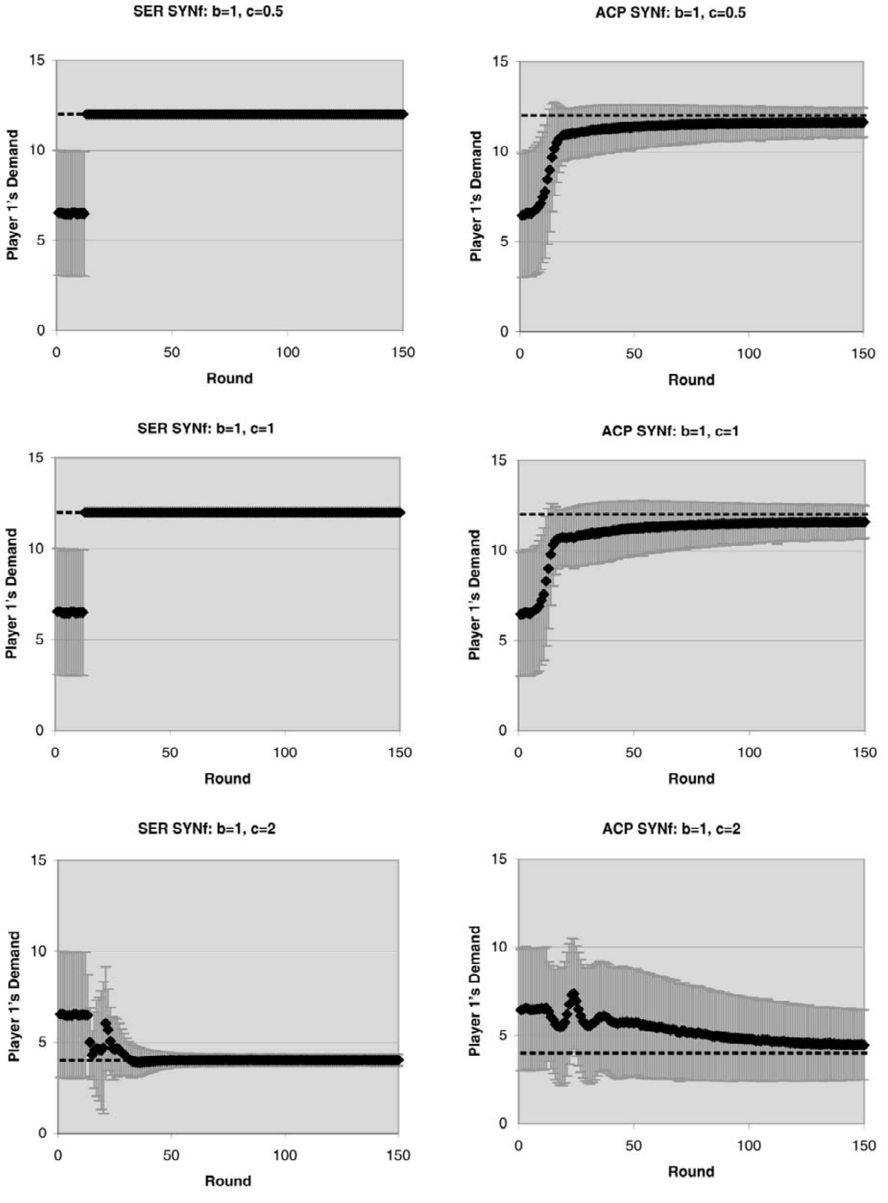

Fig. 2. Simulation results with linear utility functions, concave, linear, and convex cost functions.

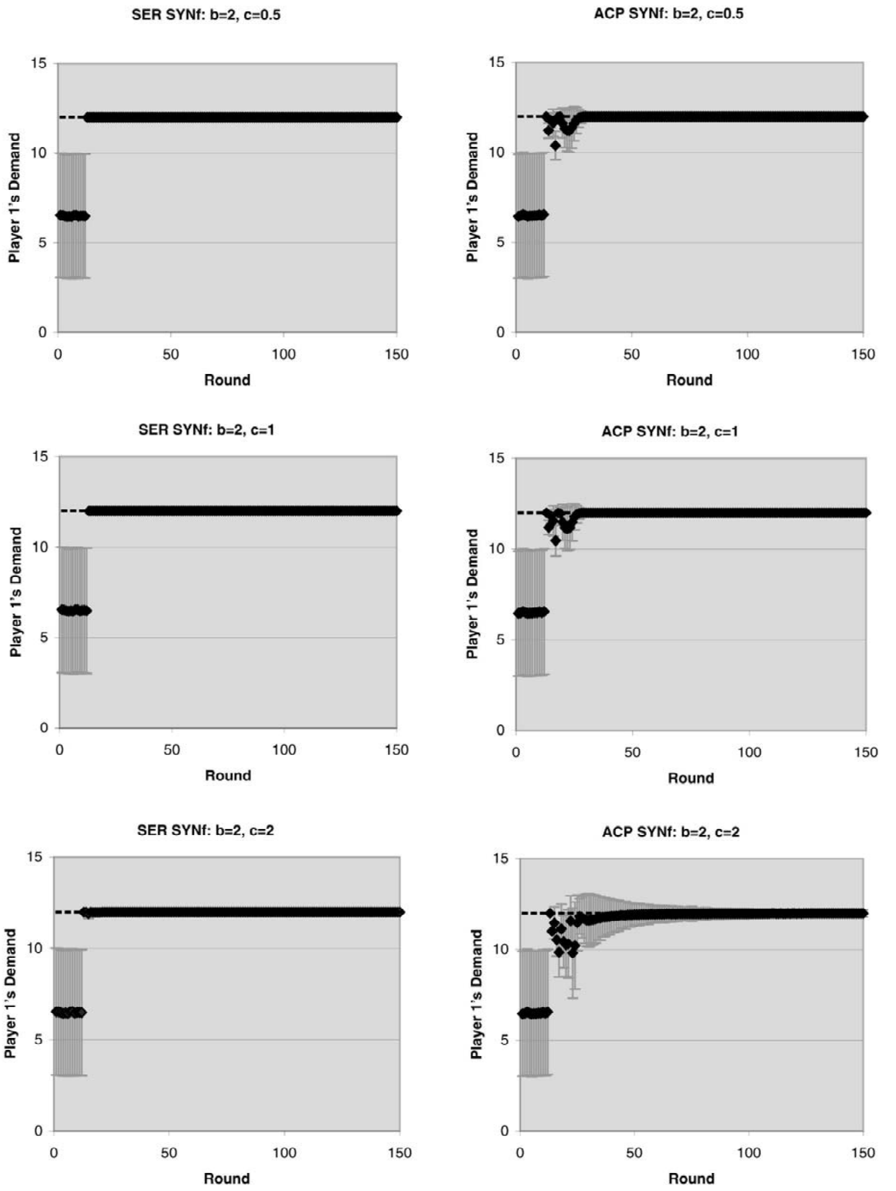

Fig. 3. Simulation results with convex utility functions, concave, linear, and convex cost functions.

Based on the Monte Carlo simulation I have the following results.

1. With concave ($b = 0.5$, e.g. Fig. 1) and linear ($b = 1$, e.g. Fig. 2) utility functions, regardless of the forms of the cost function, simulated players converge much more quickly to the stage game equilibrium under SER than under ACP. As a result the proportion of equilibrium play is significantly larger under SER than under ACP.
2. With convex ($b = 2$, e.g. Fig. 3) utility function, regardless of the forms of the cost function, simulated players under both mechanisms converge quickly to the stage game equilibrium. Convergence under SER is slightly more quickly than that under ACP. As a result the proportion of equilibrium play is weakly larger under SER than under ACP.
3. As a result of the different speed of convergence to equilibrium, with concave and linear utility functions, efficiency under SER is significantly higher than that under ACP. With convex utility functions, efficiency under SER is weakly higher than that under ACP.

Simulation results for nine different environments suggest that the experimental results on the proportion of equilibrium play are robust to variations in the environment, while experimental results on efficiency might depend on variations in the environment. In other words, even though efficiency comparison might be sensitive to the environment, the SER mechanism is more predictable than ACP because it induces robustly quicker convergence to the stage game equilibrium.

7. Conclusion

Cost sharing mechanisms have many practical applications in the real world. An increasingly important area is distributed systems like the Internet, where agents have very limited information about the payoff structure as well as the characteristics of other agents and where there is no synchronization of actions. Most current Internet routers use the average cost pricing mechanism, while this study suggests that the serial mechanism might be a better choice.

This paper reports experimental results on the serial and the average cost pricing mechanisms under five different treatments. The first is a complete information treatment designed to test the basic properties of the mechanisms. The other four treatments simulate distributed systems by giving the subjects very limited information about the game and by imposing two levels of asynchrony. The latter present a more challenging and realistic setting for the cost sharing mechanisms.

The experimental data show that under the complete information treatment both mechanisms converge well to the Nash equilibrium prediction and their performances are statistically indistinguishable. Under the limited information treatments, however, the serial mechanism performs significantly and robustly better than the average cost pricing mechanism, in terms of efficiency and convergence to equilibrium predictions regardless of the level of asynchrony. To test the robustness of the results, I conduct Monte Carlo simulation using calibrated learning algorithms in nine different environments. Simulation results indicate that the experimental results on the proportion of equilibrium play are robust to variations in the environment, while experimental results on efficiency might depend on variations in the environment.

Since both the serial and average cost pricing games are dominance-solvable in our design, these results indicate that traditional solution concepts such as Nash equilibrium or dominance-solvability might not be so useful in distributed systems. Experimental data provide empirical support for Friedman and Shenker's (1998) serially unoverwhelmed set.

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Appendix A. Experiment instructions

Instruction for Mechanism S corresponds to the serial mechanism under Round Robin. Instruction for Mechanism A corresponds to the average cost pricing mechanism under Round Robin. Instruction for Mechanism XY is for SYN for both mechanisms, as well as for player 1 in ASYN. Instruction for Mechanism XYZ is used for player 2 in ASYN for both mechanisms.

Experiment instructions—Mechanism S ID = __

Procedure

- Each participant has to make a decision in each of _ rounds.
- There are two different types: _ participants are Blue players and _ are Red players.
- The small envelope has your type and ID number. Your type remains the same for the entire experiment.
- A Blue player always meets a Red player and a Red player always meets a Blue player.
- In each round, a Blue is matched with a Red. You will be matched with each participant of the other type only once.
- In each round, a Blue and a Red simultaneously choose a number out of the following numbers: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, which denotes **your demand** of output.
- **Your payoff = (your valuation − your share of the cost)** × 10.
- One unit of output is worth 16.1 points (unit value) to a Blue player, and 20.1 points (unit value) to a Red player. Therefore,

  your valuation = (**your unit value**) × (**your demand**).

- Cost of producing x units is x^2. The smaller demander will pay half of the cost of producing twice the amount of the smaller demand, i.e. you pay the per capita cost of production as if everyone demanded the same amount as you did. Therefore, if your demand is smaller than your opponent's demand, your share of the cost is the following

$$\textbf{smaller demander's cost} = 1/2 \times (2 \times \text{smaller demand})^2 = 2 \times (\text{smaller demand})^2$$

  If you demand a larger amount than your opponent, you are the larger demander. You will pay the rest of the cost of production. Therefore,

$$\textbf{larger demander's cost} = \underbrace{(\text{smaller} + \text{larger demand})^2}_{\text{total cost of production}} - \underbrace{2 \times (\text{smaller demand})^2}_{\text{smaller demander's cost}}$$

- For example, if a Blue demands 2 units and a Red demands 1 unit, then the total demand is 3 units. The calculation of payoffs for the two types are tabulated as follows:

	Blue	Red
Your demand	2 (larger)	1 (smaller)
Your valuation	$16.1 \times 2 = 32.2$	$20.1 \times 1 = 20.1$
Your share of the cost	$(1+2)^2 - 2 \times 1^2 = 7$	$2 \times 1^2 = 2$
Your payoff	$(32.2 - 7) \times 10 = 252$	$(20.1 - 2) \times 10 = 181$

- The above information is summarized by the payoff tables in your folder.

Payoff table

- The payoff table summarizes both your payoff and your opponent's payoff. A Blue player chooses which row to play. A Red player chooses which column to play. Your payoff is determined by both your choice and your opponent's choice. The first number in each cell (in blue) denotes the payoff to a Blue player. The second number in each cell (in red) denotes the payoff to a Red player.
- For example, if a Blue demands 2 units, and a Red demands 1 unit, you can find the payoff to each participant on the second row and the first column of the payoff matrix. The cell contains two numbers: the first number is 252, which is Blue's payoff; the second number is 181, which is Red's payoff.

Information

- At the end of each round, each participant is informed of the following results of the round:
  –your own demand
  –your opponent's demand
  –your own payoff
  –the distribution of demands of the other type in the last round.
- You will not know who your opponent was.

Total payoff

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for _ points.

Record sheet: you are required to record your demand, your opponent's demand and your payoff each round.

Review questions (write down your answers on top of your record sheet)

1. You are a _ (Blue or Red) player.
2. If you demand 2 units and your opponent demands 10 units, your payoff is _; your opponent's payoff is _. Find it from your payoff table.

Mechanism A

Red's Choices

Blue's Choices	1	2	3	4	5	6	7	8	9	10	11	12
1	141 181	131 342	121 483	111 604	101 705	91 786	81 847	71 888	61 909	51 910	41 891	31 852
2	262 171	242 322	222 453	202 564	182 655	162 726	142 777	122 808	102 819	82 810	62 781	42 732
3	363 161	333 302	303 423	273 524	243 605	213 666	183 707	153 728	123 729	93 710	63 671	33 612
4	444 151	404 282	364 393	324 484	284 555	244 606	204 637	164 648	124 639	84 610	44 561	4 492
5	505 141	455 262	405 363	355 444	305 505	255 546	205 567	155 568	105 549	55 510	5 451	-45 372
6	546 131	486 242	426 333	366 404	306 455	246 486	186 497	126 488	66 459	6 410	-54 341	-114 252
7	567 121	497 222	427 303	357 364	287 405	217 426	147 427	77 408	7 369	-63 310	-133 231	-203 132
8	568 111	488 202	408 273	328 324	248 355	168 366	88 357	8 328	-72 279	-152 210	-232 121	-312 12
9	549 101	459 182	369 243	279 284	189 305	99 306	9 287	-81 248	-171 189	-261 110	-351 11	-441 -108
10	510 91	410 162	310 213	210 244	110 255	10 246	-90 217	-190 168	-290 99	-390 10	-490 -99	-590 -228
11	451 81	341 142	231 183	121 204	11 205	-99 186	-209 147	-319 88	-429 9	-539 -90	-649 -209	-759 -348
12	372 71	252 122	132 153	12 164	-108 155	-228 126	-348 77	-468 8	-588 -81	-708 -190	-828 -319	-948 -468

Experiment instructions—Mechanism A ID = _

Procedure

- Each participant has to make a decision in each of _ rounds.
- There are two different types: _ participants are Blue players and _ are Red players.
- The small envelope has your type and ID number. Your type remains the same for the entire experiment.
- A Blue player always meets a Red player and a Red Player always meets a Blue player.
- In each round, a Blue is matched with a Red. You will be matched with each participant of the other type only once.
- In each round, a Blue and a Red simultaneously choose a number out of the following numbers: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, which denotes **your demand** of output.
- **Your payoff = (your valuation − your share of the cost)** × 10.
- One unit of output is worth 16.1 points (unit value) to a Blue player, and 20.1 points (unit value) to a Red player. Therefore,

your valuation = (your unit value) × (your demand).

- Cost of producing x units is x^2. Your share of the cost is proportional to your demand. Therefore,

$$\textbf{your share of the cost} = \frac{\text{your demand}}{\text{total demand}} \times (\text{total demand})^2 = (\text{your demand}) \times (\text{total demand}),$$

where total demand = your demand + your opponent's demand.
- For example, if a type A demands 2 units and a type B demands 1 unit, then the total demand is 3 units and the total cost of producing 3 units is 9. The calculation of payoffs for the two types are tabulated as follows:

	Blue	Red
Your demand	2	1
Your valuation	16.1 × 2 = 32.2	20.1 × 1 = 20.1
Your share of the cost	2/3 × 9 = 6	1/3 × 9 = 3
Your payoff	(32.2 − 6) × 10 = 262	(20.1 − 3) × 10 = 171

- The above information is summarized by the payoff table in your folder.

Payoff table

- The payoff table summarizes both your payoff and your opponent's payoff. A Blue player chooses which row to play. A Red player chooses which column to play. Your payoff is determined by both your choice and your opponent's choice. The first number in each cell (in blue) denotes the payoff to a Blue player. The second number in each cell (in red) denotes the payoff to a Red player.
- For example, if a Blue demands 2 units, and a Red demands 1 unit, you can find the payoff to each participant on the second row and the first column of the payoff matrix. The cell contains two numbers: the first number is 262, which is Blue's payoff; the second number is 171, which is Red's payoff.

Information

- At the end of each round, each participant is informed of the following results of the round:
  –your own demand
  –your opponent's demand
  –your own payoff
  –the distribution of demands of the other type in the last round.

- You will not know who your opponent was.

Total payoff

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for _ points.

Record sheet

- You are required to record your demand, your opponent's demand and your payoff each round.

Review questions (write down your answers on top of your record sheet).

1. You are a _ (Blue or Red) player.
2. If you demand 2 units and your opponent demands 10 units, your payoff is _; your opponent's payoff is _. Find it from your payoff table.

Mechanism S

Red's Choices

Blue's Choices	1	2	3	4	5	6	7	8	9	10	11	12
1	141 181	141 332	141 463	141 574	141 665	141 736	141 787	141 818	141 829	141 820	141 791	141 742
2	252 181	242 322	242 433	242 524	242 595	242 646	242 677	242 688	242 679	242 650	242 601	242 532
3	343 181	313 322	303 423	303 494	303 545	303 576	303 587	303 578	303 549	303 500	303 431	303 342
4	414 181	364 322	334 423	324 484	324 515	324 526	324 517	324 488	324 439	324 370	324 281	324 172
5	465 181	395 322	345 423	315 484	305 505	305 496	305 467	305 418	305 349	305 260	305 151	305 22
6	496 181	406 322	336 423	286 484	256 505	246 486	246 437	246 368	246 279	246 170	246 41	246 -108
7	507 181	397 322	307 423	237 484	187 505	157 486	147 427	147 338	147 229	147 100	147 -49	147 -218
8	498 181	368 322	258 423	168 484	98 505	48 486	18 427	8 328	8 199	8 50	8 -119	8 -308
9	469 181	319 322	189 423	79 484	-11 505	-81 486	-131 427	-161 328	-171 189	-171 20	-171 -169	-171 -378
10	420 181	250 322	100 423	-30 484	-140 505	-230 486	-300 427	-350 328	-380 189	-390 10	-390 -199	-390 -428
11	351 181	161 322	-9 423	-159 484	-289 505	-399 486	-489 427	-559 328	-609 189	-639 10	-649 -209	-649 -458
12	262 181	52 322	-138 423	-308 484	-458 505	-588 486	-698 427	-788 328	-858 189	-908 10	-938 -209	-948 -468

Experiment instructions—Mechanism XY ID = __

Procedure

- You are part of a game, in which you have to make a decision in each of 150 rounds.

- In each round, you will choose a number out of the following numbers:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Information

- At the end of **each round**, you are informed of your result of the round:
  –your own choice
  –your own payoff.

Total payoffs

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for _ points.

Record sheet: you are required to record your choice and your payoff each round.

Experiment instructions—Mechanism XYZ ID=_

Procedure

- You are part of a game, in which you have to make a decision in each of 30 rounds.
- In each round, you will choose a number out of the following numbers:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Information

- At the end of **each round**, you are informed of your result of the round:
  –your own choice
  –your own payoff.

Total payoffs

- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for _ points.

Record sheet: you are required to record your choice and your payoff each round.

References

Aadland, D., Kolpin, V., 1998. Shared irrigation costs: an empirical and axiomatic analysis. Mathematical Social Sciences 849, 203–218.
Camerer, C., Ho, T., 1999. Experience-weighted attraction learning in normal form games. Econometrica 67, 827–874.
Chen, Y., Khoroshilov, Y., 2000. Learning under limited information. University of Michigan, Manuscript.
Dorsey, R., Razzolini, L., 1999. An experimental evaluation of the serial cost sharing rule. University of Mississippi, Manuscript.
Friedman, E., Moulin, H., 1999. Three methods to share joints costs or surplus. Journal of Economic Theory 87 (2), 275–312.
Friedman, E., Shenker, S., 1998. Learning and implementation on the internet. Rutgers University, Department of Economics, New Brunswick, Manuscript.
Kolpin, V., Aadland, D., 2001. Environmental determinants of cost sharing: an application to irrigation. University of Oregon, Manuscript.
Milgrom, P., Roberts, J., 1990. Rationalizability, learning and equilibrium in games with strategic complementarities. Econometrica 58 (6), 1255–1277.
Moulin, H., Shenker, S., 1992. Serial cost sharing. Econometrica 60, 1009–1037.
Moulin, H., Shenker, S., 1994. Average cost pricing versus serial cost sharing: an axiomatic comparison. Journal of Economic Theory 64, 178–201.
Sarin, R., Vahid, F., 1999. Payoff assessments without probabilities: A simple dynamic model of choice. Games Econ. Behaviour 28, 294–309.
Shenker, S., 1990. Making greed work in networks: a game-theoretic analysis of gateway service disciplines. Xerox Research Center, Palo Alto, Manuscript.
Stoica, I., Shenker, S., Zhang, H., 1998. Core-stateless fair queueing: achieving approximately fair bandwidth allocations in high speed networks. In: Proceedings of ACM SIGCOMM, pp. 118–130.
Tauman, Y., 1988. The Aumann–Shapley prices: a survey. In: Roth, A. (Ed.), The Shapley Value. Cambridge University Press, New York.
Van Huyck, J., Battalio, R., Rankin, F., 1996. Selection dynamics and adaptive behavior without much information, Manuscript. College Station: Texas A&M University, Department of Economics.