Online Appendix for "Chinese College Admissions and School Choice Reforms: A Theoretical Analysis"

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A Evolution of the Chinese College Admissions Mechanisms

In this Appendix, we discuss the evolution of the Chinese College Admissions (CCA) mechanisms from 1949 to 2012. In this summary, we rely on several books written by educators, policy-makers and historians. In particular, Yang (2006) provides the historical and political contexts of Chinese college admissions from 1949 to 1999. Liu (2009) reports the policy debates surrounding college admissions reforms up to 2009, including survey data related to major policy reforms. In comparison, Qiu and Zhao (2011) offer practical advice for high school seniors and their parents based on recent admission statistics for each university, the admissions mechanisms, and application strategies. While Chinese college admissions have been traditionally studied by educators, Chinese economists have recently started to analyze their game-theoretic properties. We reference most of the latter in the main text. As matching mechanisms in historical documents are not described in game-theoretic terms, we provide the translation of the relevant paragraphs and our own interpretation.

For more recent information on college admissions rules and policies in various provinces, we refer the reader to the official Ministry of Education website on college admissions, `http://gaokao.chsi.com.cn/`.[1]

A.1 From Decentralized to Centralized Examinations and Admissions (1952 - 1957)

After the establishment of the People's Republic of China in 1949, Chinese universities continued to admit students via decentralized mechanisms, i.e., each university administered its own entrance

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[1]This website has remained stable since 2006. We last accessed it on October 31, 2015.

exams and admissions processes. In 1950, there were 227 universities and colleges in China, with 134,000 students (Yang 2006, p. 5).[2] Historians identified two major problems with decentralized admissions during this time period. First, as each student could be admitted to multiple universities, the enrollment-to-admissions ratio was low, ranging from 20% for some ordinary universities to 75% among the best universities in 1949 (Yang 2006, p. 6). Therefore, many ordinary universities could not fill their first-year classes. Second, after being rejected by the best universities, some qualified students missed the ordinary universities' deadlines and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances, each of which had its own centralized admissions system. This experiment achieved an improved average enrollment-to-admissions ratio of 50% for an ordinary university (Yang 2006, p. 7).

Based on the success of the alliances, the Ministry of Education decided to transition to centralized matching in 1952 by implementing the first National College Entrance Examination, also known as *gaokao*.[3] The inaugural version of this exam consisted of eight subjects (math, physics, chemistry, biology, foreign language, history and geography, politics, and Chinese), and lasted for three consecutive days, a format that persisted in essence until 2012. In 1952, the enrollment-to-admissions ratio for an ordinary university was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

Between 1952 and 1957, the Ministry of Education made several adjustments to the centralized admissions process. First, minority-serving institutions, as well as fine arts and music institutions, were allowed to include institution-specific admissions processes, such as interviews, auditions and portfolio presentations in addition to gaokao. Second, the single-track gaokao evolved into two tracks in 1954, and three tracks in 1955. The three tracks included the science and engineering track, the medicine, biology and agriculture track, and the humanities and social sciences track. In 1964, the first two tracks were recombined into a single track, forming the present-day two-track exam system. Lastly, *key universities*, such as Peking (Beijing), Tsinghua, and Jiaotong, were allowed to recruit nationwide, while *ordinary universities* were restricted to recruit within their respective province, creating a university *tier system*.[4]

[2] In reporting statistics, we exclude universities in Taiwan, Hong Kong and Macau. Also note that Chinese sources prior to 1977 typically report statistics in units of ten thousand (*wàn*).

[3] Using a national examination to select talent for various government positions had been a long tradition in China, dating back to 605 A.D. (Liu 2009, p. 2).

[4] According to Weidong Liu (interviewed by the first author on August 9, 2013), historically, tiers were created as a result of the manual admissions process. During the admissions process, each university sent four to five admissions officers to each province. Typically all admissions officers stayed at the same hotel to finish the admissions process. A province could not accommodate all university admissions officers concurrently in the same hotel because of limited hotel capacities, therefore, they dealt with one tier at a time. From the students' perspective, however, the tier system reduces the risks associated with the IA mechanism.

From a game-theoretic perspective, the centralized admissions mechanism used during this time period, "Exam-Score Based Admissions" (*fēn jí lù qǔ*), resembled a serial dictatorship mechanism: "[Admissions] should proceed in decreasing exam scores, starting with the highest score, and proceeding to the next score after [the admission of the student with] the highest score is finished. For each student, proceed based on the student's preference ranking. That is, send the student's application to his first choice. If that university decides to admit the student, it keeps his application file and marks 'Admitted' in the Admission Results column. If the university decides not to admit the student or if its quota is full, it should mark 'Not Admitted' on the student's application, and pass his file to his second-choice university (with the same process as described above). And so on." (Yang 2006, p. 76-77)

The transition from a decentralized to centralized matching approach was designed to alleviate coordination failure and excess supply. In 1956, for example, universities had a target of admitting 165,500 students, whereas 156,000 students graduated from high school that year (Yang 2006, p. 40). The government addressed this gap the next year by encouraging cadres from workplaces to apply for colleges,[5] resulting in 199,000 applicants for 120,000 spots (Yang 2006, p. 45). After a nation-wide debate of whether to go back to the decentralized admissions processes, which were used in the Soviet Union at the time, the Ministry of Education decided to continue the centralized admissions process, mainly based on its advantages of better coordination and lower transaction costs. Separate admissions processes within provinces were established after the 1957 debate.

A.2 The Leftists' Attacks on College Admissions (1958 - 1965)

Since 1958, gaokao had been attacked by leftists in the Communist Party, for its intellectual focus and lack of communist ideology. In response, the Ministry of Education stepped up the screening of student political backgrounds in the admissions process, and implemented the Guaranteed Admissions policy for cadres from proletariat families who had been through the Crash Training Schools for Workers and Farmers. Prior to 1958, these cadres were given bonus points added to their scores. After 1958, they were exempt from gaokao (Yang 2006, p. 91), resulting in a 97% admissions rate in 1958 (Yang 2006, p. 139).

To our knowledge, the first documented tiered admissions system appeared in China in 1959. "Admissions of new students should proceed in tiers. National key universities admit students first." The second tier included provincial and ministry-level key universities, whereas the third tier included all other universities and colleges in China (Yang 2006, p. 104).

After the Great Leap Forward (1958 - 1961) ended in a disastrous famine, the college admissions rate reached its lowest point prior to the Cultural Revolution, 24%, with only 107,000 students

[5] Affirmative action, in the form of adding 10-15 points per subject (out of a 100-point scale), was implemented in 1954 to increase the number of cadres in universities (Yang 2006, p. 55).

admitted among 440,000 applicants in 1962.

By 1963, the college admissions mechanism had transitioned into a hybrid of a serial dictatorship and priority matching mechanism, "Exam-Score Interval Based Admissions" (*fēn duàn lù qǔ*). Average exam scores were chunked into (typically) five-point intervals (*duàn*), e.g., [80, 100], [75, 79], [70, 74], [65, 69], etc. Admissions proceeded sequentially from the highest interval downward, clearing one interval before starting the next (*duàn duàn qīng*). Within an interval, admissions proceeded in the order of student preference rankings of universities and exam scores (Yang 2006, p. 135-136). Under this mechanism, each student could apply to five national key universities. Within each university, he could apply for three different departments. Admission decisions were made by each university. This mechanism was designed to reduce the disparity in student qualities between different departments within a university (Yang 2006, p. 150).

Meanwhile, because of the increased competitiveness in the application process, some students considered that "gaokao is a battle that determines your fate: one point [difference] in gaokao can determine whether you go to heaven [i.e., universities] or hell [i.e., becoming a farmer]" (Yang 2006, p. 171). Until recently, labor market mobility had been constrained by the Household Registration (*hù kǒu*) system. For millions of youths from rural areas, gaokao offered the only way of breaking away from the farms.

A.3 Demise of Gaokao During the Cultural Revolution (1966-1976)

The year 1966 marked the start of the ten-year Cultural Revolution, and the abolition of gaokao. In its place, farmers, workers and soldiers who had the equivalence of a high school education could be recommended for university study. The political turmoil dictated that none of the universities recruited new students for the subsequent six years. From 1972 to 1976, university admissions resumed based on a recommendation system. Students had to have completed at least two years of real-life work experience, i.e., having worked on farms, or in factories or having served in the armed forces, to be eligible. The recommendation system opened the door for rampant corruption in college admissions during this time period.

A.4 College Admissions Reforms (1977 - 2012)

With the end of the Cultural Revolution, gaokao resumed in 1977. As a result, 5.7 million applicants participated in gaokao, including many from the ten-year backlog of high school graduates together with the class of 1977, with only 4.8% of all applicants admitted into universities. In 1977, each province wrote its own exams and administered its own admissions process. Starting in 1978, gaokao again became a national exam, written by the Ministry of Education. A record 6.1 million students participated in the 1978 gaokao, with admissions rate again at 4.8%. To further curb corruption, every applicant's score was publicly posted.[6] Compared with gaokao before the Cultural Revolution, where the average admissions rate was 55.92%, the average admissions rate between 1977 and 1982 was 6.05% (Yang 2006, p. 278).

While the hybrid serial dictatorship and priority matching mechanism, "Exam-Score Interval Based Admissions," continued to be used until 1984, starting from 1985, it was gradually replaced by a priority matching mechanism, which resembled the IA mechanism but with tiers (Yang 2006, p 314-315; Liu 2009, p. 41). Under this mechanism, each university determines a minimum threshold based on exam scores, applicant ROLs, and its quota. It then receives all applications that list it as the applicants' first choice. After admitting first-choice applicants in the order of high to low exam scores up to its quota, the first round allocations are finalized and the first round is closed. After the first round, universities which have not fulfilled their quotas each review all applicants who list it as their second choice; etc. This mechanism is called the *sequential mechanism* (*shùn xù zhì yuàn*), which prioritizes students' preference orderings over their score rankings (*zhì yuàn yōu xiān*). The sequential mechanism is identical to the immediate acceptance (IA) mechanism in the matching literature.

The sequential mechanism places huge strategic importance on an applicant's first choice. Among those admitted into a key university in 2010, more than 95% of them list it as a first choice, whereas 80% of those admitted into an ordinary university list that university as a first choice (Qiu and Zhao 2011, p. 243). Therefore, if an applicant's first- and second-choice universities are too close in quality, the applicant might not get into any university in the first tier (Qiu and Zhao 2011, p. 243). An obvious problem with this mechanism is that some students with very high scores do not get into any first-tier university because they miss their first choice, leading to the popular saying that "a good score in the college entrance exam is worth less than a good strategy in the preference ranking of universities" (Nie 2007b).

To remedy the strategic manipulation inherent in the sequential mechanism, the *parallel mechanism* (*píng xíng zhì yuàn*) was first introduced into college admissions in Hunan Province in 2001. Jiangsu and Zhejiang adopted the mechanism in 2005 and 2007, respectively (Liu 2009, p. 382). The main innovation of the parallel mechanism is that students can select several "parallel" universities within each choice-band. For example, a student's first choice-band can contain four universities, A, B, C and D, in decreasing desirability. Applicants are ranked by exam scores. Starting from the applicant with the highest score to the one with the lowest score, each applicant applies for parallel universities in the order of her preference ranking, from A to D. She receives admission into the first university with an unfilled quota. After every applicant has applied to his first choice-band universities, the first round is closed. Those who are not admitted in the first round start the same process in the second round, and so on. We deem this mechanism a modified

[6]In comparison, individual gaokao scores were kept secret before the Cultural Revolution (Yang 2006, p. 269-270).

deferred acceptance mechanism.

In addition to the choice of matching mechanisms, many other important components of the college admissions process underwent changes in the 1990s and the early 21 century. First, the content of the exam, i.e., subjects that should be covered and the number of tracks, changed several times. For example, in 1999, a "$3 + X$" system was implemented, where 3 refers to the three exams required for every applicant (math, Chinese, and foreign language) and X refers to any number of exams taken from physics, chemistry, biology, geography, history, politics. Second, a controversial institutionalized feature started in the 1990s where up to 5% of high school graduates can be recommended for guaranteed admission. Third, standardized test techniques, such as an increase in multiple choice problems and machine grading, were gradually implemented in the late 80s and 90s. Fourth, as of 1985, Shanghai has been implementing its own exams. By 2006, 16 provinces moved to implement its own exams. Lastly, a computerized admissions process was first implemented in Guangxi and Tianjin in 1998. By 2001, nation-wide computerized admissions through the Internet were implemented (Liu 2009, p. 41).

Compared to the historical accounts and qualitative analysis of Chinese college admissions, game-theoretic analysis of Chinese college admissions mechanisms has been relatively new. The latter mainly focuses on two issues: the timing of preference ranking submissions (Zhong, Cheng and He 2004, Wu and Zhong 2014, Lien, Zheng and Zhong 2012, Wang and Zhong 2012) and the matching mechanisms themselves (Nie 2006, Nie 2007a, Nie 2007b, Nie and Zhang 2009, Wei 2009). We discuss both aspects in the main text of our paper. Lastly, Chiu and Weng (2009) present a theoretical model that investigates the incentives for schools to pre-commit to admitting qualified applicants who rank them as their top choices over more qualified applicants who do not. Such incentives are relevant in CCA as well (Jiang 2015).

A.5 Shanghai Mechanism: Online Q&A

We translate the following question and answer from an online Q&A forum about the parallel choices in Shanghai high school admissions, posted in May 2003.

> Question: If a student lists a school as his first choice or second choice, what difference does it make in the admission process?[7]
>
> Answer: Middle school admission principles are: based on the student exam scores and school preference ranking, place the applications accordingly, while also considering their moral, intellectual and physical aspects, choose the best from high to low scores. For each individual student, the Middle School Admissions Office will submit

[7]Translated from `http://edu.sina.com.cn/l/2003-05-15/42912.html`, retrieved on October 31, 2015.

Table 1: Chinese College Admissions Mechanisms by Province in 2012

Province	**Mechanism Type**	**Sequence**	**No. of Applicants in 2012**
Heilongjiang	sequential	(1, 1, 1, ...)	208,000
Qinghai	sequential	(1, 1, 1, ...)	38,000
Jiangsu	symmetric parallel	(3, 3, 3, ...)	500,000
Anhui	symmetric parallel	(4, 4, 4, ...)	506,000
Guangxi	symmetric parallel	(4, 4, 4, ...)	292,000
Jiangxi	symmetric parallel	(4, 4, 4, ...)	289,000
Ningxia	symmetric parallel	(4, 4, 4, ...)	60,000
Shǎnxi	symmetric parallel	(4, 4, 4, ...)	384,000
Hebei	symmetric parallel	(5, 5, 5, ...)	459,000
Hunan	symmetric parallel	(5, 5, 5, ...)	352,000
Yunan	symmetric parallel	(5, 5, 5, ...)	230,000
Zhejiang	symmetric parallel	(5, 5, 5, ...)	300,000
Tianjin	symmetric parallel	(5, 5, 5, ...)	65,000
Hainan	symmetric parallel	(6, 6, 6, ...)	54,000
Tibet	symmetric parallel	(10, 10, 10, ...)	18,000
Beijing	asymmetric parallel	(1, 3, 1, 3, 1, 3, ...)	73,000
Gansu	asymmetric parallel	(1, 3, 1, 3, 1, 3, ...)	296,000
Shandong	asymmetric parallel	(1, 4, 1, 4, 1, 4, ...)	587,000
Liaoning	asymmetric parallel	(3, 1, 8, 1, 8, ...)	245,000
Guangdong	asymmetric parallel	(3, 3, 1, ...)	692,000
Fujian	asymmetric parallel	(4, 4, 6, ...)	267,000
Shanghai	asymmetric parallel	(4, 6, 8, ...)	61,000
Xinjiang	asymmetric parallel	(4, 6, 1, 8, 1, 5, ...)	155,000
Guizhou	asymmetric parallel	(5, 5, 1, 3, 1, ...)	248,000
Jilin	asymmetric parallel	(5, 1, 7, 1, 6, ...)	165,000
Hubei	asymmetric parallel	(5, 1, 6, 1, 6, ...)	457,000
Shānxi	asymmetric parallel	(5, 1, 4, 1, 4, ...)	339,000
Sichuan	asymmetric parallel	(5, 1, 4, 1, 4, ...)	514,000
Chongqing	asymmetric parallel	(6, 1, 5, 1, 5, ...)	230,000
Henan	asymmetric parallel	(6, 6, 1, 4, ...)	855,000
Inner Mongolia	dynamic adjustment	-	206,000

Note: The sequences do not include tier 0, which is primarily for military academies.

his application in the order of his preference ranking. Only when he cannot get into his first choice, will his second choice be considered. In the admissions process of the entire district, each school has only one threshold. If a student's score is above the school threshold, whether he lists it as his first or second choice, he should be admitted.

For example, if student A's first choice is Luwan Middle School, and student B's second choice is Luwan. If A and B's scores are both above the Luwan minimum threshold, then both should be admitted into Luwan. However, if student B is already admitted by his first choice, it is impossible for him to get into Luwan. On the other hand, if the two students have different scores, e.g., A's score is low and below the Luwan threshold, while B (whose second choice is Luwan) has a high score, which is above the Luwan threshold, then A (whose first choice is Luwan) cannot be admitted into Luwan because his score is below the threshold; whereas B (whose second choice is Luwan), if not admitted by his first choice, should be admitted by Luwan, even though he listed Luwan as his second choice.

In the next section, we explain how the minimum threshold score is determined in the admissions process.

A.6 The Computer Software for CCA and the Minimum Threshold

In this section, we describe the history of computerization in CCA, and the determination of the minimum threshold for each college. This section is based on the first author's interview with Professor Weidong Liu at the Department of Computer Science and Technology, Tsinghua University, on August 9, 2013. Professor Liu is the Principal Investigator for the Chinese College Admissions Software Development Project, which was commissioned by the Ministry of Education in 1998 to decrease the time period for admissions and to reduce the error rate.

The Ministry mandates that the official version of the software should be able to accommodate regional variations; thus, the software is modularized, with major modules provided by the Tsinghua group. To adapt the software to the mechanism used in each province, the provincial officials just need to set the parameters. One key parameter is the number of parallel colleges for each choice-band, i.e., the parameter e in our notation. Each province can also add modules to accommodate province-specific policies, such as affirmative action policies.

The software has several modules: planning, student application submission, student application review, and final allocation. The final allocation is simultaneously announced by each college and the Provincial Admissions Office (PAO), which serves as an advocate for the students. The beta version of the software was implemented in 1998 in Tianjin and Guangxi. However, key elements of the manual admissions process were still present. For example, each college still sent admissions officers to the provinces using the beta version of the software. The major official versions are described as follows:

1. Version 1.0 was released in 2001 nationwide. The parallel mechanism could be accommodated in this version by setting the choice parameter greater than one (in our notation, $e > 1$).

2. Version 2.0 was released in 2003 with major security improvements.

3. Version 3.0 was released in 2006, with the USB key identification of the college identity, based on security considerations.

4. Version 4.0, implemented in 2008, supports IPv6.

5. In 2010, another version was released which enables each province to add small modules to the major modules sanctioned by the Ministry. Source code for the major modules is provided to the provinces upon request. Provincial modules can be added to reflect province-specific policies, such as affirmative action.

We next describe the minimum threshold under each mechanism. Under the sequential mechanism, three thresholds are relevant in the final allocation:

1. Tier threshold (*pī cì kòng zhì xiàn*): Each PAO determines the threshold for each tier, which is the minimum score for a student to be eligible for that tier. This threshold is determined by the quota of each college and the distribution of student scores.

2. Student application submission threshold (*tóu dàng xiàn*): Each college has its own threshold, i.e., its minimum score for a student's application to be reviewed. The Ministry allows each college to review no more than 120% of its quota, which gives the college considerable flexibility in the allocation of students among its various departments.

3. Minimum admissions threshold (*lù qǔ xiàn* $\geq$ *tóu dàng xiàn*): This is the minimum score of those finally admitted to a college.

In comparison, under the parallel mechanism, the admissions threshold for each college is determined through a simulation process. The simulations can be viewed as a negotiation process between the PAO and the colleges as to how many more applications a college can review without jeopardizing the students' welfare. This is determined by several rounds of simulations. The simulation results for each round are not released to the public.

Round 1: each college receives student applications up to 110% of its quota. The software goes through the entire allocation process and lets each college know the maximum and minimum scores in the college as well as in each of its departments. Note that the college does not see who is in each round of simulation, but only the summary statistics.

Round 2: based on the round 1 statistics, each college then proposes to adjust its percentage of applications, e.g., to 106% of its quota. The software takes the new percentages and performs another round of simulated allocations. Summary statistics are again given to each college.

$\vdots$

The simulation process continues until every college is satisfied with its summary statistics or until time has run out. In some provinces, the PAO determines the number of rounds of simulations. The minimum admissions threshold for each college is determined at the end of the simulation phase.

When the simulation phase is over, the student application review phase starts. Each college views individual applications of those students who apply for it and whose scores are above its minimum threshold, and then makes admission decisions. If an application is reviewed and the corresponding student is rejected during this phase, the student's application materials are likely to have violated some written guidelines.

B Examples and Simulations

Example 1a. (The IA mechanism is manipulable whenever the Shanghai mechanism is manipulable) Consider the following example with five students and four schools. Schools s_1, s_2, and s_4 each have a quota of one, while school s_3 has a quota of two.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_1	$\vdots$	i_5
i_1	i_3		i_1
i_2	i_4		$\vdots$
$\vdots$	$\vdots$		

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P'_{i_4}	P_{i_5}
s_1	s_2	s_1	s_2	s_2	s_1	s_4
s_4	$\vdots$	s_3	s_3	s_1	$\vdots$	$\vdots$
s_2	$\vdots$	s_2	s_1	s_3		
s_3		s_4	s_4	s_4		

The following two tables illustrate the steps of the Shanghai mechanism applied to the problem $(\succ, P)$. A student tentatively placed at a school at a particular step is outlined in a box.

Round 0	$s_1\ (q_{s_1}^{r=0}=1)$	$s_2\ (q_{s_2}^{r=0}=1)$	$s_3\ (q_{s_3}^{r=0}=2)$	$s_4\ (q_{s_4}^{r=0}=1)$
Step 1	$\boxed{i_1}, i_2$	$\boxed{i_3}, i_4$		$\boxed{i_5}$
Step 2	$\boxed{i_4}, i_1$	$\boxed{i_3}$	$\boxed{i_2}$	
Step 3				$\boxed{i_5}, i_1$
Round 1	$s_1\ (q_{s_1}^{r=1}=0)$	$s_2\ (q_{s_2}^{r=1}=0)$	$s_3\ (q_{s_3}^{r=1}=1)$	$s_4\ (q_{s_4}^{r=1}=0)$
Step 4		i_1		
Step 5	$\vdots$		$\boxed{i_1}$	

In the above tables, observe that student i_1 ends up at his last choice at problem $(\succ, P)$. Now consider the following two tables that illustrate the steps of the Shanghai mechanism when student i_1 reports P'_{i_1}, as opposed to P_{i_1}.

Round 0	$s_1\ (q_{s_1}^{r=0}=1)$	$s_2\ (q_{s_2}^{r=0}=1)$	$s_3\ (q_{s_3}^{r=0}=2)$	$s_4\ (q_{s_4}^{r=0}=1)$
Step 1	$\boxed{i_2}$	$\boxed{i_1}, i_3, i_4$		$\boxed{i_5}$
Step 2	$\boxed{i_4}, i_2$	$\boxed{i_1}$	$\boxed{i_3}$	
Step 3			$\boxed{i_2, i_3}$	

In this case, student i_1 is assigned to school s_2. Thus, the Shanghai mechanism is manipulable by student i_1 at problem $(\succ, P)$. Next, let us apply the IA mechanism to problem $(\succ, P)$. The specifications are illustrated in the following tables.

Round 0	$s_1\ (q_{s_1}^{r=0}=1)$	$s_2\ (q_{s_2}^{r=0}=1)$	$s_3\ (q_{s_3}^{r=0}=2)$	$s_4\ (q_{s_4}^{r=0}=1)$
Step 1	$\boxed{i_1}, i_2$	$\boxed{i_3}, i_4$		$\boxed{i_5}$
Round 1	$s_1\ (q_{s_1}^{r=1}=0)$	$s_2\ (q_{s_2}^{r=1}=0)$	$s_3\ (q_{s_3}^{r=1}=1)$	$s_4\ (q_{s_4}^{r=1}=0)$
Step 2	i_4		$\boxed{i_2}$	
Step 3	$\vdots$		$\boxed{i_2, i_4}$	

Observe that student i_1 ends up at s_1 (his first choice), and thus cannot gain by a misreport, but student i_4 ends up at s_3 (his third choice) at problem $(\succ, P)$. Next consider the following tables that illustrate the steps of the IA mechanism when student i_4 reports P'_{i_4}, as opposed to P_{i_4}.

Round 0	$s_1\ (q_{s_1}^{r=0}=1)$	$s_2\ (q_{s_2}^{r=0}=1)$	$s_3\ (q_{s_3}^{r=0}=2)$	$s_4\ (q_{s_4}^{r=0}=1)$
Step 1	$\boxed{i_4}, i_1, i_2$	$\boxed{i_3}$		$\boxed{i_5}$
Round 1	$s_1\ (q_{s_1}^{r=1}=0)$	$s_2\ (q_{s_2}^{r=1}=0)$	$s_3\ (q_{s_3}^{r=1}=2)$	$s_4\ (q_{s_4}^{r=1}=0)$
Step 2			$\boxed{i_2}$	i_1
Round 2	$s_1\ (q_{s_1}^{r=2}=0)$	$s_2\ (q_{s_2}^{r=2}=0)$	$s_3\ (q_{s_3}^{r=2}=1)$	$s_4\ (q_{s_4}^{r=2}=0)$
Step 3			$\boxed{i_1}$	

Now student i_4 ends up at school s_1. Thus, the IA mechanism is also manipulable at problem $(\succ, P)$.

Example 1b. (Shanghai mechanism is not manipulable when the IA mechanism is) Consider the following example with the given priority structure and the profile of preferences. Each school, s_1, s_2, and s_3, has a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_1	i_2	$\vdots$
i_2	i_3	
$\vdots$	$\vdots$	

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_2	s_2
$\vdots$	s_2	$\vdots$	s_3
	s_3		$\vdots$

Clearly, at problem $(\succ, P)$ under the IA mechanism, student i_2 can obtain a seat at s_2 by submitting P'_{i_2} as opposed to P_{i_2} which places him at s_3. Note, however, that under the Shanghai mechanism, no student can ever gain by a misreport at problem $(\succ, P)$.

Remark 4. Stability Simulations

Following the specifications in the simulation experiments reported by Erdil and Ergin (2008), we randomly generate school choice problems involving 200 students and 10 schools, each with a quota of 10. More specifically, student preferences and school priorities are created as follows:

Let the students be indexed by $i = 1, ..., 200$, and the schools be indexed by $x = 1, ..., 10$, where we assume that $q_x = 20$ for each school x. For each student i and school x, we represent the locations of the student and the school in a two-dimensional map by points $l^i, l^x \in \mathbb{R}^2$, respectively. Let $i = 0$ denote an artificially created dummy student with average tastes.

Let Z_{ix} be i.i.d. normally distributed random variables with mean zero and variance one for $i = 0, 1, ..., 200$; $x = 1, ..., 10$. We define the utility of student $i = 1, ..., 200$ for school $x = 1, ..., m$ by:

$$U_{ix} = -\beta d(l^i, l^x) + (1-\beta)[\alpha Z_{0x} + (1-\alpha) Z_{ix}]$$

where $d(.,.)$ denotes the usual Euclidean distance, and $\alpha, \beta \in [0, 1]$ are fixed common parameters. The parameter α captures the correlation in student preferences and the parameter β captures student preference sensitivity to locational proximity. We assume that each student is in the walk zone of the school located closest to her residence. Each school's priority order consists of only two indifference classes determined by its walk zone.

Given parameters α and β, in each simulation experiment, we go through the following steps:

1. Generate locational parameters of students and schools by using the i.i.d. uniform distribution on $[0, 1] \times [0, 1]$;

2. Determine the strict priority structure from the locational parameters;
3. Generate the i.i.d normal variables Z_{ix} and determine the utility values from the above formula. Define the preference of each student i by: $xP_iy \Leftrightarrow U_{ix} > U_{iy}$, where all schools are acceptable;
4. Once the problem is generated, calculate the outcome of φ^e and compute the number of justified envy situations, i.e., the number of blocking pairs.
5. Calculate the average number of justified envy situations across 100 problems.

Figure 1 provides simulation results on the average number of blocking pairs (justified envy situations) against parameter e as e increases from 1 to 8 for various α and β combinations. (Note that since there are 10 schools, $e \geq 9$ coincides with DA.)

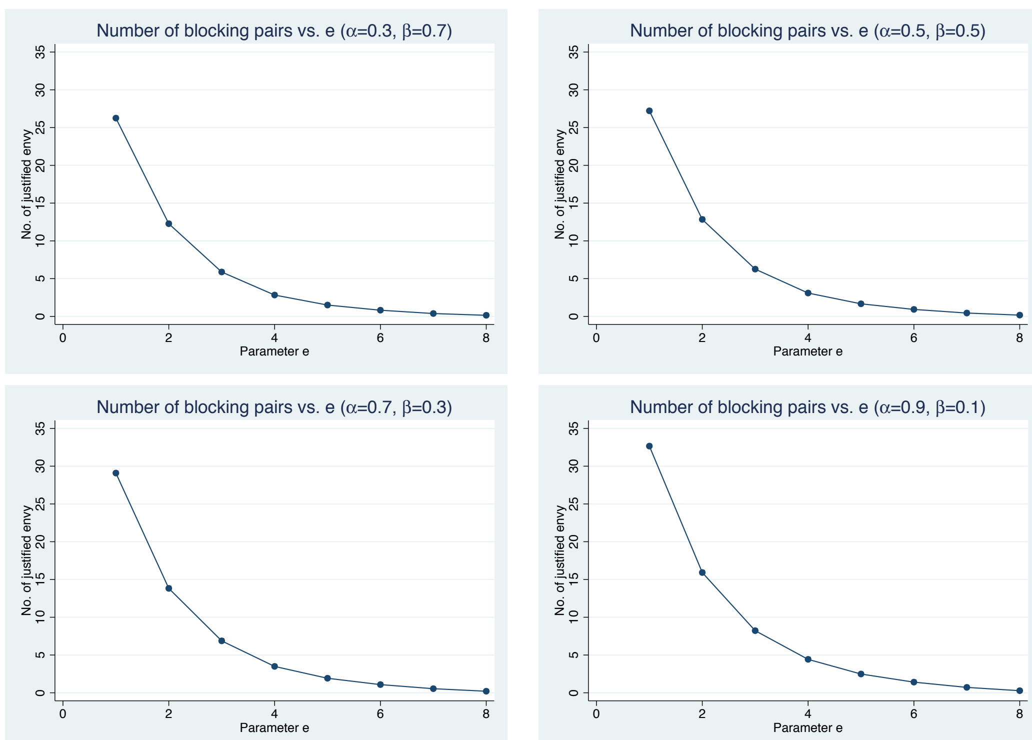

Figure 1: Average number of blocking pairs versus bandwidth e in simulations

Regardless of the specific choices, we find that the average number of justified envy situations exponentially decreases with an increasing e parameter, consistent with our theoretical predictions. Interestingly, switching from the IA mechanism ($e = 1$) to the Shanghai mechanism ($e = 2$) always leads to a more than 50% drop in the number of justified envy situations. Even though one cannot make conclusive problemwise comparisons between two members whose e parameter is not a multiple of the other, these simulations suggest that a higher e parameter leads to a lower average number of justified envy situations.

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