Supplemental Material to

When Does Learning in Games

Generate Convergence to Nash Equilibria?

The Role of Supermodularity in an Experimental Setting

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This material includes an alternative convergence level measure and results using that measure, efficiency in simulations, as well as experimental instructions.

1 Supplemental Results

As a particular convergence level does not capture the change in equilibrium play over time, we define an alternative measure, which captures the change in equilibrium play induced by the mechanism and reduces cohort effects.

DEFINITION 1 *The convergence-level change, $\Delta L(\tau)$ is measured by the difference in the proportion of Nash equilibrium play in the last and first τ rounds, i.e., $\Delta L(\tau) = L_b(T - \tau + 1, T) - L_b(1, \tau)$, where $0 < \tau < T/2$, and T is the total number of rounds.*

Result 1 highlights the convergence level achieved towards the end of the game. However, it does not indicate whether players have learned equilibrium strategies. Therefore, we look at the improvement in convergence over time as we change the parameters. We use the difference in convergence level between the first and last twenty rounds, $\Delta L(20) = L_b(41, 60) - L_b(1, 20)$.

[Table 1 about here.]

Table 1 reports the convergence-level change between the first and last 20 rounds, $\Delta L(20)$, to subgame-perfect Nash equilibrium (top two panels) and ϵ-Nash equilibrium (bottom two panels) for each session under each of the six different treatments, as well as the alternative hypotheses and the corresponding p-values of one-tailed permutation tests. We notice that the β-effects in Result 1 persist (although sometimes they are only weakly significant). However, the α-effects disappear, as higher α weakly increases the level of convergence in the first 20 rounds as well.[1]

[1]For example, the permutation test of the null hypothesis of equal proportion against H_1: $\alpha 10\beta 20 < \alpha 20\beta 20$ yields a p-value of 0.091.

[Table 2 about here.]

Table 2 reports the change in efficiency between the first and last 20 rounds. We note that efficiency change between the first and last 20 rounds is not significantly different across treatments.

We now present our efficiency results in simulations. In the simulated data, the following relative efficiency ranking of the treatments in round 500 is significant:

$$\alpha 20\beta 40 > \alpha 20\beta 18 > \alpha 10\beta 20 > \alpha 20\beta 20 > \alpha 20\beta 00 > \alpha 10\beta 00.$$

At round 500, efficiency of treatment $\alpha 10\beta 00$ is approximately 78%, that of $\alpha 20\beta 00$ approaches 93%, while that of other four treatments is over 98%.

[Table 3 about here.]

Table 3 reports results of t-tests comparing efficiency in different treatments. Efficiency is determined solely by the quantity produced, which in turn is a function of p_2. Therefore, efficiency rankings match those for p_2-convergence (Result 5). Another interesting welfare result is the overall budget balance. After 500 rounds, the magnitude of budget imbalance is significantly closer to zero for all treatments except for $\alpha 10\beta 00$. In fact, decreasing α from 20 to 10 significantly increases the deficit for both $\beta 00$ and $\beta 20$. Finally, $\alpha 20\beta 40$ is the only treatment that has an overall budget surplus after 500 rounds, although this surplus is approaching zero.

2 Experiment Instructions

Instructions for the $\alpha 20\beta 20$ treatment is attached. Instructions for other treatments are identical except for the parameters involving α and β.

Experiment Instructions – Mechanism A20 B20

Introduction

- You are about to participate in a decision process in which one of numerous alternatives is selected in each of 60 rounds. This is part of a study intended to provide insight into certain features of decision processes. If you follow the instructions carefully and make good decisions you may earn a considerable amount of money. You will be paid in cash at the end of the experiment.

- *During the experiment, we ask that you please do not talk to each other.* If you have a question, please raise your hand and an experimenter will assist you.

Procedure

- You will be randomly assigned to one of two groups: the Blue group or the Red Group. There will be 6 players in each group. You will stay in the same group for the entire experiment.

- In each of 60 rounds, you will be randomly matched with a player from the other group. You will not know the identity of your Match. Your payoff each round depends only on the decisions made by you and your Match.

- In each of 60 rounds, Red will produce a quantity, Q. Red gets a revenue of $240 \cdot Q + 250$ and pays the production cost of $\frac{5}{4} \cdot Q^2$. Red's production imposes a loss of $\frac{5}{2} \cdot Q^2$ on Blue.

- In order to compensate Blue's loss, prior to production, Blue and Red simultaneously announce a price, P_{Blue} and P_{Red}, respectively. Red will pay a tax of $10 \cdot P_{Blue} \cdot Q$. Blue will receive a compensation of $10 \cdot P_{Red} \cdot Q$. Note that Blue's announcement, P_{Blue}, determines Red's tax rate; and Red's announcement, P_{Red}, determines Blue's compensation rate.

- If P_{Blue} and P_{Red} are not the same, each will pay a penalty proportional to $(P_{Blue} - P_{Red})^2$.

- Each round consists of two stages: the Announcement Stage and the Production Stage.
  - Announcement Stage: Blue selects P_{Blue}, an integer between 0 and 40. At the same time, Red selects P_{Red}, also an integer between 0 and 40.
  - Production Stage: Red then selects the quantity for that period, Q, an integer between 0 and 50. *This quantity is affected by* P_{Blue}. When Red chooses Q, Red's terminal will display a payoff table listing Red's payoff for each Q.

Payoffs

- **Per Round Payoffs: Red**

$$\text{Payoff}_{Red} = \underbrace{240 \cdot Q + 250}_{\text{Revenue}} - \underbrace{\frac{5}{4} \cdot Q^2}_{\text{Production Cost}} - \underbrace{10 \cdot P_{Blue} \cdot Q}_{\text{Tax}} - \underbrace{20 \cdot (P_{Blue} - P_{Red})^2}_{\text{Penalty}}$$

Revenue: Red receives 240 points for each unit Red produces plus 250 points.

Production Cost: This term represents the cost of producing Q units.

Tax: Red pays a tax to compensate Blue. The tax rate, P_{Blue}, is announced by Blue.

Penalty: Red is penalized for any difference between P_{Blue} and P_{Red}.

- **Per Round Payoffs: Blue**

$$\text{Payoff}_{Blue} = \underbrace{10 \cdot P_{Red} \cdot Q}_{\text{Compensation}} - \underbrace{\frac{5}{2} \cdot Q^2}_{\text{Loss}} - \underbrace{20 \cdot (P_{Blue} - P_{Red})^2}_{\text{Penalty}}$$

Compensation: Blue receives a compensation. The compensation rate, P_{Red}, is announced by Red.

Loss: This term represents Blue's loss due to Red's production.

Penalty: Blue is penalized for any difference between P_{Blue} and P_{Red}.

- There will be 60 rounds. There will be no practice rounds. From the first round, you will be paid for each decision you make.
- Your total payoff is the sum of your payoffs in all rounds.
- The exchange rate is $1 for ____________ points.

Information At the end of each **round**, you are informed of your result of the round:

- Your Price
- The Price of your Match for that round
- The Quantity selected that round
- Your Payoff

We encourage you to earn as much cash as you can. Are there any questions?

Change in Proportion of Nash Price 1 (p_1^*)							Permutation Tests	
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	H_1	p-value
$\alpha 10\beta 00$	-0.025	-0.017	0.083	-0.017		0.006	$\alpha 10\beta 00 < \alpha 20\beta 00$	0.302
$\alpha 20\beta 00$	0.000	0.050	0.000	0.042	0.000	0.018	$\alpha 20\beta 00 < \alpha 20\beta 20$	0.004***
$\alpha 20\beta 18$	0.100	0.083	0.050	0.092		0.081	$\alpha 20\beta 00 < \alpha 20\beta 18$	0.016**
$\alpha 10\beta 20$	0.242	0.017	0.050	-0.067	0.192	0.087	$\alpha 10\beta 20 < \alpha 20\beta 20$	0.155
$\alpha 20\beta 20$	0.225	0.192	0.083	0.042	0.233	0.155	$\alpha 20\beta 18 < \alpha 20\beta 20$	0.103
$\alpha 20\beta 40$	0.100	0.383	0.158	0.092		0.183	$\alpha 20\beta 20 < \alpha 20\beta 40$	0.381
Change in Proportion of Nash Price 2 (p_2^*)							Permutation Tests	
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	H_1	p-value
$\alpha 10\beta 00$	0.000	0.017	0.017	0.008		0.010	$\alpha 10\beta 00 < \alpha 20\beta 00$	0.183
$\alpha 20\beta 00$	0.033	0.067	-0.042	0.042	0.050	0.030	$\alpha 20\beta 00 < \alpha 20\beta 20$	0.044**
$\alpha 20\beta 18$	0.108	0.250	0.000	0.167		0.131	$\alpha 20\beta 00 < \alpha 20\beta 18$	0.054*
$\alpha 10\beta 20$	0.275	0.025	0.100	0.017	0.592	0.202	$\alpha 10\beta 20 < \alpha 20\beta 20$	0.492
$\alpha 20\beta 20$	0.217	0.058	0.333	0.008	0.400	0.203	$\alpha 20\beta 18 < \alpha 20\beta 20$	0.238
$\alpha 20\beta 40$	0.208	0.467	0.208	0.117		0.250	$\alpha 20\beta 20 < \alpha 20\beta 40$	0.349

Change in Proportion of ϵ-Nash Price 1 (ϵ-p_1^*)							Permutation Tests	
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	H_1	p-value
$\alpha 10\beta 00$	-0.058	0.058	0.167	0.075		0.060	$\alpha 10\beta 00 < \alpha 20\beta 00$	0.222
$\alpha 20\beta 00$	-0.025	0.300	0.067	0.042	0.233	0.123	$\alpha 20\beta 00 < \alpha 20\beta 20$	0.099*
$\alpha 20\beta 18$	0.192	0.425	0.150	0.267		0.258	$\alpha 20\beta 00 < \alpha 20\beta 18$	0.087*
$\alpha 10\beta 20$	0.367	0.033	0.142	0.092	0.433	0.213	$\alpha 10\beta 20 < \alpha 20\beta 20$	0.349
$\alpha 20\beta 20$	0.283	0.333	0.183	0.033	0.442	0.255	$\alpha 20\beta 18 < \alpha 20\beta 20$	0.492
$\alpha 20\beta 40$	0.158	0.300	0.283	-0.017		0.181	$\alpha 20\beta 20 < \alpha 20\beta 40$	0.779
Change in Proportion of ϵ-Nash Price 2 (ϵ-p_2^*)							Permutation Tests	
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	H_1	p-value
$\alpha 10\beta 00$	0.000	0.092	0.050	0.092		0.058	$\alpha 10\beta 00 < \alpha 20\beta 00$	0.183
$\alpha 20\beta 00$	0.200	0.217	-0.100	0.117	0.183	0.123	$\alpha 20\beta 00 < \alpha 20\beta 20$	0.032**
$\alpha 20\beta 18$	0.483	0.492	0.025	0.333		0.333	$\alpha 20\beta 00 < \alpha 20\beta 18$	0.064*
$\alpha 10\beta 20$	0.425	0.142	0.192	0.083	0.583	0.285	$\alpha 10\beta 20 < \alpha 20\beta 20$	0.508
$\alpha 20\beta 20$	0.283	0.192	0.333	0.142	0.450	0.280	$\alpha 20\beta 18 < \alpha 20\beta 20$	0.683
$\alpha 20\beta 40$	0.308	0.583	0.317	0.033		0.310	$\alpha 20\beta 20 < \alpha 20\beta 40$	0.373

Note: Significant at: * 10% level; ** 5% level; *** 1% level.

Table 1: Level-of-Convergence Change: Last 20 Rounds-First 20 Rounds

Change in Efficiency Induced: Last 20 Rounds-First 20 Rounds								
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	H_1	p-value
$\alpha10\beta00$	0.024	0.045	0.126	0.256		0.113	$\alpha10\beta00 < \alpha20\beta00$	0.198
$\alpha20\beta00$	0.181	0.295	0.105	0.194	0.083	0.171	$\alpha20\beta00 < \alpha20\beta20$	0.333
$\alpha20\beta18$	0.323	0.286	0.111	0.173		0.223	$\alpha20\beta00 < \alpha20\beta18$	0.214
$\alpha10\beta20$	0.238	0.147	0.194	0.018	0.220	0.163	$\alpha10\beta20 < \alpha20\beta20$	0.397
$\alpha20\beta20$	0.204	0.188	0.173	0.177	0.212	0.191	$\alpha20\beta18 < \alpha20\beta20$	0.793
$\alpha20\beta40$	0.119	0.316	0.417	0.127		0.245	$\alpha20\beta20 < \alpha20\beta40$	0.222

Note: Significant at: * 10% level; ** 5% level; *** 1% level.

Table 2: Change in Efficiency in Experimental Data

Treatment	Efficiency	H_1	p-value
$\alpha10\beta00$	0.7804		
$\alpha20\beta00$	0.9280	$\alpha20\beta00 > \alpha10\beta00$	0.000***
$\alpha20\beta18$	0.9831	$\alpha20\beta18 > \alpha10\beta20$	0.000***
$\alpha10\beta20$	0.9813	$\alpha10\beta20 > \alpha20\beta20$	0.003***
$\alpha20\beta20$	0.9800	$\alpha20\beta20 > \alpha20\beta00$	0.000***
$\alpha20\beta40$	0.9862	$\alpha20\beta40 > \alpha20\beta18$	0.000***

Table 3: Results of t-tests comparing efficiency in simulations of experimental treatments. Values are for round 500 and based on 1500 simulated games.

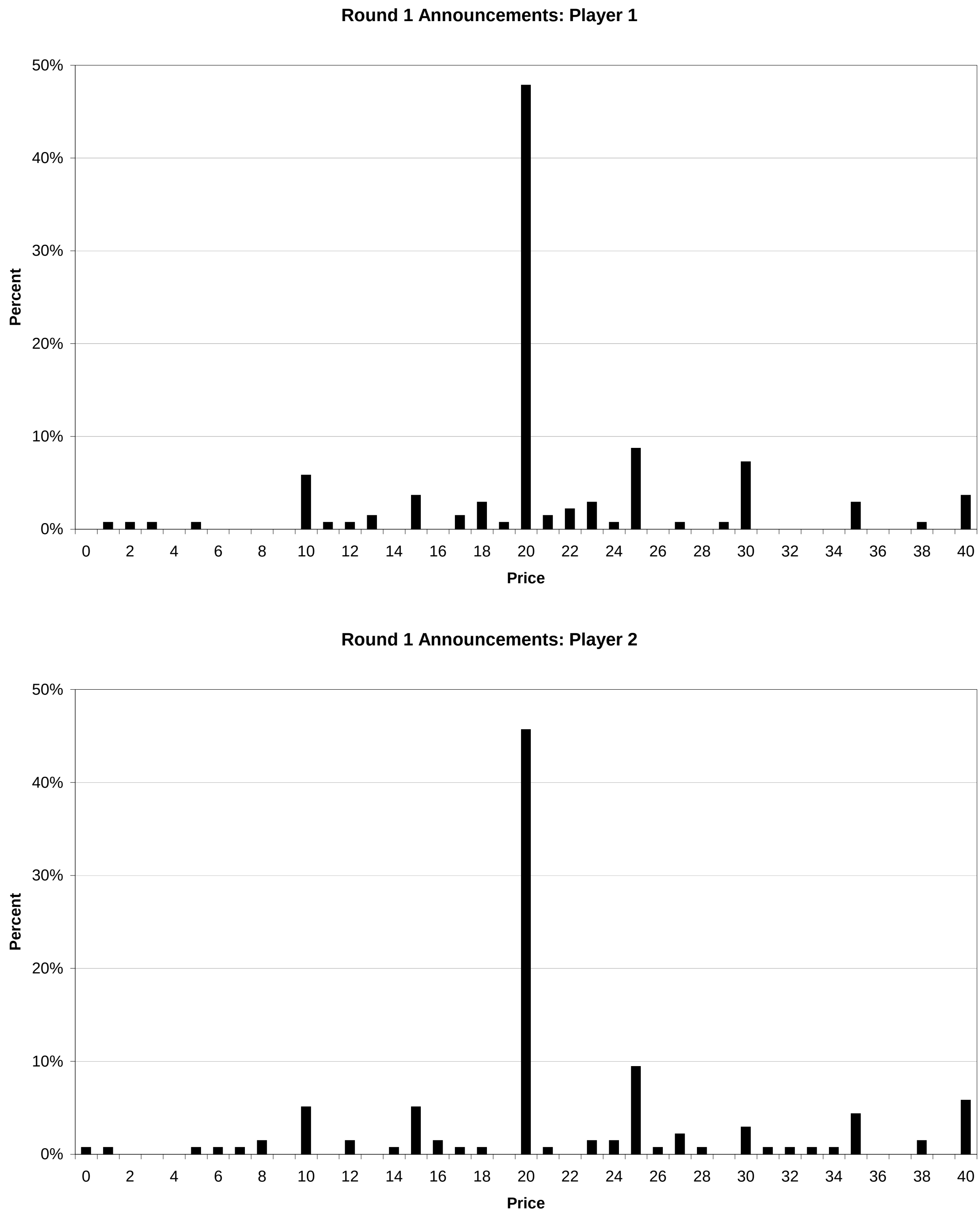

Figure 1: Distribution of first round choices