Chinese College Admissions and School Choice Reforms: A Theoretical Analysis*

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Abstract

Each year approximately 10 million high school seniors in China compete for 6 million seats through a centralized college admissions system. Within the last decade, many provinces have transitioned from a 'sequential' to a 'parallel' mechanism to make their admissions decisions. In this study, we characterize a parametric family of application-rejection assignment mechanisms, including the sequential, Deferred Acceptance, and parallel mechanisms in a nested framework. We show that all of the provinces that have abandoned the sequential mechanism have moved towards less manipulable and more stable mechanisms. We also show that existing empirical evidence is consistent with our theoretical predictions.

Keywords: college admissions, school choice, Chinese parallel mechanism, immediate acceptance, deferred acceptance

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Confucius said, "Emperor Shun was a man of profound wisdom. [. . .] Shun considered the two extremes, but only implemented the moderate [policies] among the people." - *Moderation*, Chapter 6[1]

1 Introduction

School choice and college admissions mechanisms are some of the most important and widely-debated education policies in the world. The past two decades have witnessed major innovations in this domain. For example, shortly after Abdulkadiroğlu and Sönmez (2003) was published, New York City high schools replaced their allocation mechanism with a capped version of the student-proposing deferred acceptance (DA) mechanism (Gale and Shapley 1962, Abdulkadiroğlu, Pathak and Roth 2005b). In 2005, the Boston Public School Committee voted to replace its Boston school choice mechanism with the DA mechanism (Abdulkadiroğlu, Pathak, Roth and Sönmez 2005a) after it had been shown the manipulation vulnerability of its existing mechanism (Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006, Chen and Sönmez 2006).

Like school choice in the United States, college admissions policies in China have been subject to debate and reform. The historical decentralized admissions system was fraught with several weaknesses. First, since each student could be admitted into multiple universities, the enrollment-to-admissions ratio was low, ranging from 20% for ordinary universities to 75% among the best universities (Yang 2006, p. 6). Furthermore, some qualified students who were rejected by the top universities missed the application and examination deadlines for ordinary universities and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances to centralize the admissions process. Based on the success of these alliances,[2] the Ministry of Education decided to transition to a centralized matching mechanism and implemented the first National College Entrance Examination, also known as *gaokao*, in August 1952.

Gaokao formed the foundation for the current admissions system. In recent years, roughly 10 million high school seniors compete for 6 million seats at various universities in China each year. The matching of students to universities has profound implications for the education and labor market outcomes for these students. Through its regional variations and its evolution over time, the Chinese system also provides matching theorists and experimentalists with a wealth of field

[1] *Moderation* (*zhōng yōng*) is one of the four most influential classics in ancient Chinese philosophy. Emperor Shun, who ruled China from 2255 BC to 2195 BC, was considered one of the wisest emperors in Chinese history.

[2] This experiment achieved an improved average enrollment-to-admissions ratio of 50% for an ordinary university (Yang 2006, p. 7). The enrollment to admissions ratio for an ordinary university in 1952 was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

observations to enrich our understanding of matching mechanisms (see Online Appendix A for a historical account of the Chinese college admissions system). In this paper, we provide a systematic theoretical characterization of the major Chinese college admissions (CCA) mechanisms.

The CCA mechanisms are centralized matching processes via standardized tests. Each province implements an independent matching process from one of the three classes: sequential, parallel, or asymmetric parallel. The *sequential mechanism,* or Boston or Immediate Acceptance (IA) mechanism, had been the only mechanism used in Chinese student assignments both at the high school and college level (Nie 2007b).[3] However, this mechanism has a serious limitation: "a good score in the college entrance exam is worth less than a good strategy in the ranking of colleges" (Nie 2007a). Indeed, one parent explains the problem as follows (Nie 2007b):

> My child has been among the best students in his school and school district. He achieved a score of 632 in the college entrance exam last year. Unfortunately, he was not accepted by his first choice. After his first choice rejected him, his second and third choices were already full. My child had no choice but to repeat his senior year.

To alleviate this problem of high-scoring students not being accepted by any universities, the *parallel mechanism* was proposed by Zhenyi Wu.[4] In the parallel mechanism, students select several "parallel" colleges within each choice-band. For example, a student's first choice-band may contain a set of three colleges, A, B, and C while her second choice-band may contain another set of three colleges, D, E, and F (in decreasing desirability within each band). Once students submit their choices, colleges process the student applications, using a mechanism where students gain priority for colleges they have listed in their first band over other students who have listed the same college in the second band. Assignments for parallel colleges listed in the same band are considered temporary until all choices of that band have been considered. Thus, this mechanism lies between the IA mechanism, where every choice is final, and the DA mechanism, where every choice is temporary until all seats are filled.

In 2001, Hunan became the first province to transition to the parallel mechanism in its tier 0 admissions to military academies, which precedes the admissions to other four-year colleges. In 2002, Hunan further allowed parallel choice-bands among tiers 2, 3 and 4 colleges. In 2003, Hunan implemented a full version of the mechanism, allowing 3 parallel colleges in the first choice-band,

[3] In China, this mechanism is executed sequentially across tiers in decreasing prestige. In other words, each college belongs to a tier, and within each tier, the IA mechanism is used. When assignments in the first tier are finalized, the assignment process in the second tier starts, and so on. In this paper, we do not explicitly model tiers or other restrictions students face in practice.

[4] Zhenyi Wu was the Director of Undergraduate Admissions at Tsinghua University from 1999 to 2002. Wu discussed the problems with the sequential mechanisms and outlined the parallel mechanism in interviews published in *Beijing Daily* (June 13, 2001), and *Guangming Daily* (July 26, 2001), respectively.

5 in the second choice-band, 5 in the third choice-band, 5 in the fourth choice-band, and so on.[5] By 2012, various versions of the parallel mechanism have been adopted by 28 out of 31 provinces.

The *parallel mechanism* is widely perceived to improve allocation outcomes for students. As a parent in Beijing puts it:[6]

> My child really wanted to go to Tsinghua University. However, [...], in order not to take any risks, we unwillingly listed a less prestigious university as her first choice. Had Beijing allowed parallel colleges in the first choice[-band], we could at least give [Tsinghua] a try.

While students and parents have responded favorably to parallel mechanisms, the properties and outcomes of these mechanisms have not been systematically studied theoretically or empirically. This paper attempts to provide a theoretical analysis of the CCA mechanisms by considering two questions. First, do the parallel mechanisms better serve the interests of students than the sequential (or IA) mechanism? Second, when the number of parallel choices within a choice-band varies, how do manipulation incentives and mechanism stability change? We investigate these questions theoretically in this paper and experimentally in a companion paper (Chen and Kesten 2015).

In our investigation, we use a more general priority structure than that used in the context of college admissions, as both college admissions and school choice in China have transitioned from the IA to parallel mechanisms. In the latter context, students applying for middle schools are prioritized based on their residence, whereas students applying for high schools are prioritized based on their municipal-wide exam scores. In the context of school choice, similar manipulations under the IA mechanism are documented and analyzed in He (2014) using school choice data from Beijing. To our knowledge, Shanghai was the first city to adopt the parallel mechanism for its high school admissions.[7]

To study the performance of the different mechanisms formally, we first undertake a theoretical analysis using a parametric family of *application-rejection* mechanisms where each member is characterized by some positive number $e \in \{1, 2, \ldots, \infty\}$ of parallel and periodic choices through which the application and rejection process continues before assignments are finalized.

In our framework, as parameter e varies, we move from the IA mechanism ($e = 1$) to the Chinese parallel mechanisms ($e \in [2, \infty)$), and to the DA mechanism ($e = \infty$). In this framework,

[5]Information regarding the Hunan reform was obtained from two documents, *Constructing College Applicants' Highway towards Their Ideal Universities: Five years of Practice and Exploration of the Parallel Mechanism Implementation in Gaokao in Hunan* (2006), and *Summary of the Parallel Mechanism Implementation During the 2008 Gaokao in Hunan* (2008). The latter was circulated among the 2008 Ten-Province Collaborative Meeting of the Provincial Examination Institute Directors. We thank Tracy Liu and Wei Chi for sharing these documents and their interview notes with Guoqing Liu, Director of the Hunan Provincial Admissions Office in the early 2000s.

[6]Li Li. "Ten More Provinces Switch to Parallel College Admissions Mechanism This Year." *Beijing Evening News*, June 8, 2009.

[7]`http://edu.sina.com.cn/l/2003-05-15/42912.html`, retrieved on October 31, 2015.

moving from one extreme member to the other entails trade-offs in terms of strategic immunity and stability.[8] Using techniques developed by Pathak and Sönmez (2013), we provide property-based rankings of the members of this family by showing that, whenever any given member can be manipulated by a student, any member with a smaller e number can also be manipulated, but not vice versa (Theorems 1 and 3). Thus, the parallel mechanism used in Tibet ($e = 10$) is *less manipulable* than any other parallel or IA mechanism currently in use. In fact, with the exception of Beijing, Gansu and Shangdong, all of the provinces that have adopted a parallel mechanism have transitioned to a less manipulable assignment system.

Our analysis also shows that, when $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$, any stable equilibrium of the application-rejection mechanism (e) is also a stable equilibrium of the application-rejection mechanism (e'), but not vice versa (Theorems 2 and 4). Thus, the parallel mechanism used in Hainan ($e = 6$) is *more stable* than the version used in Jiangsu ($e = 3$).[9] Indeed, we find that every newly adopted parallel mechanism is more stable than the IA mechanism it replaced.

Although it is well-known that the dominant strategy equilibrium outcome of the DA mechanism Pareto dominates any equilibrium outcome of the IA mechanism (Ergin and Sönmez, 2006), we find no clear dominance of the DA mechanism over any Chinese parallel mechanism (Proposition 4). Moreover, we show that a parallel mechanism provides students with a certain sense of "insurance" by allowing them to list their equilibrium assignments under the IA mechanism as a safety option while listing more desirable options higher up in their preferences. This strategy leads to an outcome at least as good as that of the IA mechanism (Proposition 5). Notably, such insurance does not come at any *ex ante* welfare cost in a stylized setting (Proposition 7 in the Appendix).

The rest of this paper is organized as follows. Section 2 formally introduces the school choice problem and the family of mechanisms. Section 3 presents the theoretical results. Section 4 summarizes existing empirical evidence that describes patterns in the mechanism outcomes over time and compares these with our theoretical predictions. Section 5 concludes. The Appendix contains the proofs, whereas the Online Appendices A and B contain the institutional history of Chinese college admissions, and the auxiliary theoretical results, respectively.

[8] A mechanism is stable if the resulting matching is non-wasteful and there is no unmatched student-school pair (i, s) such that i would rather be assigned to school s where he has higher priority than at least one student currently assigned to it.

[9] Nie and Zhang (2009) investigate the theoretical properties of a variant of the parallel mechanism where each applicant has three parallel colleges, i.e., $e = 3$ in our notation, and characterize the equilibrium when applicant beliefs are i.i.d. draws from a uniform distribution. Wei (2009) considers the parallel mechanism where each college has an exogenous minimum score threshold drawn from a uniform distribution. Under this scenario, she demonstrates that increasing the number of parallel options cannot make an applicant worse off.

2 The school choice problem and the three mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) is comprised of a number of students, each of whom is to be assigned a seat at one of a number of schools. Further, each school has a maximum capacity, and the total number of seats in the schools is no less than the number of students. We denote the set of students by $I = \{i_1, i_2, \ldots, i_n\}$, where $n \geq 2$. A generic element in I is denoted by i. Likewise, we denote the set of schools by $S = \{s_1, s_2, \ldots, s_m\} \cup \{\emptyset\}$, where $m \geq 2$ and $\emptyset$ denotes a student's outside option, or the so-called null school. A generic element in S is denoted by s. Each school, s, has a number of available seats, q_s, or the **quota** of s. Let $q_\emptyset = \infty$. For each school, there is a strict priority order for all students, and each student has strict preferences over all schools. The priority orders are determined according to both state or local laws as well as certain criteria set by school districts. We denote the priority order for school s by $\succ_s$, and the preferences of student i by P_i. Let R_i denote the at-least-as-good-as relation associated with P_i. Formally, we assume that R_i is a linear order, i.e., a complete, transitive, and anti-symmetric binary relation on S. That is, for any $s, s' \in S$, $s \; R_i \; s'$ if and only if $s = s'$ or $s \; P_i \; s'$. For convenience, we sometimes write $P_i : s_1, s_2, s_3, \ldots$ to denote that, for student i, school s_1 is his first choice, school s_2 his second choice, school s_3 his third choice, etc.

A **school choice problem**, or simply a problem, is a pair $(\succ = (\succ_s)_{s \in S}, P = (P_i)_{i \in I})$ consisting of a collection of priority orders and a preference profile. Let $\mathcal{R}$ be the set of all problems. A **matching** μ is a list of assignments such that each student is assigned to one school and the number of students assigned to a particular school does not exceed the quota of that school. Formally, a matching is a function $\mu : I \to S$ such that for each $s \in S$, $|\mu^{-1}(s)| \leq q_s$. Given $i \in I$, $\mu(i)$ denotes the assignment of student i at μ. Given $s \in S$, $\mu^{-1}(s)$ denotes the set of students assigned to school s at μ. Let $\mathcal{M}$ be the set of all matchings. A matching μ is **non-wasteful** if no student prefers a school with an unfilled quota to his assignment. Formally, for all $i \in I$, $s \; P_i \; \mu(i)$ implies $|\mu^{-1}(s)| = q_s$. A matching μ is **Pareto efficient** if there is no other matching which makes all students at least as well off and at least one student better off. Formally, there is no $\alpha \in \mathcal{M}$ such that $\alpha(i) \; R_i \; \mu(i)$ for all $i \in I$ and $\alpha(j) \; P_j \; \mu(j)$ for some $j \in I$.

A closely related problem to the school choice problem is the *college admissions problem* (Gale and Shapley 1962). In the college admissions problem, schools have preferences over students, whereas in a school choice problem, schools are objects to be consumed. A key concept in college admissions is "stability," i.e., there is no unmatched student-school pair (i, s) such that student i prefers school s to his assignment, and school s either has not filled its quota or prefers student i to at least one student who is assigned to it. The natural counterpart of stability in our context is defined by Balinski and Sönmez (1999). The **priority of student i for school s is violated** in a given matching μ (or alternatively, student i justifiably envies student j for school s) if i would rather be assigned to s to which some student j with lower $s-$priority than i has been assigned, i.e., $s\ P_i\ \mu(i)$ and $i \succ_s j$ for some $j \in I$. A matching is **stable** if it is non-wasteful and no student's priority for any school is violated.

A **school choice mechanism**, or simply a mechanism φ, is a systematic procedure that chooses a matching for each problem. Formally, it is a function $\varphi : \mathcal{R} \to \mathcal{M}$. Let $\varphi(\succ, P)$ denote the matching chosen by φ for problem $(\succ, P)$ and let $\varphi_i(\succ, P)$ denote the assignment of student i at this matching. A mechanism is Pareto efficient (stable) if it always selects Pareto efficient (stable) matchings. A mechanism φ is **strategy-proof** if it is a dominant strategy for each student to truthfully report his preferences. Formally, for every problem $(\succ, P)$, every student $i \in I$, and every report P_i', $\varphi_i(\succ, P)\ R_i\ \varphi_i(\succ, P_i', P_{-i})$.

Following Pathak and Sönmez (2013), a mechanism ϕ is *manipulable by student* j at problem $(\succ, P)$ if there exists P_j' such that $\phi_j(\succ, P_j', P_{-j})\ P_j\ \phi_j(\succ, P)$. Thus, mechanism ϕ is said to be *manipulable* at a problem $(\succ, P)$ if there exists some student j such that ϕ is manipulable by student j at $(\succ, P)$. We consider mechanism φ to be **more manipulable** than mechanism ϕ if (i) at any problem ϕ is manipulable, then φ is also manipulable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is manipulable but ϕ is not. We consider mechanism φ to be **more stable** than mechanism ϕ if (i) at any problem ϕ is stable, φ is also stable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is stable but ϕ is not.[10] [11]

We now describe the three mechanisms that are central to our study. The first two are the IA and the DA mechanisms, while the third one is a stylized version of the simplest parallel mechanism.

2.1 Immediate Acceptance Mechanism (IA)

The Immediate Acceptance mechanism was the prevalent college admissions mechanism in China in the 1980s and 1990s. In the context of school choice, it is commonly referred as the Boston mechanism. The outcome of the *Immediate Acceptance (IA) mechanism* can be calculated via the following algorithm for a given problem:

Step 1: For each school s, consider only those students who have listed it as their first choice. Up to q_s students among them with the highest $s-$priority are assigned to school s.

Step k, $k \geq 2$: Consider the remaining students. For each school s with q_s^k available seats, consider only those students who have listed it as their k-th choice. Those q_s^k students among them with the highest $s-$priority are assigned to school s.

[10] See Kesten (2006 and 2011) for similar problem-wise property comparisons across and within mechanisms for matching problems.

[11] An *ex ante* interpretation of the relative stability metric introduced here could mean that a more stable mechanism is more likely to produce a stable matching before the actual realization of student preferences and school priorities.

The algorithm terminates when there are no students left to assign. Importantly, the assignments in each step are final. Based on this feature, an important critique of the IA mechanism highlighted in the literature is that it gives students incentives to misrepresent their preferences, as a student who has high priority for a school may lose her priority advantage for that school if she does not list it as her first choice (see e.g., Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006, Chen and Sönmez 2006, and He 2014).

2.2 Deferred Acceptance Mechanism (DA)

By contrast, the student-optimal stable mechanism (Gale and Shapley 1962) finds the stable matching that is most favorable to each student. Its outcome can be calculated via the following *Deferred Acceptance (DA) algorithm* for a given problem:

Step 1: Each student applies to her favorite school. For each school s, up to q_s applicants who have the highest $s-$priority are tentatively assigned to school s. The remaining applicants are rejected.

Step k, $k \geq 2$: Each student rejected from a school at step $k-1$ applies to her next favorite school. For each school s, up to q_s students who have the highest $s-$priority among the new applicants and those tentatively on hold from an earlier step, are tentatively assigned to school s. The remaining applicants are rejected.

The algorithm terminates when each student is tentatively placed to a school. Note that DA assignments in each step are temporary until the last step. The DA mechanism has several desirable theoretical properties, most notably in terms of incentives and stability. Under the DA mechanism, it is a dominant strategy for students to state their true preferences (Roth 1982, Dubins and Freedman 1981). Furthermore, it is stable. Although it is not Pareto efficient, it is the most efficient among the stable school choice mechanisms.

In practice, the DA mechanism has been the leading mechanism for school choice reforms. For example, the DA mechanism has been adopted by New York City and Boston public school systems, which had suffered from congestion and incentive problems, respectively, from their previous assignment systems (Abdulkadiroğlu et al. 2005a, Abdulkadiroğlu et al. 2005b).

2.3 The Chinese Parallel Mechanisms

As mentioned in the introduction, a Chinese parallel mechanism was first implemented in Hunan tier 0 college admissions in 2001. From 2001 to 2012, variants of the mechanism have been adopted by 28 provinces to replace IA mechanisms (Wu and Zhong 2014).

While there are many regional variations in CCA, from a game theoretic perspective, they differ in two main dimensions. The first dimension is the timing of preference submission. Some mechanism variations require submission before the exam (2 provinces), others after the exam but *before* knowing the exam scores (3 provinces), and others *after* knowing the exam scores (26 provinces).[12] The second dimension is the actual matching mechanisms. In 2012, while the IA mechanism was still used in 2 provinces, variants of the parallel mechanisms have been adopted by 28 provinces, while the remaining province, Inner Mongolia, uses an admissions process which resembles a dynamic implementation of the parallel mechanism.

In this study, we investigate the properties of the family of mechanisms used for Chinese school choice and college admissions. We begin with a stylized version in its simplest form, with two parallel choices per choice-band. A more general description is contained in Section 3.

- An application to the first ranked school is sent for each student.
- Throughout the allocation process, a school can hold no more applications than its quota.

  If a school receives more applications than its quota, it retains the students with the highest priority up to its quota and rejects the remaining students.
- Whenever a student is rejected from her first-ranked school, her application is sent to her second-ranked school. Whenever a student is rejected from her second-ranked school, she can no longer make an application in this round.
- Throughout each round, whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the ones with the highest priority up to the quota are retained.
- The allocation is finalized every two choices. That is, if a student is rejected by her first two choices in the initial round, then she participates in a new round of applications together with other students who have also been rejected from their first two choices, and so on. At the end of each round, the assigned students and the slots assigned to them are removed from the system.

[12] Zhong, Cheng and He (2004) demonstrate that, while there does not exist a Pareto ranking of the three variants in the preference submission timing, the first two mechanisms can sometimes achieve Pareto efficient outcomes. Furthermore, experimental studies confirm the *ex ante* efficiency advantage of the IA mechanism with pre-exam preference ranking submissions in both small (Lien, Zheng and Zhong 2012) and large markets (Wang and Zhong 2012). Lastly, using a data set from Tsinghua University, Wu and Zhong (2014) find that, while students admitted under the IA mechanism with pre-exam preference ranking submissions have, on average, lower entrance exam scores than those admitted under other mechanisms, they perform as well or even better in college than their counterparts admitted under other timing mechanisms.

The assignment process ends when no more applications can be rejected. We refer to this mechanism as the *Shanghai mechanism*.[13]

In the next section, we offer a formal definition of the parallel mechanisms and characterize the theoretical properties of this family of matching mechanisms.

3 A parametric family of mechanisms

In this section, we investigate the theoretical properties of a symmetric family of application-rejection mechanisms. Given student preferences, school priorities, and school quotas, we begin by outlining a parametric *application-rejection algorithm* that indexes each member of the family by a *permanency-execution period* e:

Round t $=0$**:**

- Each student applies to his first choice. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e-th choice school, applies to his next choice. If a student has been rejected from all his first e choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest of the applicants are rejected.
- The round terminates whenever each student is either assigned to a school or is unassigned in this round, i.e., he has been rejected by all his first e choice schools. At this point, all tentative assignments become final and the quota of each school is reduced by the number of students permanently assigned to it.

In general,

Round t ≥ 1**:**

[13] In the Online Appendix A, we provide a translation of an online Q&A about the Shanghai parallel mechanism used for middle school admissions to illustrate how the parallel choices work.

- Each unassigned student from the previous round applies to his $te+1$-st choice school. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest of the applicants are rejected.

In general,

- Each rejected student, who is yet to apply to his $te+e$-th choice school, applies to his next choice. If a student has been rejected from all his first $te+e$ choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest of the applicants are rejected.

- The round terminates whenever each student is either assigned to a school or is unassigned in this round, i.e., he has been rejected by all his first $te+e$ choice schools. At this point, all tentative assignments become final and the quota of each school is reduced by the number of students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point, all the tentative assignments become final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* (e), denoted by φ^e. This family of mechanisms nests the IA and the DA mechanisms as extreme cases, the Chinese parallel mechanisms as intermediate cases, and the Chinese asymmetric parallel mechanisms as extensions (see Section 3.3).[14]

Remark 1 *The application-rejection mechanism* (e) *is equivalent to*

(i) the IA mechanism when $e = 1$,

(ii) the Shanghai mechanism when $e = 2$,

(iii) the Chinese parallel mechanism when $2 \leq e < \infty$, *and*

(iv) the DA mechanism when $e = \infty$.

Remark 2 *It is easy to verify that all members of the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$, *are non-wasteful. Hence, the outcome of an application-rejection mechanism is stable for a given problem if and only if it does not result in a priority violation.*

[14]From a modeling vantage point, our main analysis could have alternatively been based on the more general setting of Section 3.3. However, as will be seen, the main findings about both families of mechanisms are essentially driven by the number of choices considered within the initial round (rather than any other round); therefore, we have adopted the simpler modeling approach to facilitate the exposition and illustration of ideas.

We now examine the properties of this family of application-rejection mechanisms.

Proposition 1 *Within the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$,
(i) there is exactly one member that is Pareto efficient. This is the IA mechanism;
(ii) there is exactly one member that is strategy-proof. This is the DA mechanism; and
(iii) there is exactly one member that is stable. This is the DA mechanism.

All proofs are relegated to the Appendix.

3.1 Property-specific comparisons of application-rejection mechanisms

As Proposition 1 shows, an application-rejection (e) mechanism is manipulable if $e < \infty$. Hence, when faced with a mechanism other than the DA mechanism, students must determine their optimal strategy, especially which e schools they should list on top of their rank-ordered lists. More specifically, since priorities matter only *within* a round, getting assigned to one of her first e choices is crucial for a student.

When $e < \infty$, a successful strategy for a student is one that ensures that he is assigned to his "target school" at the end of the initial round, i.e., round 0. If a student is not assigned to a first choice school in round 0, the IA mechanism could be more costly than a parallel mechanism such as the Shanghai mechanism, which offers a "second chance." On the other hand, the DA mechanism completely eliminates any possible loss of priority advantage. The three-way tension among incentives, stability, and welfare that emerges under this class is rooted in this observation.

We next provide an incentive-based ranking of the family of application-rejection mechanisms.

Theorem 1 (Manipulability) *For any* e, φ^e *is more manipulable than* $\varphi^{e'}$ *where* $e' > e$.

While our proof of Theorem 1 is contained in the Appendix, the Online Appendix B provides two examples. Example 1a shows that the IA mechanism is manipulable when the Shanghai mechanism is also manipulable, whereas Example 1b shows that the Shanghai mechanism is not manipulable when the IA mechanism is.

Corollary 1 *Among application-rejection mechanisms, the IA mechanism is the most manipulable and the DA mechanism is the least manipulable member.*

Corollary 2 *Any Nash equilibrium of the preference revelation game associated with* φ^e *is also a Nash equilibrium of that of* $\varphi^{e'}$ *where* $e' > e$.

Remark 3 *Notwithstanding the manipulability of all application-rejection mechanisms except the DA, it is still in the best interest of each student to report his within-round choices in their true order. More precisely, for a student facing φ^e, any strategy that does not truthfully list the first e choices for consideration in the initial round is dominated by an otherwise identical strategy that lists them in their true order. Similarly, not listing a set of e choices for consideration in a subsequent round is also dominated by an otherwise identical strategy that lists them in their true order.*

Note that Corollary 2 implies that the set of Nash equilibrium strategies corresponding to the preference revelation games associated with members of the application-rejection family has a nested structure.[15] Thus, when we make problemwise comparisons across members of the application-rejection family (e.g., see Proposition 2), such comparisons might as well be made across equilibria of the two different members.

We next investigate how member mechanisms of the family rank based on stability. An immediate observation is that, under an application-rejection (e) mechanism, no student's priority for one of his first e choices is ever violated.[16] This is true as all previous assignments are tentative until the student is rejected from all his first e choices. However, it should not be assumed that mechanisms become more stable as parameter e increases.

Proposition 2 (Stability) *Let $e' > e$.*

(i) If $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is more stable than φ^e.

(ii) *If $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is not more stable than φ^e.*

Remark 4 *As a complementary remark, the following is also true: Let $e' = ke$. Given any problem, take any student who has justified envy under mechanism $\varphi^{e'}$. Then for the same problem under mechanism φ^e, either the student again has justified envy or he causes a corresponding student to have justified envy. In Online Appendix B, we report results from numerical simulations which show that the average number of justified envy situations decreases exponentially as parameter e increases.*

Corollary 3 *The DA mechanism is more stable than the Shanghai mechanism, which is more stable than the IA mechanism.*

Corollary 4 *Any other (symmetric) application-rejection mechanism is more stable than the IA mechanism.*

[15]A similar observation is made by Haeringer and Klijn (2008) for the revelation games under the IA mechanism when the number of school choices a student can make (in her preference list) is limited by a quota.

[16]In other words, an application-rejection (e) mechanism is stable with respect to the first e choices. Similarly, a student never has justified envy toward another student who is assigned within the same or at a later round.

Proposition 2 indicates that, within the parallel mechanisms, if the number of choices considered in each round by one mechanism is a multiple of the number considered by another mechanism, the mechanism that allows for more choices within a choice-band is the more stable one. Intuitively, if $e' = ke$, then every k round of φ^e can be embedded in a single round of $\varphi^{e'}$. Thus, whenever a φ^e assignment respects a student's priorities in any round (i.e., it is stable), the $\varphi^{e'}$ assignment necessarily does so.

With Proposition 2 and Theorem 1, we can now compare stability properties of certain members across equilibria.

Theorem 2 (Stable Equilibria) *Let $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$. Any equilibrium of φ^e that leads to a stable matching is also an equilibrium of $\varphi^{e'}$ and leads to the same stable matching. However, the converse is not true, i.e., there are stable equilibria of $\varphi^{e'}$ that may be neither equilibria nor stable under φ^e.*

Theorem 2 shows that the set of stable equilibrium profiles (i.e., the equilibrium profiles that lead to a stable matching under students' true preferences) for a given application-rejection mechanism φ^e is (strictly) smaller than that of $\varphi^{e'}$ whenever e' is a multiple of e. This implies, for example, that the Shanghai mechanism admits a larger set of stable equilibrium profiles than the IA mechanism.

A common, albeit questionable, metric often used by practitioners as a measure of students satisfaction is based on considering the number of students assigned to their first choices.[17] We see that, the IA mechanism is the most generous in terms of first choice accommodation, whereas the DA mechanism is the least.

Proposition 3 (Choice accommodation) *Within the class of application-rejection mechanisms,*

(i) φ^e *assigns a higher number of students to their first choices than* $\varphi^{e'}$, *where* $e < e'$.

(ii) φ^e *assigns a higher number of students to their first* e *choices than* $\varphi^{e'}$, *where* $e \neq e'$.

Corollary 5 *Within the class of application-rejection mechanisms, the IA mechanism maximizes the number of students receiving their first choices.*

Corollary 6 *Within the class of application-rejection mechanisms, the Shanghai mechanism maximizes the number of students receiving their first or second choices.*

[17]For example, in evaluating the outcome of the IA mechanism, Cookson Jr. (1994) reports that 75% of all students entering the Cambridge public school system at the K-8 levels gained admission to the school of their first choice. Similarly, the analysis of the Boston and NYC school district data by Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) and Abdulkadiroğlu, Pathak and Roth (2009) also reports the number of first choices of students.

Note that Proposition 3 is based on the observation that upon the termination of the initial round of φ^e every assigned student receives one of his first e choices and every unassigned student has already been rejected from all of his first e choices. Then contrasting the initial rounds of φ^e and $\varphi^{e'}$ implies the above finding. This result might help understand why mechanisms such as the IA or the Shanghai mechanism that prioritize first, or first and second, choices may be more appealing to authorities.

However, one needs to be cautious when interpreting Proposition 3, as students may change strategies across mechanisms or choose not to report their true preferences (except under the DA mechanism). To address this issue, we next investigate the properties of Nash equilibrium outcomes for the family of application-rejection mechanisms.

3.2 Equilibria of the Induced Preference Revelation Games: *Ex post* equilibria

Ergin and Sönmez (2006) show that every Nash equilibrium outcome of the preference revelation game induced by the IA mechanism leads to a stable matching under students' true preferences. They further show that any given stable matching can be supported as a Nash equilibrium of this game. These results have a clear implication. Since the DA mechanism is strategy-proof and chooses the most favorable stable matching for students, the IA mechanism can only be *at best* as good as the DA mechanism in terms of the resulting welfare. Put differently, there is a clear welfare loss associated with the IA mechanism relative to the DA mechanism.

To analyze the properties of the equilibrium outcomes of the application-rejection mechanisms, we next study the Nash equilibrium outcomes induced by the preference revelation games under this family of mechanisms. It turns out that the DA mechanism does not generate a clear welfare gain relative to the Chinese parallel mechanisms.

Proposition 4 (*Ex post* equilibria) *Consider the preference revelation game induced by φ^e under complete information.*

(i) If $e = 1$, then, for every problem, every Nash equilibrium outcome of this game is stable and thus Pareto dominated by the DA mechanism under the true preferences.

(ii) If $e \notin \{1, \infty\}$, there exist problems where the Nash equilibrium outcomes of this game, in undominated strategies, are unstable and Pareto dominate the DA mechanism under the true preferences.[18]

Unlike the IA mechanism, we find an ambiguous result when we do the welfare comparison between the equilibria of the DA and the Chinese parallel mechanisms. To understand this result,

[18]Note that the DA mechanism also admits Nash equilibria that lead to unstable matchings that Pareto dominate the DA outcome under the true preferences. However, any such equilibria necessarily involve a dominated strategy.

consider the IA outcome. Under the IA mechanism, only students' first choices matter for equilibrium play. As such, if a student has justified envy, then a profitable deviation exists. By contrast, under a parallel mechanism, the first e choice assignments are tentative until the initial round terminates. Thus, a student with justified envy is not guaranteed his desired school via a potential deviation where he ranks that school as first choice. In fact, such a move may trigger a rejection chain that may cause him to be rejected from his desired school. Indeed, by carefully choosing second and/or lower choices, students can coordinate on mutually beneficial school assignments, even though this may cause others to be justifiably envious.

The fact that the IA and parallel mechanisms both admit multiple equilibria precludes a direct equilibrium-wise comparison between the two mechanisms. Nonetheless, we are interested in knowing whether there is any validity to the widespread belief (also expressed in a quote in the introduction) that a parallel mechanism better serves student interests than does the IA mechanism. The next proposition offers a formal argument that a parallel mechanism may indeed be more favorable for each student relative to the IA mechanism.

Proposition 5 (Insurance under the Parallel Mechanisms) *Let μ be any equilibrium outcome under the IA mechanism. Under φ^e, if each student i lists $\mu(i)$ as one of his first e choices and also lists any schools he truly prefers over $\mu(i)$ as higher-ranked choices, then each student's parallel mechanism assignment is at least as good as his IA mechanism assignment.*[19]

Remark 5 *Since Proposition 5 hinges on part (i) of Proposition 4, we cannot generalize it to any two application-rejection mechanisms. For example, let μ be an equilibrium outcome of the Shanghai mechanism. If each student lists his assignment at μ as one of his first e choices as above, then the resulting outcome of φ^e with any $e > 2$ need not be weakly preferred by each student to that of the Shanghai mechanism.*[20]

From a practical point of view, Proposition 5 implies that, whatever school a student is "targeting" under the IA mechanism, he would be at least as well off under a parallel mechanism by simply including his target school among his first e choices while ranking better options higher up in his preferences, provided that other students do the same. In other words, the Chinese parallel mechanisms may allow students to retain their would-be assignments under the IA mechanism as "insurance" options while keeping more desirable options within reach. Practitioners seem to understand this aspect of the parallel mechanism. For example, the official Tibetan gaokao website starts with the following introduction to its admissions mechanism:

[19]We stipulate that the e-th choice is the last choice when $e = \infty$. For expositional simplicity, we also assume that student i has $e - 1$ truly better choices than $\mu(i)$.

[20]To illustrate this point for the Shanghai vs. the DA mechanisms, for example, let μ correspond to an unstable equilibrium outcome that Pareto dominates the DA matching under truthtelling.

> To reduce the risks applicants bear when submitting their college rank-ordered lists, and to reduce the applicants' psychological pressure, the Tibet Autonomous Region [will] implement the parallel mechanism among ordinary colleges in 2012.[21]

We can also interpret Proposition 5 as a statement regarding the degree of student coordination. Let μ^{DA} be the DA outcome under a given profile of students' true preferences. This is indeed also an equilibrium outcome of the IA mechanism for a profile of reports where each student lists his DA assignment as his first choice. Nevertheless, as our experimental analysis confirms, it is unlikely to expect to observe students coordinating on one such equilibrium (Chen and Kesten 2015). In practice, the use of such strategies may as well entail potentially large costs for students in cases of miscoordination. Proposition 5 suggests that if each student includes his DA assignment among his first e choices under φ^e, and is otherwise truthful about the choices he declares to be more desirable, he will be guaranteed an assignment no worse than what he would receive under the DA mechanism. Notably, this conclusion does not depend on whether or not the profile of student reports constitutes an equilibrium of φ^e: the outcome of φ^e always Pareto dominates that of the DA mechanism. Interestingly, if such a profile is a disequilibrium under φ^e, then the outcome of φ^e strictly Pareto dominates that of the DA mechanism, making at least one student strictly better off under φ^e in comparison to the DA mechanism. In this sense, the Chinese parallel mechanisms may facilitate coordination on desirable outcomes and may thus reduce the risk of miscoordination under the IA mechanism. In particular, the higher the e parameter, the easier it becomes for students to include their DA assignments among their first e choices. In the extreme case of $e = \infty$, the DA assignment is necessarily one of the e choice of each student and the resulting outcome is that of the DA itself, which is also an equilibrium.

3.3 The Asymmetric Class of Chinese Parallel Mechanisms

In our previous discussion, we looked at the symmetric Chinese parallel mechanisms, where e is constant across rounds. However, 15 provinces allow for variations in the number of choices that are considered within a round. Table 1 in the Online Appendix A provides a complete list of choice sequences used in various provinces across China in 2012.

To consider asymmetric mechanisms, we first augment the application-rejection family to accommodate this asymmetry. For a given problem, let φ^S denote the *application-rejection mechanism* associated with a choice sequence $S = (e_0, e_1, e_2, \ldots)$, where the terms in the sequence respectively denote the number of choices tentatively considered in each round. (See the Appendix for a more precise description.)

[21] `http://gaokao.chsi.com.cn/gkxx/ss/201201/20120130/278496141-3.html`, retrieved on October 31, 2015.

We next investigate the incentive and stability properties of the asymmetric class of Chinese parallel mechanisms.

Theorem 3 (Manipulability of the Asymmetric Class) *An application-rejection mechanism associated with a choice sequence $S = (e_0, e_1, e_2, \ldots)$ is more manipulable than any application-rejection mechanism associated with a choice sequence $S' = (e'_0, e'_1, e'_2, \ldots)$ where $e_0 < e'_0$.*

Theorem 3 implies that a mechanism using a choice sequence of a lower number of parallel colleges in the initial round is more manipulable than a corresponding asymmetric parallel mechanism with a greater number of parallel colleges. This result underscores the importance of the initial round relative to all other rounds, a point much emphasized in the previous literature in the context of the IA mechanism.[22] Using Theorem 3, we obtain the following complete manipulability ranking among the CCA mechanisms.

Corollary 7 *The following is the manipulability order of mechanisms in various provinces of China, starting with those that are the most manipulable: $\{$Heilongjiang, Qinghai, Shandong, Gansu, Beijing$\}$ $>$ $\{$Guangdong, Jiangsu, Liaoning$\}$ $>$ $\{$Anhui, Shǎnxi, Guangxi, Jiangxi, Fujian, Ningxia, Shanghai, Xinjiang$\}$ $>$ $\{$Sichuan, Hebei, Hubei, Shānxi, Hunan, Zhejiang, Guizhou, Yunan, Jilin, Tianjin$\}$ $>$ $\{$Hainan, Henan, Chongqing$\}$ $>$ Tibet.*

While Heilongjiang, Qinghai, Shandong, Gansu, and Beijing have mechanisms that are most manipulable, Tibet stands out as the home to the least manipulable parallel mechanism. We next investigate the stability properties of the asymmetric class, beginning with a definition.

Definition 4 *A choice sequence $S = (e_0, e_1, e_2, \ldots)$ is an* <u>*additive decomposition*</u> *of another choice sequence $S' = (e'_0, e'_1, e'_2, \ldots)$ if and only if there exist indexes $t_0 < t_1 < \cdots < t_k < \cdots$ such that*

$$e'_0 = \sum_{i=0}^{t_0} e_i;\ e'_1 = \sum_{i=t_0+1}^{t_1} e_i; \cdots ; e'_k = \sum_{i=t_{k-1}+1}^{t_k} e_i, \cdots, \textit{etc.}$$

In words, if the sequence S is an additive decomposition of the sequence S', then it possible to write each term in S' as a sum of distinct but consecutive terms in S, starting with the first term and following the order of the indexes. For example, the sequence corresponding to the IA mechanism, represented by $S^{IA} = (1, 1, 1, \ldots)$, is an additive decomposition of the Shanghai sequence, represented by $S^{SH} = (2, 2, 2, \ldots)$. In fact, any sequence can be obtained from the IA sequence, as we state below.

[22] Intuitively, the reason why the ranking depends on only the number of parallel choices of the initial round is because manipulations in subsequent rounds can always be "translated" to the initial round by including the target school among the parallel choices of the initial round. Consequently, the number of choices in subsequent rounds does not matter for manipulability.

Remark 6 *The sequence corresponding to the IA mechanism is an additive decomposition of any sequence corresponding to any symmetric or asymmetric member of the application-rejection family.*

Our next proposition, which is an analogue of Proposition 2, shows that any two members of the application-rejection family represented by comparable sequences according to additive decomposition are also comparable according to their stability properties.

Proposition 6 (Stability of the Asymmetric Class) *Let φ^S and $\varphi^{S'}$ be two application-rejection mechanisms, represented by the choice sequences S and S', respectively.*

(i) If S is an additive decomposition of S', then $\varphi^{S'}$ is more stable than φ^S.

(ii) If S is not an additive decomposition of S', then $\varphi^{S'}$ is not more stable than φ^S.

Proposition 6 has a remarkable implication. Among the provinces where the IA mechanism was abandoned, all the successors are more stable.

Corollary 8 *All CCA mechanisms that replaced the IA mechanism are more stable than the IA mechanism.*

Proposition 6 also enables us to obtain cross-province stability comparisons among some of the parallel mechanisms currently in use.

Corollary 9 *The following are stability rankings among provincial parallel mechanisms.*

- *Sichuan and Shānxi are more stable than Shandong.*
- *Anhui, Shǎnxi, Guangxi, Jiangxi and Ningxia are more stable than Gansu and Beijing.*
- *Tibet is more stable than Hebei, Hunan, Zhejang, Tianjin, Yunan and Guizhou.*
- *Hainan is more stable than Jiangsu.*

The following analogue of Theorem 2 obtained from Proposition 6 and Theorem 3 enables us to compare stability properties across equilibria and is applicable to all the comparisons given in Corollary 9.

Theorem 5 (Stable Equilibria of the Asymmetric Class) *Let φ^S and $\varphi^{S'}$ be two application-rejection mechanisms, respectively represented by choice sequences S and S', where S is an additive decomposition of S' and $e_0 < e'_0$. Any equilibrium of φ^S that leads to a stable matching is also an equilibrium of $\varphi^{S'}$ and leads to the same stable matching. However, the converse is not true, i.e., there are stable equilibria of $\varphi^{S'}$ that may not be either equilibrium or stable under φ^S.*

In this section, we have shown how parallel mechanisms improve on the IA mechanism in terms of reduced manipulability, increased insurance and greater stability. In his interview with *Guangming Daily* (July 26, 2001), Zhenyi Wu, Director of Undergraduate Admissions at Tsinghua University at the time, notes this need for improvement of the IA mechanism:

> Therefore, [with the IA mechanism] applicants have two types of choices: either to apply for one's ideal college and take the risk of "falling off the cliff," or retreat and settle for a not-so-ideal college to reduce the risk, and thereby having to give up one's interests, passion and ideal.

He further notes the benefits of the parallel mechanism: "based on this method, applicants can apply for colleges based on one's aptitude and interests," which we interpret as one's true preferences. He asserts that the parallel mechanism will "induce the optimal allocation of educational resources" by reducing the number of students who repeat the last year of high school and ensuring that more [high-ranking] colleges receive a sufficient number of applications.

While he points to the overall benefits of adopting parallel mechanisms, it is not clear if practitioners understand the tradeoffs within the family of parallel mechanisms. Therefore, the initial choice of a parallel mechanism (e) is likely to be for reasons unrelated to game theoretic or welfare considerations. In Hunan Province, for example, Guoqing Liu, Director of the Hunan Provincial Admissions Office during the early 2000s, explained that the reason they set the number of parallel colleges for the first choice-band to three ($e = 3$) was because they found three "1" looking symbols, i.e., the Arabic number "1," the Roman numeral "I," and the English letter "l" (*elle*). He conjectured that this listing would make each of the three parallel colleges perceive that they were ranked number one, despite the decreased desirability from 1 (one) to l (*elle*).[23] Our theoretical analysis can thus aid practitioners by providing insights on the precise tradeoffs within the family of parallel mechanisms.

4 Empirical Evidence

In this section, we examine the empirical evidence regarding the patterns in the mechanism outcomes over time in Chinese college admissions. The empirical literature on Chinese college admission reforms test the effects of two major institutional changes on student strategies and matching outcomes: (1) the timing of the rank-ordered list (ROL) submission from pre- to post-exam (Wu and Zhong 2014), and (2) the change from the IA to the parallel mechanism. The latter is the focus of this paper.

[23] We are grateful to Tracy Xiao Liu and Wei Chi for sharing their notes from their interview with Guoqing Liu (August 14, 2013).

To our knowledge, three empirical papers evaluate the effects of switching from the IA mechanism to the parallel mechanism. Each study examines the effects by exploiting outcome variations between the final year of a retiring mechanism and the first year of a new mechanism. In the first study, Hou, Zhang and Li (2009) use survey and interview data from 2008 to examine the effects on Shanghai admissions decisions of adopting the parallel mechanism. They do so by comparing the proportion of students who decline their matches in 2008 with the proportion who do so in 2007, when the IA mechanism was in place. The second study uses province-level student admissions data to study the impact of a permanency-execution period increase on matching outcomes (Wei 2015). Wei analyzes both the change from the IA mechanism ($e = 1$) to the parallel mechanism ($e = 5$) in Hubei Province, and the change within the parallel mechanism from $e = 3$ to $e = 5$ in Hunan Province. The third study uses county-level micro data to evaluate the change from the IA mechanism ($e = 1$) to the parallel mechanism ($e = 5$) for Tier 1 admissions in Sichuan Province (Chen, Jiang and Kesten 2015). The first two studies rely on outcome data, while the third utilizes a data set containing both the student rank-ordered lists and their matching outcomes. This data set enables the authors to analyze both student strategies and matching outcomes.

Examining our theory in the context of these studies, we first develop hypotheses from the theory and then relate the empirical findings to these hypotheses. Note that we do not observe students' true preferences. Therefore, we infer their preferences based on a set of assumptions. For example, we assume that a student prefers a higher-ranked college to a lower-ranked one. Based on Theorems 1 and 3, we expect:

Hypothesis 1 (Manipulability) *There will be less manipulation of the rank-ordered list under the parallel mechanism, compared to the IA mechanism.*

Preference manipulations can exhibit themselves in a variety of ways, such as a truncation of true preferences in the ROL (Roth and Rothblum 1999), or the selection of a safe college as one's top choice, such as a local college which allocates more of its quota to local students (Chen and Sönmez 2006). To investigate student strategies, Chen et al. (2015) compare the length of the student ROL, the prestige and location of the first-choice colleges in Tiers 1, 2 and 3 admissions in 2008 and 2009. Since Sichuan Province adopted a parallel mechanism for only Tier 1 admissions in 2009, they are able to use a difference-in-differences analysis. We summarize their findings below.

Result 1 (Manipulability) *In Sichuan Province, changing the Tier 1 admissions mechanism from the IA mechanism in 2008 to the parallel mechanism ($e = 5$) in 2009 leads to a significant increase of the length of the ROL by approximate one more college, and a marginally-significant 4% decrease in the likelihood of listing a local college as the first choice (Chen et al. 2015).*

Result 1 suggests that the replacement of the IA mechanism with the parallel mechanism in the Sichuan Tier 1 admissions in 2009 results in less manipulation, as shown by a longer ROL, and more risk taking, as shown by first choice selections. Thus, it offers support for Hypothesis 1 that the parallel mechanism is manipulated less than the IA mechanism.

Furthermore, Proposition 5 states that, if a college, μ, is an equilibrium outcome under the IA mechanism, a student can list it as one of her top e choices under the parallel mechanism and any colleges she truly likes better than μ as higher ranked choices, then her assignment under the parallel mechanism is at least as good as that under the IA mechanism. This proposition suggests that the parallel mechanism offers insurance to students. Thus, we expect that students will be able to rank more prestigious colleges as their top choices under the parallel mechanism.

Hypothesis 2 (Insurance) *Students will list more prestigious colleges as their first choices under the parallel mechanism, compared to the IA mechanism.*

As student true preferences are not observable, Chen et al. (2015) use published national college rankings for 2008 and 2009 as preference proxies. This approximation is based on the assumption that most students prefer higher-ranked colleges over lower-ranked ones, as ranking is highly correlated with education quality. Since college rankings fluctuates locally from year to year, they further divide the colleges in their data set into groups of ten, based on their rankings. They then calculate the coarse ranking of colleges within each tier, normalized to [0,1] to correct for the different number of colleges in each tier. They term this normalized coarse ranking as *prestige*, and find the following:

Result 2 (Insurance) *In Sichuan Province, changing the Tier 1 admissions mechanism from the IA mechanism in 2008 to the parallel mechanism ($e = 5$) in 2009 leads to a significant 5% increase in the prestige of first-choice colleges (Chen et al. 2015).*

Related to our theoretical predictions, we see that Result 2 is consistent with Hypothesis 2, that the parallel mechanism offers students insurance compared to the IA mechanism. Next we examine matching outcomes. We derive the next hypothesis based on Proposition 3.

Hypothesis 3 (Choice Accommodation) *(a) The IA mechanism assigns a higher number of students to their reported first choices than does a parallel mechanism. (b) In comparison, the parallel mechanism (e) assigns more students to their top e choices than any other mechanism (e'), where $e \neq e'$.*

Chen et al. (2015) find the following in their study regarding choice accommodation:

Result 3 (Choice Accommodation) *In Sichuan Province, changing the Tier 1 admissions mechanism from the IA mechanism to the parallel mechanism ($e = 5$) in 2009 leads to a significant 24% decrease in admissions offers by reported top-choice colleges. In comparison, the same mechanism change leads to a statistically insignificant increase in the top-five choice accommodation in the parallel mechanism (Chen et al. 2015).*

While Result 3 is consistent with Hypothesis 3(a), we note that the empirical test for part (b) is not statistically significant. We make several interpretations regarding Result 3 in the context of our theory. First, since students list more prestigious colleges as their top choices under the parallel mechanism, it is not surprising they are therefore less likely to receive their reported top choices under this mechanism. Second, we note that theory requires that the mechanisms be applied to the same problem, which is not satisfied with field data where the mechanism change occurs in two consecutive years with different cohorts of students.

Finally, we investigate how the empirical evidence regarding the effects of mechanism change on matching stability fits with our theoretical predictions. In the context of centralized college admissions, we characterize a matching outcome to be more stable if a student has justified envy towards fewer other students. A student exhibits justified envy towards a fellow student if the latter scores lower on the college entrance exam, but is admitted into a higher ranked college. With this definition, our theory suggests the following hypothesis:

Hypothesis 4 (Stability) *Compared to the IA mechanism, any parallel mechanism will lead to (a) fewer number of students who refuse to go to their assigned college, (b) less justified envy, or (c) a higher likelihood that a student with an entrance exam score above the Tier 1 cut-off will be admitted to a Tier 1 college.*

While all three empirical studies examine mechanism stability, they measure stability in different ways. Their results are indicated below:

Result 4 (Stability) ***(a)*** *In Shanghai, the change from the IA mechanism in 2007 to the parallel mechanism ($e = 4$) in 2008 results in a 40.6% decrease in the number of students who refuse to go to the colleges they are matched with (Hou et al. 2009).*
(b) *In Sichuan Province, changing the Tier 1 admissions mechanism from the IA mechanism in 2008 to the parallel mechanism ($e = 5$) in 2009 results in a comparable proportion of students exhibiting justified envy (Chen et al. 2015).*
(c) *In Hunan Province, the change from one parallel mechanism ($e = 3$) in 2009 to a different parallel mechanism ($e = 5$) in 2010 results in a comparable proportion of students exhibiting justified envy. However, by 2013, the new parallel mechanism ($e = 5$) is found to be significantly more stable than the old parallel mechanism ($e = 3$) (Wei 2015).*

(d) *In Hubei Province, the change from the IA mechanism in 2010 to the parallel mechanism ($e = 5$) in 2011 results in a higher proportion of high-scoring students admitted by Tier 1 colleges (Wei 2015).*

The above results indicate that the parallel mechanism produces outcomes that are at least as stable as the IA mechanism. Overall, we conclude that the existing empirical evidence is consistent with our theoretical predictions on manipulability, insurance, first-choice accommodation, and stability. However, the existing evidence does not provide statistically significant support for our first e-choice accommodation predication.

5 Conclusions

School choice and college admissions decisions have profound implications for students' education and labor market outcomes. Much of the debate on school choice in the literature focuses on the IA compared to DA mechanisms. This paper attempts to expand the discussion by synthesizing these well-known mechanisms with those used for school choice and college admissions in China into a family of application-rejection mechanisms, with the IA and DA mechanisms being special cases. A key insight is that the Chinese parallel mechanism bridges the well studied IA and DA mechanisms.

Our theoretical analysis indicates that moving from one end of this family of mechanisms to the other is accompanied by a systematic change in the incentive and stability properties of the mechanisms. Our theory also suggests that the Nash equilibrium strategies corresponding to the induced preference revelation games associated with members of the application-rejection family are nested. Although the DA mechanism has been shown to dominate the equilibria of the IA mechanism under complete information, we show that no such conclusion holds for the parallel mechanism.

Examining the specific impact of the Chinese college admissions reforms, we show that all of the Chinese provinces that have moved away from the IA mechanism now have less manipulable and more stable mechanisms. Furthermore, our analysis shows that any student can ensure that she does at least as well under the parallel mechanism as under the IA mechanism. Furthermore, this insurance does not entail any *ex ante* welfare cost since the parallel mechanism allows students to communicate their preference intensities more efficiently relative to the DA mechanism.

This paper also provides a discussion of how existing empirical data fits with our theoretical predictions. We find that this empirical evidence is consistent with our theoretical predictions on manipulability, insurance, first-choice accommodation, and stability. However, the existing research does not provide statistically significant support for our first e-choice accommodation predication.

Our study represents the first systematic theoretical investigation of how different versions of the Chinese parallel mechanisms impact outcomes. This analysis yields valuable insights which enable us to treat this class of mechanisms as a family, and thus systematically study their properties and performance. More importantly, our results have policy implications for school choice and college admissions in China and other countries. In particular, the parallel mechanism might be a less radical replacement for the IA mechanism compared to the DA mechanism due to its balance of efficiency and resistance to manipulation.

In a practical sense, the choice of the number of parallel colleges (e) in a given parallel mechanism is likely to be set for reasons other than game-theoretic or welfare reasons. Our study helps policy-makers understand the consequences of the number of parallel colleges on the incentives and stability of parallel mechanisms. Of the variants of the parallel mechanisms adopted since 2001, our analysis indicates that the parallel mechanism implemented in Tibet $(e = 10)$ is the least manipulable one, whereas the partial reforms adopted in Beijing, Gansu and Shangdong are the most manipulable ones.

As college admissions reforms continue in China and other parts of the world (Westkamp 2013, Braun, Dwenger, Kübler and Westkamp 2014), theoretical, experimental and empirical analyses of ongoing reforms not only deepen our understanding of the science of market design, but also offer insights which might affect education and labor market policies.

Appendix: Proofs

Proof of Proposition 1: (Part i). It is easy to see that the IA mechanism is Pareto efficient. Now consider the following problem with four students and four schools, each with one seat. Priority orders and student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_2	$\vdots$	$\vdots$
i_2	i_3		
i_1	i_4		
i_3	i_1		

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
s_1	s_1	s_2	s_2
s_4	s_2	s_3	s_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The outcome of the application-rejection mechanism (e) for all $e \geq 2$ is the following Pareto inefficient matching:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}.$$

(Parts ii & iii). Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{e+2}\}$ and $S = \{s_1, s_2, \ldots, s_{e+2}\}$, where each school has a quota of one. Each $i_k \in I$ with $k \in \{1, 2, \ldots, e\}$ ranks school s_k first and each $i_k \in I$ with $k \in \{1, 2, \ldots, e+1\}$ has the highest priority for school s_k. The preferences of student i_{e+1} are as follows: $s_1\ P_{e+1}\ s_2\ P_{e+1}\ \ldots s_{e+1}\ P_{e+1}\ s_{e+2}$. Student i_{e+2} ranks school s_{e+1} first. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+1}. It is easy to see that he applies to school s_{e+1} in step $e+1$ of the algorithm when a lower priority student is already permanently assigned to it in round 0. Hence he is rejected from school s_{e+1} and his final assignment is necessarily worse than s_{e+1}. In this case, the outcome of the application-rejection (e) mechanism for this problem is clearly unstable. Moreover, student i_{e+1} can secure a seat at school s_{e+1} when he submits an alternative preference list in which he ranks school s_{e+1} first. ∎

Proof of Theorem 1:

We start with a useful definition. Given a preference relation P_i of a student i, let $rank_i(a)$ denote the rank of school a in student i's preferences.

Definition: Given a preference profile P, student i ranks school a at a higher e-class than student j iff

$$\left\lceil \frac{rank_i(a)}{e} \right\rceil < \left\lceil \frac{rank_j(a)}{e} \right\rceil .$$

Intuitively, a student who lists a school among his first e choices ranks that school at a higher e-class than those who do not list it as one of their first e choices; a student who lists a school among his first $e+1$ through $2e$ choices ranks that school at a higher e-class than those who do not list it as one of their first $2e$ choices; etc. The following construction will be instrumental in the proof of Theorem 1 as well as some of the subsequent proofs.

For a given problem $(\succ, P)$, the corresponding *e-augmented priority profile* $\widetilde{\succ}$ is constructed as follows. For each $a \in S$, and all $i, j \in I$, we have $i \widetilde{\succ}_a j$ if and only if either:

(1) i ranks school a at a higher e-class than j, or

(2) i and j both rank school a in the same e-class and $i \succ_a j$.

Lemma 1: Given a problem $(\succ, P)$ and the corresponding e-augmented priority profile $\widetilde{\succ}$, $\varphi^e(\succ, P) = \varphi^\infty(\widetilde{\succ}, P)$.

Proof of Lemma 1: Let J^r denote the set of students who are permanently assigned to some school at the end of round r of φ^e at problem $(\succ, P)$. We first argue that the students in J^0 receive the same assignments under the DA mechanism at problem $(\widetilde{\succ}, P)$. First observe that, by the construction of the e-augmented priority profile $\widetilde{\succ}$, a student who ranks a school in a higher e-class than does another student can never be rejected by that school under the DA mechanism at $(\widetilde{\succ}, P)$ because of the application of the other student. Since round 0 of φ^e is equivalent to applying the DA algorithm to the first e choices of all students and the assignments are made permanent at the end of round 0 of φ^e, the assignments of students in J^0 under φ^e at problem $(\succ, P)$ has to coincide with their assignments under the DA mechanism at problem $(\widetilde{\succ}, P)$. Decreasing each school's quota under φ^e before round 1 and applying the same reasoning to this round, the students in J^1 must receive the same assignments under the DA mechanism at problem $(\widetilde{\succ}, P)$. Iterating this reasoning for the subsequent rounds, we conclude that $\varphi^e(\succ, P) = \varphi^\infty(\widetilde{\succ}, P)$.

∎

Given $i \in I$ and $x \in S$, let P_i^x denote a preference relation where student i ranks school x as his first choice.

Lemma 2: Given a problem $(\succ, P)$, let $\varphi_i^E(\succ, P) = x$. Then $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$ where $e < E$.

Proof of Lemma 2: By Lemma 1, $\varphi_i^E(\succ, P) = \varphi_i^\infty(\widetilde{\succ}, P) = x$ where $\widetilde{\succ}$ is the E-augmented priority profile corresponding to $(\succ, P)$. By the strategy-proofness of the DA mechanism, $\varphi_i^\infty(\widetilde{\succ}, P) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i})$. Hence, we have:

$$\varphi_i^E(\succ, P) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i}). \tag{1}$$

On the other hand, by Lemma 1,

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^\infty(\widehat{\succ}, P_i^x, P_{-i}) \tag{2}$$

where $\widehat{\succ}$ is the E-augmented priority profile corresponding to $(\succ, P_i^x, P_{-i})$. Note that $\widehat{\succ}_{-x}$ and $\widetilde{\succ}_{-x}$ agree on all students' relative priority orderings but i and $\widehat{\succ}_x$ (weakly) improves the priority of student i for school x in comparison to $\widetilde{\succ}_x$. It thus follows from the working of the DA algorithm that:

$$\varphi_i^\infty(\widehat{\succ}, P_i^x, P_{-i}) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i}). \tag{3}$$

Last, we claim that

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^e(\succ, P_i^x, P_{-i}). \tag{4}$$

To see this, note that, when applied to $(\succ, P_i^x, P_{-i})$, the set of students who apply to school x in round 0 of φ^E is weakly larger than that in round 0 of φ^e. Since student i is not rejected from school x after applying to it in the first step under φ^E, he cannot be rejected from it under φ^e either.

Combining (1), (2), (3), and (4), we obtain $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$. ∎

Now we are ready to prove Theorem 1. Let $(\succ, P)$ be a problem such that there exists $i \in I$ and preferences P_i' where $\varphi_i^{e'}(\succ, P_i', P_{-i})\ P_i\ \varphi_i^{e'}(\succ, P)$. We show that there exists $j \in I$ and preferences P_j' such that $\varphi_j^e(\succ, P_j', P_{-j})\ P_j\ \varphi_j^e(\succ, P)$ where $e < e'$. Let $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x$. We consider two cases.

Case 1. $x\ P_i\ \varphi_i^e(\succ, P)$: Since $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x$, by Lemma 2 $\varphi_i^e(\succ, P_i^x, P_{-i}) = x$. Thus, i manipulates φ^e at $(\succ, P)$.

Case 2. $\varphi_i^e(\succ, P)\ R_i\ x$: We claim that for all $k \in I$, $\varphi_k^e(\succ, P)\ R_k\ \varphi_k^{e'}(\succ, P)$. Suppose not. Then, there exists $j \in I$ such that $\varphi_j^{e'}(\succ, P)\ P_j\ \varphi_j^e(\succ, P)$. By Lemma 2, $\varphi_j^e(\succ, P_j^{\varphi_j^{e'}(\succ,P)}, P_{-i}) = \varphi_j^{e'}(\succ, P)$ and thus j manipulates φ^e at $(\succ, P)$. Hence the claim is true. Moreover, since $\varphi_i^e(\succ, P)$ $R_i\ x$ and $x\ P_i\ \varphi_i^{e'}(\succ, P)$, by transitivity we have $\varphi_i^e(\succ, P)\ P_i\ \varphi_i^{e'}(\succ, P)$. This, together with the preceding claim, implies that $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$.

Next, consider the rounds of $\varphi^{e'}$ when applied to problem $(\succ, P)$. Let $y = \varphi_i^{e'}(\succ, P)$. Let r be the round at the end of which student i is (permanently) assigned to school y. We claim that $r \geq 1$. Suppose for a contradiction that $r = 0$. Then, since $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x\ P_i\ y = \varphi_i^{e'}(\succ, P)$, student i ranks school x at the same (and the highest) e'-class at both (P_i', P_{-i}) and P. Let $\widetilde{\succ}$ and $\widehat{\succ}$ be the e'-augmented priority profiles corresponding to $(\succ, P_i', P_{-i})$ and $(\succ, P)$, respectively. Thus, by Lemma 1, $\varphi_i^\infty(\widetilde{\succ}, P_i', P_{-i}) = x$ and $\varphi_i^\infty(\widehat{\succ}, P) = y$. Let P_i^{xy} be a relation where i ranks x first and y second. By the strategy-proofness of the DA mechanism, $\varphi_i^\infty(\widetilde{\succ}, P_i^{xy}, P_{-i}) = x$. Note that student i ranks school x at the same (and the highest) e'-class at both (P_i', P_{-i}) and P, $\widehat{\succ}_x = \widetilde{\succ}_x$. Thus, $\varphi_i^\infty(\widehat{\succ}_x, \widetilde{\succ}_{-x}, P_i^{xy}, P_{-i}) = x$. Recall that $\varphi_i^\infty(\widehat{\succ}, P) = y$. By the strategy-proofness of the DA mechanism, $\varphi_i^\infty(\widehat{\succ}, P_i^{xy}, P_{-i}) = y$. But then, at both $(\widehat{\succ}_x, \widetilde{\succ}_{-x}, P_i^{xy}, P_{-i})$ and $(\widehat{\succ}, P_i^{xy}, P_{-i})$, the preference profiles are the same and student i lists school x as a first choice. Since the priority order for x is also identical at both problems, the DA mechanism should give i the same assignment for both problems. But this is a contradiction. Thus, $r \geq 1$, as claimed.

Let $z_0 = \varphi_i^e(\succ, P)$. Since $\varphi_i^e(\succ, P)\ P_i\ \varphi_i^{e'}(\succ, P)$ and $\varphi^{e'}$ is nonwasteful, there exists $j_1 \in \varphi^{e'}(\succ, P)(z_0) \backslash \varphi^e(\succ, P)(z_0)$. Since $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$, we must have $\varphi_{j_1}^e(\succ, P)\ P_{j_1}$ $\varphi_{j_1}^{e'}(\succ, P)$. Letting $z_1 = \varphi_{j_1}^e(\succ, P) \neq z_0$, there exists $j_2 \in \varphi^{e'}(\succ, P)(z_1) \backslash \varphi^e(\succ, P)(z_1)$. Since I is finite, iterating this reasoning, we obtain a set $J = \{i, j_1, \ldots, j_k\}$ of students with $k \geq 1$, each of whom is assigned to a distinct school from the set $A = \{z_0, z_1, \ldots, z_k = y\}$ at $\varphi^e(\succ, P)$. Reconsidering the $\varphi^{e'}$ algorithm when applied to problem $(\succ, P)$, each student in J must then be assigned to the corresponding school in A in the same round. Otherwise, the school from the set A that admits a student at a later round will still have a vacant position in all previous rounds, which contradicts the fact that the student from the set J assigned to it at $\varphi^e(\succ, P)$ is better off compared to

$\varphi_i^{e'}(\succ, P)$. In other words, all Pareto improving assignment exchanges from $\varphi^{e'}(\succ, P)$ to $\varphi^{e}(\succ, P)$ must involve students who receive their (permanent) assignments in the same round. Hence, each student in J is (permanently) assigned to the corresponding school in A in round $r \geq 1$.

Consider round r of the $\varphi^{e'}$ algorithm when applied to problem $(\succ, P)$. Let J^r be the set of students such that (1) they each receive their (permanent) assignments at the end of round r, and (2) they each are better off at $\varphi^e(\succ, P)$ compared to $\varphi^{e'}(\succ, P)$.[24] Let $j^* \in J^r$ be the *last* student in J^* to apply to his assignment at $\varphi^e(\succ, P)$ in that round and let $z^* = \varphi_{j^*}^{e'}(\succ, P)$. Let k^* be the student who is kicked out from z^* at that step. Note that k^* necessarily exists since a student from J^r has already been kicked out from z^* at a previous step in that round. Thus, $z^* \; P_{k^*} \; \varphi_{k^*}^{e'}(\succ, P)$. Moreover, by the choice of j^*, $k^* \notin J^r$. If student k^* receives his (permanent) assignment at the end of round r, then $\varphi_{k^*}^{e}(\succ, P) = \varphi_{k^*}^{e'}(\succ, P)$. Otherwise, student k^* receives his (permanent) assignment at a later round than r and by the argument in the preceding paragraph pertaining to students who are better off at $\varphi^e(\succ, P)$, $z^* \; P_{k^*} \; \varphi_{k^*}^{e}(\succ, P)$.

Finally, since school z^* has a vacancy before round $r \geq 1$, it follows that $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^*$. Then by Lemma 2, $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = \varphi_{k^*}^{e}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^* \; P_{k^*} \; \varphi_{k^*}^{e}(\succ, P)$. Hence, student k^* manipulates φ^e at $(\succ, P)$.

We next prove that $\varphi^{e'}$ may not be manipulable when φ^e is. We first fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{e+2}\}$ and $S = \{s_1, s_2, \ldots, s_{e+1}\}$ where each school has a quota of one. Each student $i \in I$ has the following preferences: $s_1 \; P_i \; s_2 \; P_i \ldots s_e \; P_i \; s_{e+1} \; P_i \; \emptyset$. There is a single priority order for each school given as follows: for each $s \in S$, suppose $i_k \succ_s i_{k'}$ whenever $k < k'$, i.e., i_1 has the highest priority, i_2 has the second highest priority and so on. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+2}. It is easy to see that he is unassigned in round 0 and is assigned to his last choice (i.e., the null school) at step $e+2$ of round 1 after being rejected from school s_{e+1}. If student i_{e+2} were to report school s_{e+1} as his first choice, he would clearly be assigned to it in round 0. Hence, φ^e is manipulable by student i_{e+1} at this problem. It is easy to see that no student can manipulate $\varphi^{e'}$ via a preference misreport at this problem. ∎

Proof of Proposition 2:

(Part i). Let $e' = ke$. If $k = \infty$, Proposition 1 implies that the DA mechanism is more stable than φ^e for any $e < \infty$. Let $k \in \mathbb{N}$. We show that if $\varphi^{e'}$ is unstable at a problem, then so is φ^e. We prove the contrapositive of this statement. Let $(\succ, P)$ be a problem at which $\varphi^e(\succ, P)$ is stable. We show that $\varphi^e(\succ, P) = \varphi^{e'}(\succ, P)$.

Consider mechanism φ^e when applied to problem $(\succ, P)$. Since $\varphi^e(\succ, P)$ is stable, any student unassigned in round 0 (who was rejected from all his first $e-$choices) must have a lower priority

[24]Note that the set J^r is well-defined by the argument made in the previous paragraph.

at his first $e-$choice schools than every student who obtained a seat at any such school in round 0. Similarly, since $\varphi^e(\succ, P)$ is stable, any student unassigned in round 1 (who was rejected from all his first $2e-$choices) must have a lower priority at his first $2e-$choice schools than every student who obtained a seat at any such school in round 0 or round 1. In general, any student unassigned in round $k-1$ must have a lower priority at his first $ke-$choice schools than every student who obtained a seat at any such school in round $k-1$ or any previous round. But this implies that any student who is unassigned at the end of round $k-1$ of φ^e is also unassigned at the end of round 0 of $\varphi^{e'}$, as he applies to, and gets rejected from, the same set of schools in the same order under both mechanisms. Similarly, any student who is assigned to some school s in some round $t \leq k-1$ of φ^e is also assigned to school s in round 0 of $\varphi^{e'}$ as he cannot be rejected in favor of a student who does not list school s among his first $(t+1)e-$choices. Then the students who participate in rounds k through $2k-1$ of φ^e are the same as those who participate in round 1 of $\varphi^{e'}$ and, by the same argument, they apply to, and get rejected from, the same set of schools in the same order under both mechanisms. Iterating this reasoning, we conclude that $\varphi^{e'}(\succ, P) = \varphi^e(\succ, P)$.

The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not.

(Part ii). Since $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, there exists $t \in \mathbb{N}$ such that $te < e' < (t+1)e$. Consider the following problem $(\succ, P)$. Let $I = \{i_1, i_2, \ldots, i_{te+e'+2}\}$ and $S = \{s_1, s_2, \ldots, s_{te+e'+1}\}$ where $q_s = 1$ for all $s \in S$. Each $i_j \in I \backslash \{i_{te+1}, i_{te+e'+2}\}$ top-ranks school s_j and has the highest priority for it. The remaining two students' preferences are as follows. $P_{i_{te+1}} : s_1, s_2, \ldots, s_{te+1}, \emptyset$ and $P_{i_{te+e'+3}} : s_{te+2}, s_{te+3}, \ldots, s_{te+e'+1}, s_{te+1}, \emptyset$. Let $i_{te+e'+2} \succ_{s_{te+1}} i_{te+1}$.

It is not difficult to calculate that for each $i_j \in I \backslash \{i_{te+1}, i_{te+e'+3}\}$, $\varphi^e_{i_j}(\succ, P) = \varphi^{e'}_{i_j}(\succ, P) = s_j$, $\varphi^e_{i_{te+1}}(\succ, P) = \varphi^{e'}_{i_{te+e'+3}}(\succ, P) = \emptyset$, and $\varphi^{e'}_{i_{te+1}}(\succ, P) = \varphi^e_{i_{te+e'+3}}(\succ, P) = s_{te+1}$. Clearly, $\varphi^e(\succ, P)$ is stable whereas $\varphi^{e'}(\succ, P)$ is not. The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not. ∎

Proof of Theorem 2: The first statement follows from the proof of Theorem 1, Corollary 2, and Proposition 2. For the second statement, we construct a problem under which $\varphi^{e'}$ has a stable equilibrium which neither is an equilibrium nor leads to a stable matching under φ^e. Consider the following problem $(\succ, P)$. Let $I = \{i_1, i_2, \ldots, i_{e'}\}$ and $S = \{s_1, s_2, \ldots, s_{e'}\}$, where $q_s = 1$ for all $s \in S$. Each $i \in I$ ranks school s_1 first, s_2 second, ..., and $s_{e'}$ last. For each school $s \in S$, i_1 has the highest priority, i_2 has the second-highest priority, ..., and $i_{e'}$ has the lowest priority. At the unique stable matching, μ, i_1 is assigned to s_1, i_2 is assigned to s_2, ..., and $i_{e'}$ is assigned to $s_{e'}$. Consider the following profile of reports. Each student but student $i_{e'}$ reports truthfully, while student $i_{e'}$ only switches the positions of s_e and s_{e+1} and is truthful otherwise. These reports constitute an equilibrium under $\varphi^{e'}$ and lead to μ. However, the same profile is not an equilibrium under φ^e since $i_{e'}$ is now assigned to s_{e+1} and, any $i \in \{i_{e+1}, \ldots, i_{e'-1}\}$ can profitably deviate by replacing any one of his first e choices with s_{e+1}. Nor does this profile lead to a stable matching since any $i \in \{i_{e+1}, \ldots, i_{e'-1}\}$ can form a blocking pair with school s_{e+1}. ∎

Proof of Proposition 3:

Part (i). Fix a problem $(\succ, P)$. Take any two mechanisms φ^e and $\varphi^{e'}$ with $e' > e$. We contrast round 0 of φ^e with that of $\varphi^{e'}$. For any school $s \in S$, the set of students who apply to s in round 0 of $\varphi^{e'}$ is weakly larger than the set of students who apply to s in round 0 of φ^e. This implies that any student who is assigned to his first choice at the end of round 0 of $\varphi^{e'}$ is also assigned to his first choice at the end of round 0 of φ^e but not vice versa. In other words, a student who is assigned to his first choice under φ^e, may be rejected from that school under $\varphi^{e'}$ due to the application of a higher priority student who ranks it as one of his $e+1$ through e' choices.

Part (ii). Fix a problem$(\succ, P)$. Suppose $e' < e$. Consider any student-say i- who is assigned to one of his first e choices-say s- under $\varphi^{e'}$ but not under φ^e. Since assignments under φ^e are final after the first e choices have been considered (or alternatively, since the equivalent DA algorithm constructed in Lemma 1 prioritizes the first e choices), student i's slot at s is filled by another student who also ranks s as one of his first e choices. Thus, the number of students who receive one of their first e choices cannot decrease under φ^e.

Suppose $e' > e$. Take any student-say j- who is assigned to one of his first e choices under $\varphi^{e'}$. Note that the corresponding $e-$augmented priority profile for this problem gives (weakly) higher priority to student j for all his first e choices than the corresponding $e'-$augmented priority profile. Then by Lemma 1 and the stability of the DA mechanism, student j must be assigned to one of his first e choices under φ^e as well. ∎

Proof of Proposition 4: Part (i) is established in Theorem 1 of Ergin and Sönmez (2006). We prove part (ii). Let $I = \{i_1, i_2, i_3, i_4\}$ and $S = \{s_1, s_2, s_3, s_4\}$, where each school has a quota of one. Consider the following priority profile $\succ$ and true preferences $P = (P_1, P_2, P_3, P_4)$ of students.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_3	i_1	i_2	i_2
i_2	i_3	i_1	i_4
i_1	i_2	i_3	i_1
i_4	i_4	i_4	i_3

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
s_1	s_1	s_3	s_4
s_3	s_2	s_1	s_1
s_2	s_4	s_2	s_2
s_4	s_3	s_4	s_3

The DA outcome for problem $(\succ, P)$ is the following matching:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_3 & s_2 & s_1 & s_4 \end{pmatrix}.$$

Consider a strategy profile $Q = (Q_{i_1}, Q_{i_2}, Q_{i_3}, Q_{i_4})$ where $Q_{i_1} = P_{i_1}, Q_{i_3} = P_{i_3}, Q_{i_4} = P_{i_4}$and $s_2\ Q_{i_2}\ s_4\ Q_{i_2}\ s_1\ Q_{i_2}\ s_3$. For problem $(\succ, Q)$, the outcome of the application-rejection mechanism (e), for any $e \geq 2$, is the unstable and Pareto efficient matching:

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_1 & s_2 & s_3 & s_4 \end{pmatrix},$$

where μ' Pareto dominates μ. To see that Q is indeed an equilibrium profile (in undominated strategies), it suffices to consider possible deviations by student i_2. For any preferences in which he ranks s_1 first, he gets rejected from s_1 at the third step. If he ranks s_2 first, clearly his assignment does not change. If he ranks s_3 (s_4) first, he is assigned to s_3 (s_4). ∎

Proof of Proposition 5: Let $(\succ, P)$ be the problem where P is the list of true student preferences. By Proposition 2, μ is stable under $(\succ, P)$. Let P' be a preference profile where each $i \in I$ lists $\mu(i)$ as his e-th choice and such that for any $s \in S$, $s\ P'_i\ \mu(i)$ implies $s\ P_i\ \mu(i)$. We show that for each $i \in I$, $\varphi^e_i(\succ, P)\ R_i\ \mu(i)$ for any e. Suppose to the contrary that student i remains unassigned at the end of round 0. This means that school $\mu(i)$ is full at the end of round 0, and in particular, there is $j \neq i$ such that $\varphi^e_j(\succ, P) = \mu(i) \neq \mu(j)$ and $j \succ_{\mu(i)} i$. Then, since $\mu(i)\ P_j\ \mu(j)$ and $j \succ_{\mu(i)} i$, μ is not stable under $(\succ, P)$. ∎

Ex ante Equilibria: Incomplete information view

Abdulkadiroğlu, Che and Yasuda (2011) [henceforth, ACY] study an incomplete information model of school choice that captures two salient features from practice: correlated ordinal preferences and coarse school priorities. More specifically, they consider a highly special setting where students share the same ordinal preferences but different and unknown cardinal preferences and where schools have no priorities, i.e., priorities are determined via a random lottery draw after students submit preference rankings. Nonetheless, the DA outcome coincides with a purely random allocation in this stylized setting.[25] ACY focus on the symmetric Bayesian Nash equilibria under the IA mechanism and show that every student is at least weakly better off in any such equilibrium than under the DA mechanism. This result suggests that there may be a welfare loss for every student under the DA mechanism relative to the IA mechanism in such circumstances.[26]

[25] Since this model assumes no priorities, any stable mechanism always induces an equal weighted lottery over all feasible allocations. In this restricted setting, the DA and the well-known top trading cycles mechanisms (Abdulkadiroğlu and Sönmez (2003)) both coincide with a random serial dictatorship mechanism.

[26] However, this finding is not robust to changes in the priority structure. Indeed, Troyan (2012) shows that when school priorities are introduced into the same setting, the IA mechanism no longer dominates the DA mechanism in terms of *ex ante* welfare.

We next investigate whether or not the *ex ante* dominance of the IA mechanism in this restricted setting prevails when compared with a Chinese parallel mechanism.[27] It turns out the answer is negative.

Proposition 7 (Ex ante equilibrium) *In the Bayesian setting of ACY,*

(i) each student is weakly better off in any symmetric equilibrium of the Shanghai mechanism than she is in the DA mechanism, and

(ii) no ex ante Pareto ranking can be made between the IA and the Shanghai mechanisms, i.e., there exist problems where some student types are weakly better off at the equilibrium under the Shanghai mechanism than they are under the IA mechanism and vice versa.

Part (i) says that, just like the IA mechanism, the Shanghai mechanism also leads to a clear welfare gain over the DA mechanism in the same setting. This shows that in special settings, just like the IA mechanism, the Shanghai mechanism may also allow students to communicate their preference intensities. Part (ii) shows the non-dominance of the IA mechanism over the Shanghai mechanism in the same Bayesian setting.

To gain a clear insight into the *ex ante* welfare issues, we focus on the IA and the DA mechanisms together with the Shanghai mechanism, the simplest member of the Chinese parallel mechanisms. We show that, in the same setting as ACY, there may be students who are better off in a Bayesian equilibrium of the Shanghai mechanism than in one of the IA equilibria. The following example illustrates the intuition.

Let there be four students of three types, with values $\{\mathbf{v}_L, \mathbf{v}_M, \mathbf{v}_H\}$, two from the low type and one each from the medium and high types, and four schools $\{s_0, s_1, s_2, s_3\}$, each with one seat. There are no priorities *a priori*, students have common ordinal preferences, and each student type has the von Neumann Morgenstern (vNM) utility values given in the following table.

	$\mathbf{v}_L$	$\mathbf{v}_M$	$\mathbf{v}_H$
s_0	.9	.53	.36
s_1	.09	.36	.35
s_2	.01	.11	.29
s_3	0	0	0

First, consider the IA mechanism with random tie-breaking. Type $\mathbf{v}_L$ students have a dominant strategy of ranking schools truthfully. Given that, a type $\mathbf{v}_M$ student has a best response of ranking s_1 as his first choice (regardless of what a type $\mathbf{v}_H$ does). Furthermore, given all these strategies,

[27] As noted earlier, out of the 31 provinces in China, two of them, Beijing and Shanghai, require students to submit preference rankings before taking the college entrance exam.

a type $\mathbf{v}_H$ student has a best response of ranking s_2 as his first choice. This constitutes the unique equilibrium under the IA mechanism, where the type $\mathbf{v}_H$ student obtains an expected utility of .29.

Now let us consider the Shanghai mechanism with random tie-breaking. Type $\mathbf{v}_L$ students again have a dominant strategy of ranking schools truthfully. Given that, a type $\mathbf{v}_M$ student has a best response of ranking schools truthfully (regardless of what type $\mathbf{v}_H$ does). Furthermore, given all these strategies, a type $\mathbf{v}_H$ student has a best response of respectively ranking s_1 and s_2 as his first and second choices (see the proof of Proposition 4 part (ii) for details). This constitutes the unique equilibrium under the Shanghai mechanism, where the type $\mathbf{v}_H$ student now obtains an expected utility of .32.

The reason why some students may prefer the Shanghai mechanism to the IA mechanism, unlike the case against the DA mechanism, can be explained as follows. Under the IA mechanism, students' first choices are crucial and thus students target a single school at equilibrium. Under the Shanghai mechanism, the first two choices are crucial and students target a pair of schools. This difference, however, may enable a student to guarantee a seat at an unpopular school under the Shanghai mechanism by ranking it as his second choice and still give him some chance to obtain a more preferred school by ranking it as his first choice. For example, in the above scenario, the type $\mathbf{v}_H$ student "gains priority" at school s_2, her sure outcome in the IA mechanism, when others do not include it in their first two choices, and also has a chance of ending up at s_1.[28]

Although this example assumes students have complete information about their cardinal preferences, it is possible to use the same insight to show the non-dominance of the IA over the Shanghai mechanism in a Bayesian setting.

Proof of Proposition 7:

Part (i). We start by adopting the ACY model. Let $S = \{s_0, s_1, \ldots, s_m\}$ with $m \geq 1$ be the set of schools (without the outside option). Each student privately draws vNM utility values $v = (v_0, \ldots, v_m)$ from a finite set $\mathcal{V} = \{(v_0, \ldots, v_m) \in [0,1]^m | v_0 > v_1 \ldots > v_m\}$ with probability $f(\mathbf{v})$, which is common knowledge. Without loss of generality, we assume that $\sum_{s \in S} q_s = n = |I|$. Let Π be the set of all ordinal preferences over S, and $\Delta(\Pi)$ the set of probability distributions over Π. A symmetric Bayesian strategy is a mapping $\sigma : \mathcal{V} \to \Delta(\Pi)$.

In showing the dominance of the Shanghai mechanism over the DA mechanism, we use exactly the same proof strategy as ACY. Following ACY, the probability that any student is assigned to school $s \in S$ is given by

[28] Loosely speaking, the IA lottery (i.e., the IA mechanism with random tie-breaking) when compared with the Shanghai lottery (i.e., the Shanghai mechanism with random tie-breaking) can be seen as a weighted average over more extreme choices (when the lotteries are non-degenerate). In the above example, for instance, a low type student faces a lottery between his first and last choices under the IA mechanism. This is because, if he misses his first choice, his second and third choices will already be taken. On the other hand, the Shanghai lottery always puts positive weight on the first *and* the second choices. At the other extreme, the DA lottery is an equal weighted average over all choices.

$$P_s^{DA} = \frac{q_s}{n}.$$

For any equilibrium strategy $\sigma \in \{\sigma^*(v)\}_{\mathbf{v}\in\mathcal{V}}$, let $P_s^{SHA}(\sigma)$ be the probability that a student is assigned to school s if he plays σ when all other students play σ^*. Then, in equilibrium, for each $s \in S$,

$$\sum_{\mathbf{v}\in\mathcal{V}} nP_s^{SHA}(\sigma^*(\mathbf{v}))f(\mathbf{v}) = q_s.$$

Suppose a type $\widetilde{\mathbf{v}} \in \mathcal{V}$ student chooses to play $\sigma^*(\mathbf{v})$ with probability $f(\mathbf{v})$. Denote that strategy by $\widetilde{\sigma}$. Then he is assigned to $s \in S$ with probability

$$P_s^{SHA}(\widetilde{\sigma}) = \sum_{\mathbf{v}\in\mathcal{V}} P_s^{SHA}(\sigma^*(\mathbf{v}))f(\mathbf{v}) = \frac{q_s}{n} = P_s^{DA}.$$

That is, by playing $\widetilde{\sigma}$, which is not necessarily an equilibrium strategy, a student can guarantee himself the same random assignment as that he would obtain under the DA mechanism.

Part (ii). We start by showing that the specified strategies for the complete information example given in the text indeed constitute the unique equilibrium of the Shanghai mechanism. Let $u_i(s)$ denote the vNM utility of student i for school s and σ_i denote a (pure) strategy of student i. Suppose students 1 and 2 are of the low type, while students 3 and 4 are respectively of the medium and high types. Let $EU_i^{SHA}(\sigma^*)$ be the expected utility of student i at the specified strategy profile, i.e., when $\sigma_i^* = s_0s_1s_2s_3$ for $i = 1, 2, 3$ and $\sigma_4^* = s_1s_2s_0s_3$. Then we have $EU_i^{SHA} = \frac{1}{3}u_i(s_0) + \frac{1}{6}u_i(s_1) + \frac{1}{6}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA} = \frac{1}{2}u_4(s_1) + \frac{1}{2}u_4(s_2) = .32$.

Clearly, for any student, ranking s_3 at any position but the bottom is dominated. Moreover, σ_1^* and σ_2^* are dominant strategies. We first claim that σ_3^* is a best response to σ_1^* and σ_2^*, regardless of what 4 chooses. To show this, we fix σ_1^* and σ_2^*, and consider three possibilities for σ_4^*.

1. $\sigma_4^* = s_0s_1s_2s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .24 > EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .23$.[29]

2. $\sigma_4^* = s_1s_2s_0s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .22 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .21$.

3. $\sigma_4^* = s_0s_2s_1s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .23 > EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .19$.

Last, we claim that σ_4^* is a best response to σ_1^*, σ_2^*, and σ_3^*. Indeed, $EU_4^{SHA}(\sigma_4^*) = .32 > EU_4^{SHA}(\sigma_4 = s_0s_2s_1s_3) = .31 > EU_4^{SHA}(\sigma_4 = s_0s_1s_2s_3) = .25$. Thus, we have confirmed that profile σ^* constitutes the unique equilibrium of the Shanghai mechanism.

[29] Upon fixing σ_1^* and σ_2^*, we calculate that $EU_i^{SHA}(\sigma_3 = s_1s_2s_0s_3, \sigma_4^* = s_0s_1s_2s_3) = \frac{1}{4}u_i(s_0) + \frac{1}{3}u_i(s_1) + \frac{1}{12}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA}(\sigma_3 = s_1s_2s_0s_3, \sigma_4^* = s_0s_1s_2s_3) = \frac{1}{4}u_4(s_1) + \frac{3}{4}u_4(s_2)$.

We next prove part (ii) of Proposition 3 building on the example given in the main text. Let $I = \{1, 2, 3, 4\}$, $S = \{s_0, s_1, s_2, s_3\}$, and $\mathcal{V} = \{\mathbf{v}_L, \mathbf{v}_M, \mathbf{v}_H\}$ (as in the example) with probabilities $p_L = \frac{3}{4} - \frac{\varepsilon}{2}$, $p_M = \frac{1}{4} - \frac{\varepsilon}{2}$, and $p_H = \varepsilon$, where $\varepsilon > 0$ can be chosen arbitrarily close to zero. Consider the following strategies under the IA mechanism: $\sigma^{IA}(\mathbf{v}_L) = s_0 s_1 s_2 s_3$, $\sigma^{IA}(\mathbf{v}_M) = s_1 s_0 s_2 s_3$, and $\sigma^{IA}(\mathbf{v}_H) = s_2 s_0 s_1 s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrium for a sufficiently small ε.

Since an exact analysis would be unnecessarily lengthy and cumbersome, we provide a summary argument. For a low type student, it is still a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, the following table provides the possible realizations for the types of the remaining three students and a corresponding best response of a high type student to the particular realization in each case. With an abuse of notation, let $|\mathbf{v}_x|$ denote the number of students of type $\mathbf{v}_x$. Note that we do not display those realizations involving a high type student as they do not impact equilibrium verification when ε is sufficiently close to zero.

Realization	Probability	Best response	Payoff loss from $\sigma^{IA}(\mathbf{v}_H)$	Minimum gain from $\sigma^{IA}(\mathbf{v}_H)$
$\|\mathbf{v}_L\| = 3$	.42	$\sigma = s_1$	$-.06$	-
$\|\mathbf{v}_L\| = 2, \|\mathbf{v}_M\| = 1$	.42	$\sigma^{IA}(\mathbf{v}_H)$	-	.11
$\|\mathbf{v}_L\| = 1, \|\mathbf{v}_M\| = 2$	.14	$\sigma^{IA}(\mathbf{v}_H)$	-	.11
$\|\mathbf{v}_M\| = 3$	.02	$\sigma^{IA} = s_0$	$-.07$	-

For example, the first row of the table represents the case when all three students are of the low type, which occurs with probability $p_L^3 \cong .42$. In this case, a high type student maximizes his payoff by ranking s_1 first, from which he receives a payoff of .35. But since $\sigma^{IA}(\mathbf{v}_H)$ is not a best response to this realization, a high type student receives only .29 by playing $\sigma^{IA}(\mathbf{v}_H)$. The second row represents the case when two students are of a low type and one is of a medium type, which occurs with probability $3p_L^2 p_M \cong .42$. In this case, $\sigma^{IA}(\mathbf{v}_H)$ is a best response of a high type student to this realization, from which he receives a payoff of .29. The next-best action of a high type to this realization is playing $\sigma = s_1$, by which he receives $\frac{.35}{2} \cong .18$. Hence playing $\sigma^{IA}(\mathbf{v}_H)$ gives him an extra payoff of at least .11 over any other strategy. The rest of the table is filled in similarly. It follows from the table that the expected utility loss of a high type student due to playing $\sigma^{IA}(\mathbf{v}_H)$ when it is not a best response is more than offset by his gain from playing $\sigma^{IA}(\mathbf{v}_H)$ when it is a best response.

We next consider a medium type student. Fixing the strategies of the other students as above, the following table provides the possible realizations for the types of the remaining three students and the corresponding best responses of a medium type student to the particular realization in each case. Once again, we do not display those realizations involving a high type student.

Realization	Probability	Best response	Payoff loss from $\sigma^{IA}(\mathbf{v}_M)$	Minimum gain from $\sigma^{IA}(\mathbf{v}_M)$
$\lvert\mathbf{v}_L\rvert = 3$	.42	$\sigma^{IA}(\mathbf{v}_M)$	-	.12
$\lvert\mathbf{v}_L\rvert = 2, \lvert\mathbf{v}_M\rvert = 1$	.42	$\sigma^{IA}(\mathbf{v}_M)$	-	.04
$\lvert\mathbf{v}_L\rvert = 1, \lvert\mathbf{v}_M\rvert = 2$	.14	$\sigma = s_0$	$-.15$	-
$\lvert\mathbf{v}_M\rvert = 3$	.02	$\sigma = s_0$	$-.44$	-

It follows from the table that the expected utility loss of a medium type student due to playing $\sigma^{IA}(\mathbf{v}_M)$ when it is not a best response is more than offset by his gain from playing $\sigma^{IA}(\mathbf{v}_M)$ when it is a best response. Thus, $(\sigma^{IA}(\mathbf{v}_L), \sigma^{IA}(\mathbf{v}_M), \sigma^{IA}(\mathbf{v}_H))$ is a Bayesian equilibrium under the IA mechanism. In particular, $EU^{IA}_{\mathbf{v}_H} \cong .29$.

Next we consider the following strategies under the Shanghai mechanism: $\sigma^{SHA}(\mathbf{v}_L) = \sigma^{SHA}(\mathbf{v}_M) = s_0s_1s_2s_3$ and $\sigma^{SHA}(\mathbf{v}_H) = s_1s_2s_0s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrim for a sufficiently small ε. For a low type student, it is a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, for any particular realization (that does not involve a high type student), a high type student faces three students that are playing $\sigma^{SHA}(\mathbf{v}_L)$. As calculated above for the example with complete information, it is then a best response for him to play $\sigma^{SHA}(\mathbf{v}_H)$. Similarly, for a medium type student, it is also a best response for him to play $\sigma^{SHA}(\mathbf{v}_H)$ for any particular realization that does not involve a high type student. Thus, $(\sigma^{SHA}(\mathbf{v}_L), \sigma^{SHA}(\mathbf{v}_M), \sigma^{SHA}(\mathbf{v}_H))$ is a Bayesian equilibrium under the Shanghai mechanism. In particular, $EU^{SHA}_{\mathbf{v}_H} \cong .32 > EU^{IA}_{\mathbf{v}_H}$. ∎

Description of the Algorithm for the Asymmetric Class of Parallel Mechanisms:

Let $S = (e_0, e_1, e_2, \ldots)$ be a given choice sequence.

Round t $=0$**:**

- Each student applies to his first choice. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e_0-th choice school, applies to his next choice. If a student has been rejected from all his first e_0-choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.
- The round terminates whenever each student is either assigned to some school or remains unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

In general,

Round t ≥ 1**:**

- Each unassigned student from the previous round applies to his $\sum_{i=0}^{t-1} e_i + 1$-st choice school. Each school x considers its applicants. Those students with the highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

  In general,

- Each rejected student, who is yet to apply to his $\sum_{i=1}^{t} e_i$-th choice school, applies to his next choice. If a student has been rejected from all his first $\sum_{i=1}^{t} e_i$-choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

- The round terminates whenever each student is either assigned to some school or remains unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* S and denoted by φ^S.

Proof of Theorem 3: Clearly, Theorem 1 shows this result for the special case when all the terms in a choice sequence are identical. It is fairly straightforward to check that the proof of Theorem 1 depends on only the number of choices that are considered in round 0 and not on the number of choices considered in any subsequent round of the application-rejection algorithm. Hence, the same proof still applies once Lemmas 1 and 2 are appropriately modified for the extended class. For brevity, we omit these details. ∎

Proof of Proposition 6: Since the proof is analogous to that of Proposition 3, for brevity we describe only the necessary modifications.

Part (i). If S is an additive decomposition of S', it is straightforward to show analogously to the proof of part (i) of Proposition 2 that, at any problem when the outcome of φ^S is stable, the outcome of $\varphi^{S'}$ is exactly the same stable matching. For the converse, a simple variant of the same example could be used to show that φ^S can choose an unstable matching for a problem $\varphi^{S'}$ that chooses a stable matching.

Part (ii). Suppose that S is not an additive decomposition of S'. Let t be the smallest index such that $e_t \neq e'_t$. Similarly to the proof of part (ii) of Theorem 1, one can construct a problem where a priority violation occurs for the $\sum_{i=0}^{t} e_i + 1$-st choice of a student, which leads to an unstable matching under φ^S but not under $\varphi^{S'}$.

We next describe the construction of a problem where the outcome of φ^S is stable while that of $\varphi^{S'}$ is not. Since S is not an additive decomposition of S', there exists an index t such that $\sum_{i=0}^{t} e'_i \neq \sum_{i=0}^{l} e_i$ for any l. We then choose the largest k and the smallest k' such that $\sum_{i=0}^{k} e_i < \sum_{i=0}^{t} e'_i < \sum_{i=0}^{k'} e_i$. Once again, using a variant of the problem in the proof of part (ii) of Theorem 1, one can construct a problem where a priority violation occurs for the $\sum_{i=0}^{t} e'_i + 1$-st choice of a student, which leads to an unstable matching under $\varphi^{S'}$ but not under φ^S. ∎

Proof of Theorem 4: The first statement follows from Theorem 3 and Proposition 6. The second statement can be shown similarly to the proof of Theorem 2, replacing e and e' respectively with e_0 and e'_0. ∎

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