

**From Boston to Chinese Parallel to Deferred Acceptance:
Theory and Experiments on a Family of School Choice Mechanisms ***

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Abstract

We characterize a parametric family of application-rejection school choice mechanisms, including the Boston and Deferred Acceptance mechanisms as special cases, and spanning the parallel mechanisms for Chinese college admissions, the largest centralized matching in the world. Moving from one extreme member to the other results in systematic changes in manipulability, stability and welfare properties. Neither the ex-post dominance of the DA over the Boston equilibria, nor the ex-ante dominance of the Boston equilibria over the DA in stylized settings extends to the parallel mechanisms. In the laboratory, participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the Chinese parallel and then the Boston mechanisms. Furthermore, while the DA is significantly more stable than the Chinese parallel mechanism, which is more stable than Boston, efficiency comparisons vary across environments.

Keywords: school choice, Boston mechanism, Chinese parallel mechanism, deferred acceptance, experiment

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1 Introduction

School choice has been one of the most important and widely-debated education policies in the past two decades (Hoxby 2003), with game theory playing a major role in the adoption of school choice mechanisms. Shortly after Abdulkadiroğlu and Sönmez (2003) was published, New York City public schools decided to replace its allocation mechanism with a capped version of the student-proposing deferred acceptance (DA) mechanism (Gale and Shapley 1962, Abdulkadiroğlu, Pathak and Roth 2005b). Later, presented with theoretical analysis (Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006) and experimental evidence (Chen and Sönmez 2006) that one of the most popular school choice mechanisms, the Boston mechanism, is vulnerable to strategic manipulation and thus might not result in efficient allocations,¹ the Boston Public School Committee voted to replace the existing Boston school choice mechanism with the deferred acceptance mechanism in 2005 (Abdulkadiroğlu, Pathak, Roth and Sönmez 2005a).

Despite the concern regarding potential manipulation, some recent literature on school choice has provided a more optimistic view of the Boston mechanism and highlighted some virtues of the Boston mechanism. Under certain restrictions, Abdulkadiroğlu, Che and Yasuda (2011) and Miralles (2009) emphasize possible *ex ante* welfare advantages of the Boston mechanism compared to the DA. Featherstone and Niederle (2008) confirm these predictions in the laboratory. In a similar vein, Kojima and Ünver (2010) offer axiomatic characterizations of the Boston mechanism, whereas Kesten (2011) shows that, contrary to the DA, the Boston mechanism is immune to manipulation attempts by schools through concealing capacity.

In this paper, we strive to better understand the sources of these different points of view regarding the Boston mechanism and its comparison to the DA. Specifically, we ask: how do the efficiency-incentive-stability trade-offs change when transitioning from the Boston mechanism to a mechanism such as the DA? A class of mechanisms that we believe could provide key insights to this question is one pioneered in Shanghai for high school admissions,² and later adopted by 22 provinces in Chinese college admissions. In the latter con-

¹He (2012) presents an interesting analysis of strategic manipulations under the Boston mechanism using school choice data from Beijing.

²This mechanism was adopted in Shanghai for high school admissions prior to 2003, <http://edu.sina.com.cn/1/2003-05-15/42912.html>, retrieved on January 7, 2013.

text, they are called the *parallel mechanisms*.

Like school choice in the United States, college admissions are among the most intensively debated public policies in the past thirty years in China. After the establishment of the People's Republic of China in 1949, Chinese universities continued to admit students via decentralized mechanisms. Historians identified two major problems with decentralized admissions during this time period. From the perspectives of the universities, as each student could be admitted into multiple universities, the enrollment to admissions ratio was low, ranging from 20% for some ordinary universities to 75% among the best universities in 1949 (Yang 2006, p. 6). From the students' perspectives, however, after being rejected by the best universities, some qualified students missed the application and examination deadlines of ordinary universities and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances, with centralized admissions within each alliance. Based on the success of the alliances,³ the Ministry of Education decided to transition to centralized matching by implementing the first National College Entrance Examination, also known as *gaokao*, in August 1952.

In recent years, each year approximately 10 million high school seniors compete for 6 million seats at various universities in China. To our knowledge, this annual event is the largest centralized matching process in the world. The matching of students to universities has profound implications for the education and labor market outcomes of these students. For matching theorists and experimentalists, the regional variations of matching mechanisms and their evolution over time provide a wealth of field observations which can enrich our understanding of matching mechanisms (see Appendix A for a historical account of Chinese college admissions). This paper provides a systematic theoretical characterization and experimental investigation of the major Chinese college admissions (CCA) mechanisms.

The CCA mechanisms are centralized matching processes via standardized tests, with each province implementing an independent matching process. These matching mechanisms fall into three classes: sequential, parallel, and partial parallel. The *sequential mechanism* is a priority matching mechanism similar to the Boston mechanism (Nie 2007b), but executed sequentially across tiers in decreasing prestige. In the sequential mechanism, each

³This experiment achieved an improved average enrollment to admissions ratio of 50% for an ordinary university (Yang 2006, p. 7). The enrollment to admissions ratio for an ordinary university in 1952 was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

college belongs to a tier. Within each tier, the Boston mechanism is used. When assignments in the first tier are finalized, the assignment process in the second tier starts, and so on. A common complaint about the sequential mechanism is that “a good score in the college entrance exam is worth less than a good strategy in the ranking of colleges” (Nie 2007a). In response to the college admissions reform survey conducted by the Beijing branch of the National Statistics Bureau in 2006, a parent complained:

My child has been among the best students in his school and school district. He achieved a score of 632 in the college entrance exam last year. Unfortunately, he was not accepted by his first choice. After his first choice rejected him, his second and third choices were already full. My child had no choice but to repeat his senior year.⁴

While the sequential mechanism used to be the only mechanism used in CCA prior to 2003, to alleviate the problem of high-scoring students not accepted by any universities and the pressure to manipulate preference rankings under the sequential mechanism, the parallel mechanism has been adopted by 22 provinces by 2012. In the parallel mechanism, students can place several “parallel” colleges for each choice. For example, a student’s first choice can contain four colleges, A, B, C and D, in decreasing desirability. Colleges consider student applications, where allocations among the parallel colleges are temporary until a student is rejected from all the parallel colleges he has listed. Thus, this mechanism lies between the Boston mechanism, where every step is final, and the DA, where every step is temporary until all seats are filled.⁵

In China, the *parallel mechanism* is widely perceived to improve allocation outcomes. For example, using survey and interview data from Shanghai in 2008, the first year when Shanghai adopted the parallel mechanism for college admissions, Hou, Zhang and Li (2009) find a 40.6% decrease in the number of students who refused to go to the universities they were matched with, compared to the year before when the sequential mechanism was in place.

⁴Source: http://www.bjstats.gov.cn/lcdcxt/tjfx/tjbg/200606/t20060626_44830.htm, retrieved on January 7, 2013. This report is also cited in Nie (2007b).

⁵An alternative interpretation of the parallel mechanism is that it approximates serial dictatorship with tiers (Wei 2009). Note, however, in the college admissions context when colleges have identical preferences, serial dictatorship and DA are equivalent.

An interview with a parent in Beijing also underscores the incentives to manipulate the first choice under the sequential versus the parallel mechanisms:⁶

My child really wanted to go to Tsinghua University. However, . . . , in order not to take any risks, we unwillingly listed a less prestigious university as her first choice. Had Beijing allowed parallel colleges as the first choice, we could at least give [Tsinghua] a try.

Transitioning from sequential to the parallel mechanisms, five provinces⁷ have adopted a hybrid between the sequential and parallel mechanisms, called the *partial parallel mechanism*. In Beijing, for example, a student can list one college as her first choice which retains the sequential nature, but three parallel colleges as her second choice. While variants of the parallel (and the partial parallel) mechanisms, each of which differs in the number of parallel choices, have been implemented in different provinces, to our knowledge, they have not been systematically studied theoretically or tested in the laboratory. In particular, when the number of parallel choices varies, how do manipulation incentives, allocation efficiency and stability change? In this paper, we investigate this question both theoretically and experimentally. We call the entire class of parallel (and partial parallel) mechanisms the *Chinese parallel mechanisms*, the simplest member of this class the *Shanghai mechanism*.

To study the performance of the different mechanisms more formally, we first provide a theoretical analysis and present a parametric family of *application-rejection* mechanisms where each member is characterized by some positive number $e \in \{1, 2, \dots, \infty\}$ of parallel and periodic choices through which the application and rejection process continues before assignments are finalized.

As parameter e varies, we go from the familiar Boston mechanism ($e = 1$) to the Chinese parallel mechanisms ($e \in [2, \infty)$), and from those to the DA ($e = \infty$). In this framework, we find that, as one moves from one extreme member of this family to the other, the experienced trade-offs are in terms of strategic immunity, stability, and welfare. We provide property-based rankings of the members of this family using some techniques recently developed by Pathak and Sönmez (forthcoming). We show that any given member is *more manipulable*

⁶Li Li. “Ten More Provinces Switch to Parallel College Admissions Mechanism This Year.” *Beijing Evening News*, June 8, 2009.

⁷In 2012, provinces using the partial parallel mechanism include Beijing, Chongqing, Gansu, Jilin and Sichuan.

than a member with a higher e number. As far as stability is concerned, while the ranking is more subtle within the general class of mechanisms, the DA is *more stable* than the Shanghai mechanism, which is more stable than the Boston mechanism.⁸ On the welfare side, the member of the family indexed by e is the one that maximizes the number of students receiving one of their (reported) e choices. However, no ranking can be made based on Pareto efficiency.⁹

In the literature the comparison of the equilibrium outcomes of the Boston mechanism with the (dominant strategy equilibrium) outcome of the DA have lead to different conclusions depending on what is assumed about the information participants hold. Under the assumption of complete information, the DA outcome Pareto dominates that in any of the Boston equilibria (Ergin and Sönmez 2006). Under the incomplete information assumption, however, this conclusion is totally reversed if students are also assumed to have common ordinal preferences and no priorities (Abdulkadiroğlu et al. 2011). It turns out that neither Boston nor the DA have any such dominance over a Chinese parallel mechanism, such as the Shanghai mechanism, regardless of the informational view.

Since the theoretical welfare ranking in this family of mechanisms assumes truth-telling, which is a dominant strategy only under the DA, it is important to assess the behavioral responses to members of this family. Furthermore, because of the multiplicity of Nash equilibrium outcomes in this family of mechanisms, empirical evaluations of the performance of these mechanisms in controlled laboratory settings will inform policymakers in school choice or college admissions reform.

For these reasons, we evaluate three members of this family in two environments in the laboratory. In both environments, we find that participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the Chinese parallel and then the Boston mechanisms. Consistent with theory, the DA achieves a significantly higher pro-

⁸More precisely, if the outcome of the Boston mechanism is stable at a given problem, then the outcome of the Shanghai mechanism is also stable at the same problem, while the converse statement is not necessarily true.

⁹Nie and Zhang (2009) investigate the theoretical properties of a variant of the parallel mechanism where each applicant has three parallel colleges, i.e., $e = 3$ in our notation, and characterize the equilibrium when applicant beliefs are i.i.d draws from a uniform distribution. Using an example, the authors demonstrate that increasing the number of parallel options, e , does not necessarily result in Pareto improvements for all students, which we generalize in Theorem 3. Wei (2009) considers the parallel mechanism where each college has an exogenous minimum score threshold drawn from a uniform distribution. Under this scenario, she demonstrates that increasing the number of parallel options cannot make an applicant worse off.

portion of stable outcomes than the Chinese parallel, which achieves a significantly higher proportion of stable outcomes than Boston. However, the efficiency comparison is sensitive to the environment. While theory is silent on equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than the unstable ones. To our knowledge, our paper presents the first experimental evaluation of the Chinese parallel mechanism relative to Boston and the DA, as well as equilibrium selection in school choice mechanisms.

The rest of this paper is organized as follows. Section 2 formally introduces the school choice problem and the family of mechanisms. Section 3 presents the theoretical results. Section 4 describes the experimental design. Section 5 summarizes the results of the experiments. Section 6 concludes.

2 School choice problem and the three mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) is comprised of a number of students each of whom is to be assigned a seat at one of a number of schools. Further, each school has a maximum capacity, and the total number of seats in the schools is no less than the number of students. We denote the set of students by $I = \{i_1, i_2, \dots, i_n\}$, where $n \geq 2$. A generic element in I is denoted by i . Likewise, we denote the set of schools by $S = \{s_1, s_2, \dots, s_m\} \cup \{\emptyset\}$, where $m \geq 2$ and \emptyset denotes a student's outside option, or the so-called null school. A generic element in S is denoted by s . Each school has a number of available seats. Let q_s be the number of available seats at school s , or the **quota** of s . Let $q_\emptyset = \infty$. For each school, there is a strict priority order of all students, and each student has strict preferences over all schools. The priority orders are determined according to state or local laws as well as certain criteria of school districts. We denote the priority order for school s by \succ_s , and the preferences of student i by P_i . Let R_i denote the at-least-as-good-as relation associated with P_i . Formally, we assume that R_i is a linear order, i.e., a complete, transitive, and anti-symmetric binary relation on S . That is, for any $s, s' \in S$, $s R_i s'$ if and only if $s = s'$ or $s P_i s'$. For convenience, we sometimes write $P_i : s_1, s_2, s_3, \dots$ to denote that, for student i , school s_1 is his first choice, school s_2 his second choice, school s_3 his third choice, etc.

A **school choice problem**, or simply a problem, is a pair $(\succ = (\succ_s)_{s \in S}, P = (P_i)_{i \in I})$ consisting of a collection of priority orders and a preference profile. Let \mathcal{R} be the set of

all problems. A **matching** μ is a list of assignments such that each student is assigned to one school and the number of students assigned to a particular school does not exceed the quota of that school. Formally, it is a function $\mu : I \rightarrow S$ such that for each $s \in S$, $|\mu^{-1}(s)| \leq q_s$. Given $i \in I$, $\mu(i)$ denotes the assignment of student i at μ and given $s \in S$, $\mu^{-1}(s)$ denotes the set of students assigned to school s at μ . Let \mathcal{M} be the set of all matchings. A matching μ is **non-wasteful** if no student prefers a school with unfilled quota to his assignment. Formally, for all $i \in I$, $s P_i \mu(i)$ implies $|\mu^{-1}(s)| = q_s$. A matching μ is **Pareto efficient** if there is no other matching which makes all students at least as well off and at least one student better off. Formally, there is no $\alpha \in \mathcal{M}$ such that $\alpha(i) R_i \mu(i)$ for all $i \in I$ and $\alpha(j) P_j \mu(j)$ for some $j \in I$.

A closely related problem to the school choice problem is the *college admissions problem* (Gale and Shapley 1962). In the college admissions problem, schools have preferences over students whereas in a school choice problem, schools are merely objects to be consumed. A key concept in college admissions is “stability,” i.e., there is no unmatched student-school pair (i, s) such that student i prefers school s to his assignment, and school s either has not filled its quota or prefers student i to at least one student who is assigned to it. The natural counterpart of stability in our context is defined by Balinski and Sönmez (1999). The **priority of student i for school s is violated** at a given matching μ (or alternatively, student i justifiably envies student j for school s) if i would rather be assigned to s to which some student j who has lower s -priority than i , is assigned, i.e., $s P_i \mu(i)$ and $i \succ_s j$ for some $j \in I$. A matching is **stable** if it is non-wasteful and no student’s priority for any school is violated.

A **school choice mechanism**, or simply a mechanism φ , is a systematic procedure that chooses a matching for each problem. Formally, it is a function $\varphi : \mathcal{R} \rightarrow \mathcal{M}$. Let $\varphi(\succ, P)$ denote the matching chosen by φ for problem (\succ, P) and let $\varphi_i(\succ, P)$ denote the assignment of student i at this matching. A mechanism is Pareto efficient (stable) if it always selects Pareto efficient (stable) matchings. A mechanism φ is **strategy-proof** if it is a dominant strategy for each student to truthfully report her preferences. Formally, for every problem (\succ, P) , every student $i \in I$, and every report P'_i , $\varphi_i(\succ, P) R_i \varphi_i(\succ, P'_i, P_{-i})$.

Following Pathak and Sönmez (forthcoming), a mechanism ϕ is *manipulable by student j* at problem (\succ, P) if there exists P'_j such that $\phi_j(\succ, P'_j, P_{-j}) P_j \phi_j(\succ, P)$. Thus, mecha-

nism ϕ is said to be *manipulable* at a problem (\succ, P) if there exists some student j such that ϕ is manipulable by student j at (\succ, P) . Mechanism φ is **more manipulable** than mechanism ϕ if (i) at any problem ϕ is manipulable, φ is also manipulable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is manipulable but ϕ is not. Mechanism φ is **more stable (more efficient)** than mechanism ϕ if (i) at any problem ϕ is stable (Pareto efficient), φ is also stable (Pareto efficient); and (ii) the converse is not always true, i.e., there is at least one problem at which φ is stable (Pareto efficient) but ϕ is not.¹⁰

We now describe the three mechanisms that are central to our study. The first two are the familiar Boston and the DA mechanisms, while the third one is a stylized version of the simplest parallel mechanism.

2.1 Boston Mechanism (BOS)

Our first mechanism is the most common school choice mechanism observed in practice. Its outcome can be calculated via the following algorithm for a given problem:

Step 1: For each school s , consider only those students who have listed it as their first choice. Up to q_s students among them with the highest s -priority are assigned to school s .

Step k , $k \geq 2$: Consider the remaining students. For each school s with q_s^k available seats, consider only those students who have listed it as their k -th choice. Those q_s^k students among them with the highest s -priority are assigned to school s .

The algorithm terminates when there are no students left. Importantly, note that the assignments in each step are final. Based on this feature, an important critique of the Boston mechanism highlighted in the literature is that it gives students strong incentives to misrepresent their preferences. Because a student who has high priority for a school may lose her priority advantage for that school if she does not list it as her first choice, the Boston mechanism forces students to make hard and risky strategic choices (see e.g., Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006, Chen and Sönmez 2006, and He 2012).

¹⁰See Kesten (2006, 2011) for similar problem-wise property comparisons across and within mechanisms for matching problems.

2.2 Deferred Acceptance Mechanism (DA)

A second matching mechanism is the student-optimal stable mechanism (Gale and Shapley 1962), which finds the stable matching that is most favorable to each student. Its outcome can be calculated via the following *deferred acceptance (DA) algorithm* for a given problem:

Step 1: Each student applies to her favorite school. For each school s , up to q_s applicants who have the highest s -priority are tentatively assigned to school s . The remaining applicants are rejected.

Step k , $k \geq 2$: Each student rejected from a school at step $k - 1$ applies to her next favorite school. For each school s , up to q_s students who have the highest s -priority among the new applicants and those tentatively on hold from an earlier step, are tentatively assigned to school s . The remaining applicants are rejected.

The algorithm terminates when each student is tentatively placed to a school. Note that, in the DA, assignments in each step are temporary until the last step. The DA has several desirable theoretical properties, most notably in terms of incentives and stability. Under the DA, it is a dominant strategy for students to state their true preferences (Roth 1982, Dubins and Freedman 1981). Furthermore, it is stable. Although it is not Pareto efficient, it is the most efficient among the stable school choice mechanisms.

In practice, the DA has been the leading mechanism for school choice reforms. For example, the DA has been adopted by New York City and Boston public school systems, which had suffered from congestion and incentive problems from their previous assignment systems, respectively (Abdulkadiroğlu et al. 2005a, Abdulkadiroğlu et al. 2005b).

2.3 The Chinese Parallel Mechanisms

The Chinese parallel mechanism was first implemented as a high school admissions mechanism in Shanghai prior to 2003. From 2003 to 2012, variants of the mechanism have been adopted by 22 (5) provinces as the parallel (partial parallel) college admissions mechanisms to replace the sequential mechanisms (Wu and Zhong 2012).

While there are many regional variations in CCA, from a game theoretic perspective, however, they differ in two main dimensions which impact the students' strategic decisions

during the application process. The first dimension is the timing of preference submission, including before the exam (2 provinces), after the exam but *before* knowing the exam scores (3 province), and *after* knowing the exam scores (26 provinces).¹¹ The second dimension is the actual matching mechanisms used in each province. The sequential mechanism used to be the only college admissions mechanism used in China. In 2012, while the sequential mechanism was still used in 3 provinces, variants of the parallel (and partial parallel) mechanism have been adopted by 27 provinces, while the remaining province, Inner Mongolia, uses an admissions process which resembles a dynamic implementation of the parallel mechanism. A brief description of the evolution of Chinese college admissions mechanisms from 1949 to 2012 is contained in Appendix A.

In this study, we investigate the properties of the family of mechanisms used for Chinese college admissions. We now describe a stylized version of the Chinese parallel mechanisms in its simplest version, with two parallel choices, adapted for the school choice context. A more general description is contained in Section 3.

- An application to the first ranked school is sent for each student at the first step.
- Throughout the allocation process, a school can hold no more applications than its quota.

If a school receives more applications than its quota, it retains the students with the highest priority up to its quota and rejects the remaining students.

- Whenever a student is rejected from her first choice school, her application is sent to her second choice school. Whenever a student is rejected from her second choice school, she can no longer make an application in this round.
- Throughout each round, whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among

¹¹Zhong, Cheng and He (2004) demonstrate that, while there does not exist a Pareto ranking of the three variants in the preference submission timing, the first two mechanisms can sometimes achieve Pareto efficient outcomes. Furthermore, experimental studies confirm the *ex ante* efficiency advantage of the Boston mechanism with pre-exam preference ranking submissions in both small (Lien, Zheng and Zhong 2012) and large markets (Wang and Zhong 2012). Lastly, using a data set from Tsinghua University, Wu and Zhong (2012) find that, while students admitted under the sequential mechanism with pre-exam preference ranking submissions have on average lower entrance exam scores than those admitted under other mechanisms, they perform as well or even better in college than their counterparts admitted under other timing mechanisms.

the retained and new applications, the ones with the highest priority up to the quota are retained.

- The allocation is finalized every *two* choices. That is, if a student is rejected by her first two choices in the initial round, then she participates in a new round of applications together with other students who have also been rejected from their first two choices, and so on. At the end of each round the assigned students and the slots assigned to them are removed from the system.

The assignment process ends when no more applications can be rejected. We refer to this mechanism as the *Shanghai mechanism*.¹²

In the next section, we offer a formal definition of the parallel mechanisms and characterize the theoretical properties of this family of matching mechanisms.

3 Theoretical Analysis: A parametric family of mechanisms

In this section, we investigate the theoretical properties of the family of application-rejection mechanisms. Given student preferences, school priorities, and school quotas, consider the following parametric *application-rejection algorithm* that indexes each member of the family by a *permanency-execution period* e :

Round $t = 0$:

- Each student applies to her first choice. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e -th choice school, applies to his next choice. If a student has been rejected from all his first e choices, then he remains unassigned in this round and does not make any applications until the next round.

¹²In Appendix A, we provide a translation of an online Q&A about the Shanghai parallel mechanism used for middle school admissions to illustrate how the parallel choices work.

Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round, i.e., he has been rejected by all his first e choice schools. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

In general,

Round $t \geq 1$:

- Each unassigned student from the previous round applies to his $te+1$ -st choice school. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his $te + e$ -th choice school, applies to his next choice. If a student has been rejected from all his first $te + e$ choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.
- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round, i.e., he has been rejected by all his first $te + e$ choice schools. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* (e)

and denoted by φ^e . This family of mechanisms nests the Boston and the DA mechanisms as extreme cases, the Chinese parallel mechanisms as intermediate cases, and the Chinese partial parallel mechanisms as an extension (see Section 3.3).

Remark 1 *The application-rejection mechanism (e) coincides with*

- (i) *the Boston mechanism when $e = 1$,*
- (ii) *the Shanghai mechanism when $e = 2$,*
- (iii) *the Chinese parallel mechanism when $2 \leq e < \infty$, and*
- (iv) *the DA mechanism when $e = \infty$.*

Remark 2 *It is easy to verify that all members of the family of application-rejection mechanisms, i.e., $e \in \{1, 2, \dots, \infty\}$, are non-wasteful. Hence, the outcome of an application-rejection mechanism is stable for a given problem if and only if it does not result in a priority violation.*

Next is our first observation about the properties of this family of mechanisms.

Proposition 1 *Within the family of application-rejection mechanisms, i.e., $e \in \{1, 2, \dots, \infty\}$,*

- (i) *there is exactly one member that is Pareto efficient. This is the Boston mechanism;*
- (ii) *there is exactly one member that is strategy-proof. This is the DA mechanism; and*
- (iii) *there is exactly one member that is stable. This is the DA mechanism.*

All proofs and examples are relegated to Appendix B.

3.1 Property-specific comparisons of application-rejection mechanisms

As Proposition 1 shows, an application-rejection (e) mechanism is manipulable if $e < \infty$. Hence, when faced with a mechanism other than the DA, students should make careful judgments to determine their optimal strategies, and in particular, when deciding which e schools to list on top of their preference lists. More specifically, since priorities matter for determining the assignments only *within* a round and have no effect on the assignments of past rounds, getting assigned to one of the first e choices is extremely crucial for a student.

In other words, a successful strategy for a student is one that ensures that he is assigned to his “target school” at the end of the initial round, i.e., round 0. In this sense, missing the

first choice in the Boston mechanism could be more costly to a student than in a Chinese parallel mechanism such as the Shanghai, which offers a “second chance” to the student before he loses his priority advantage. On the other hand, at the other extreme of this class lies the DA, which completely eliminates any possible loss of priority advantage for a student. The three-way tension among incentives, stability, and welfare that emerges under this class is rooted in this observation.

We next provide an incentive-based ranking of the family of application-rejection mechanisms.

Theorem 1 (Manipulability) *For any e , φ^e is more manipulable than $\varphi^{e'}$ where $e' > e$.*

Corollary 1 *Among application-rejection mechanisms, Boston is the most manipulable and the DA is the least manipulable member.*

Corollary 2 *Any Nash equilibrium of the preference revelation game associated with φ^e is also a Nash equilibrium of that of $\varphi^{e'}$ where $e' > e$.*

Remark 3 *Notwithstanding the manipulability of all application-rejection mechanisms except the DA, it is still in the best interest of each student to put his within-round choices in their true order. More precisely, for a student facing φ^e , any strategy that does not list the first e choices, that are considered in the initial round, in their true order, is dominated by the otherwise identical strategy that lists them in their true order. Similarly, not listing a set of e choices considered in a subsequent round is also dominated by an otherwise identical strategy that lists them in their true order.*

Corollary 2 says that the set of Nash equilibrium strategies corresponding to the preference revelation games associated with members of the application-rejection family has a nested structure.¹³ A useful interpretation is that when making problemwise comparisons across the members of the application-rejection family (e.g., see Theorem 2), such comparisons might as well be made across equilibria of two different members.

¹³A similar observation is made by Haeringer and Klijn (2008) for the revelation games under the Boston mechanism when the number of school choices a student can make (in her preference list) is limited by a quota.

Corollary 2 also implies that for any given problem, the DA has the largest set of equilibrium profiles within the entire family, whereas Boston has the smallest. Consequently, this observation entails that coordination issues may become more serious for the Chinese parallel mechanisms as e increases. Interestingly, this means that the DA would be subject to a more difficult coordination problem than Boston if not for its strategy-proofness property.

We now turn to investigate a possible ranking of the members of the family based on stability. An immediate observation is that under an application-rejection (e) mechanism, no student's priority for one of his first e choices is ever violated. This is simply because all previous assignments are tentative in the application-rejection algorithm until the student is rejected from all his first e choices. This observation hints that one might expect mechanisms to become more stable as parameter e grows. The next result shows that this may not always be the case.

Theorem 2 (Stability) *Let $e' > e$.*

- (i) *If $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is more stable than φ^e .*
- (ii) *If $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is not more stable than φ^e .*

Corollary 3 *The DA is more stable than the Shanghai mechanism, which is more stable than the Boston mechanism.*

Corollary 4 *Any other application-rejection mechanism is more stable than the Boston mechanism.*

Corollary 5 *The only mechanisms which are more stable than the Shanghai mechanism are φ^4 , φ^6 , \dots , and φ^∞ .*

Theorem 2 indicates that while it is possible to rank all three special members of the family of application-rejection mechanisms, i.e., $e \in \{1, 2, \infty\}$, according to the stability of their outcomes, within the Chinese parallel mechanisms, however, there may not be a problemwise systematic ranking in general. Nevertheless, if the number of choices considered in each round by one mechanism is a multiple of that of the other mechanism, in this case the mechanism that allows for more choices is the more stable one.

When true preferences are not observable, a plausible metric for evaluating student welfare is based on considering the number of students assigned to their first choices.¹⁴ As a way to assess student welfare, we next present a ranking of the mechanisms within the family based on the number of first choices they assign. It turns out that the Boston is the most generous in terms of first choice accommodation, whereas the DA is the least. Hence, the next result shows that, while there may not be a systematic ranking within the family of application-rejection mechanisms based on Pareto efficiency, the reduction in the scope of manipulation with an increasing e parameter may come at the cost of a diminishing number of first choice assignments.

Theorem 3 (Welfare) (i) *Boston is more efficient than any other application-rejection (e) mechanism. Mechanisms other than Boston cannot be ranked based on efficiency. More precisely, when $1 \neq e < e'$, φ^e is not more efficient than $\varphi^{e'}$ nor vice versa.*

(ii) *φ^e assigns a higher number of students to their first choices than $\varphi^{e'}$ where $e < e'$.*

(iii) *φ^e assigns a higher number of students to their first e choices than $\varphi^{e'}$ where $e \neq e'$.*

Corollary 6 *The Boston mechanism maximizes the number of students receiving their first choices.*

Corollary 7 *The Shanghai mechanism maximizes the number of students receiving their first or second choices.*

Nonetheless, one needs to be cautious when interpreting Theorem 3. Since all members of the family with the exception of the DA violate strategy-proofness, student preference submission strategies may also vary across mechanisms. To address this issue, in the next section we investigate the properties of Nash equilibrium outcomes of the family of application-rejection mechanisms from both complete and incomplete information perspectives.

¹⁴For example, in evaluating the outcome of the Boston mechanism, Cookson Jr. (1994) reports that 75% of all students entering the Cambridge public school system at the K-8 levels gained admission to the school of their first choice. Similarly, the analysis of the Boston and NYC school district data by Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) and Abdulkadiroğlu, Pathak and Roth (2009) also report the number of first choices of students.

3.2 Equilibria of the Induced Preference Revelation Games

3.2.1 Ex post equilibria: Complete information view

Ergin and Sönmez (2006) show that, under complete information, every Nash equilibrium outcome of the preference revelation game induced by the Boston mechanism leads to a stable matching under students' true preferences, and that any given stable matching can be supported as a Nash equilibrium of this game. Hence, if one assumes that all students are strategic and able to coordinate their actions to achieve an equilibrium outcome, then this result has a clear implication. Since the DA is strategy-proof and chooses the most favorable stable matching for students, the Boston mechanism can *at best* be as good as the DA in terms of the resulting welfare. Put differently, there is a clear welfare loss associated with the Boston mechanism relative to the DA.

To analyze the properties of the equilibrium outcomes of the application-rejection mechanisms, we next study the Nash equilibrium outcomes induced by the preference revelation games under this family of mechanisms. It turns out that the DA does not generate a clear welfare gain relative to the Chinese parallel mechanisms.

Proposition 2 (Ex post equilibria) *Consider the preference revelation game induced by φ^e under complete information.*

- (i) *If $e = 1$, then every Nash equilibrium outcome of this game is stable under the true preferences and thus it is Pareto dominated by the DA.*
- (ii) *If $e \notin \{1, \infty\}$, there exist Nash equilibrium outcomes of this game which are unstable under the true preferences and Pareto dominate the DA.*

Proposition 2 shows that the welfare comparison between the equilibria of the DA and the Chinese parallel mechanisms is ambiguous. On the other hand, the fact that both the Boston and the Chinese parallel mechanisms admit multiple equilibria, precludes a direct equilibrium-wise comparison between the two mechanisms. Nevertheless, a curious question at this point is then whether there could be any validity to the widespread belief (also expressed in a quote in the introduction) that the parallel mechanisms may better serve the

interests of students than the sequential mechanisms, which are based on the Boston mechanism. The next result provides a formal sense in which a Chinese parallel mechanism may indeed be more favorable for each student relative to the Boston.

Proposition 3 (Insurance under the Parallel Mechanisms) *Let μ be an equilibrium outcome under the Boston mechanism. Under φ^e if each student i lists $\mu(i)$ as his e -th choice and any $(e - 1)$ schools he truly likes better than $\mu(i)$ as higher-ranked choices (not necessarily in their true order), then each student's assignment is at least as good as that under Boston.*¹⁵

Remark 4 *It is worth noting that Proposition 3 does not generalize to any two application-rejection mechanisms as this result crucially hinges on part (i) of Proposition 2. For example, let μ be an equilibrium outcome of the Shanghai. If each student lists his assignment at μ as his e -th choice similarly to the above, then the resulting outcome of φ^e with any $e > 2$ need not be weakly preferred to that of Shanghai by each student.*¹⁶

From a practical point of view, Proposition 3 says that whatever school a student is “targeting” under the Boston mechanism, he would be at least as well off under a parallel mechanism by simply including it among his first e choices while ranking better options higher up in his preferences, provided that other students are doing the same. In other words, the Chinese parallel mechanisms may allow students to retain their Boston assignments as “insurance” options while keeping more desirable options within reach.

3.2.2 Ex ante Equilibria: Incomplete information view

Abdulkadiroğlu et al. (2011) [henceforth, ACY] study an incomplete information model of school choice that captures two salient features from practice: correlated ordinal preferences and coarse school priorities. More specifically, they consider a stylized setting where students share the same ordinal preferences but different and unknown cardinal preferences

¹⁵We stipulate that the e -th choice is the last choice when $e = \infty$. For expositional simplicity, we also assume that student i has $e - 1$ truly better choices than $\mu(i)$. The proposition still holds if the statement were to be modified so that each student i lists $\mu(i)$ as *one of* his first e choices.

¹⁶To illustrate this point for the Shanghai vs. the DA, for example, let μ correspond to an unstable equilibrium outcome that Pareto dominates the DA matching under truth-telling.

and schools have no priorities, i.e., priorities are determined via a random lottery draw after students submit preference rankings. They focus on the symmetric Bayesian Nash equilibria under Boston and show that every student is at least weakly better off in any such equilibrium than in the dominant strategy equilibrium of the DA. This striking finding suggests that there may be a clear welfare loss to every student under the DA relative to Boston in such circumstances.

As noted earlier, out of the 31 provinces in China, two of them, Beijing and Shanghai, require students to submit preference rankings before taking the college entrance exam. Hence the ACY setting can be seen as a sensible, albeit not perfect model to capture behavior in these provinces. Thus we next investigate whether or not the *ex ante* dominance of Boston also prevails when compared with other members of the application-rejection family. It turns out the answer is negative.

To gain a clear insight into the *ex ante* welfare issues we focus on the familiar Boston and the DA together with Shanghai, the simplest member of the Chinese parallel mechanisms. We show that, in the same setting as ACY, there may be students who are better off in a Bayesian equilibrium of Shanghai than in one of Boston. The following example illustrates the intuition.

Let there be four students of three types, with values $\{v_L, v_M, v_H\}$, two from the low type and one each from the medium and high types, and four schools $\{s_0, s_1, s_2, s_3\}$, each with one seat. There are no priorities *a priori*, students have common ordinal preferences, and each student type has the von Neumann Morgenstern (vNM) utility values given in the following table.

| | v_L | v_M | v_H |
|-------|-------|-------|-------|
| s_0 | .9 | .53 | .36 |
| s_1 | .09 | .36 | .35 |
| s_2 | .01 | .11 | .29 |
| s_3 | 0 | 0 | 0 |

First, consider Boston with random tie-breaking. Type v_L students have a dominant strategy of ranking schools truthfully. Given that, type v_M student has a best response of ranking s_1 as his first choice (regardless of what type v_H does). And, given all these strategies, type v_H student has a best response of ranking s_2 as his first choice. This constitutes

the unique equilibrium under the Boston mechanism, where type v_H student obtains an expected utility of .29.

Now let us consider the Shanghai mechanism with random tie-breaking. Type v_L students again have a dominant strategy of ranking schools truthfully. Given that, type v_M student has a best response of ranking schools truthfully (regardless of what type v_H does). And, given all these strategies, type v_H student has a best response of respectively ranking s_1 and s_2 as his first and second choices (see the proof of Proposition 4 part (ii) in Appendix B for details). This constitutes the unique equilibrium under the Shanghai mechanism, where type v_H student now obtains an expected utility of .32.

The reason why some students may prefer the Shanghai to the Boston, unlike the case against the DA, as in this example, can be intuitively explained as follows. Under the Boston mechanism, students' first choices are crucial and thus students target a single school at equilibrium. Under the Shanghai mechanism, the first two choices are crucial and students target a pair of schools. This difference, however, may enable a student to guarantee a seat at an unpopular school under the Shanghai by ranking it as his second choice and still give him some chance to obtain a more preferred school by ranking it as his first choice. For example, in the above scenario, type v_H student "gains priority" at school s_2 , her sure outcome in Boston, when others do not include it in their first two choices and enjoys as well a positive chance of ending up at s_1 .¹⁷

Although we have assumed in the above example that students have complete information about their cardinal preferences, it is possible to use the same insight to show the non-dominance of Boston over Shanghai in a Bayesian setting.

Proposition 4 (Ex ante equilibrium) *In the Bayesian setting of ACY (see the Appendix B for a formal treatment),*

- (i) *each student is weakly better off in any symmetric equilibrium of Shanghai than she is in the DA, and*

¹⁷Loosely speaking, the Boston lottery (i.e., Boston with random tie-breaking) when compared with the Shanghai lottery (i.e., Shanghai with random tie-breaking) can be seen as a weighted average over more extreme choices (when the lotteries are non-degenerate). In the above example, for instance, a low type student faces a lottery between his first and last choices under Boston. This is because if he misses his first choice, his second and third choices will already be taken. On the other hand, the Shanghai lottery always puts positive weight on the first *and* the second choices. At the other extreme, the DA lottery is an equal weighted average over all choices.

(ii) *no ex ante Pareto ranking can be made between Boston and Shanghai, i.e., there exists problems where some student types are weakly better off at the equilibrium under Shanghai than they are under Boston and vice versa.*

Part (i) of Proposition 4 says that just like the Boston mechanism, the Shanghai mechanism also leads to a clear welfare gain over the DA in the same setting. This shows that just like the Boston mechanism, the Shanghai mechanism may also allow students to communicate their cardinal utilities, thereby resolving their conflicting interests more efficiently than the DA.

3.3 The Extended Class of the Chinese Parallel Mechanisms

Thus far our analysis focused on the Chinese parallel mechanisms where the same number of student choices were considered periodically, i.e., the parameter e has been constant across rounds. In fact, in some Chinese provinces the college admission mechanisms allow for variations in the number of choices that are considered within a round. For example, in Anhui province, the number of parallel choices are set to $e = 4, 4, 4, 6, \dots$ in 2012. We next augment and extend the application-rejection family to accommodate for the extended class.

Given a problem, consider the following *application-rejection algorithm* that is associated with a choice sequence (e_1, e_2, e_3, \dots) , where the terms in this sequence denote the number of choices to be tentatively considered in each round.

Round $t = 0$:

- Each student applies to her first choice. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e_1 -th choice school, applies to his next choice. If a student has been rejected from all his first e_1 -choices, then he remains unassigned in this round and does not make any applications until the next round.

Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

In general,

Round $t \geq 1$:

- Each unassigned student from the previous round applies to his $\sum_{i=1}^t e_i + 1$ -st choice school. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his $\sum_{i=1}^{t+1} e_i$ -th choice school, applies to his next choice. If a student has been rejected from all his first $\sum_{i=1}^{t+1} e_i$ -choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest x -priority are tentatively assigned to school x up to its quota. The rest are rejected.
- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* (e_1, e_2, e_3, \dots) .

Theorem 4 (Extended Parallel Class) *An application-rejection mechanism associated with a choice sequence (e_1, e_2, e_3, \dots) is more manipulable than any application-rejection mechanism associated with a choice sequence $(e'_1, e'_2, e'_3, \dots)$ with $e_1 < e'_1$.*

Theorem 4 says that a mechanism using a choice sequence of fewer number of parallel colleges in the initial round is more manipulable than a corresponding partial parallel mechanism with a greater number of such parallel colleges. This result in turn underscores the importance of the initial round relative to all other rounds, a point much emphasized in the previous literature in the context of the Boston mechanism.¹⁸ Theorem 4 implies, for example, that the partial parallel mechanism used in Beijing, with choices of $(1, 3, 1, 3, \dots)$, is more manipulable than the parallel mechanism used in Anhui province, with choices of $(4, 4, 4, 6, \dots)$.

4 Experimental Design

We design our experiment to compare the performance of the Boston, Shanghai and the DA mechanisms based on the theoretical characterization of the family of application-rejection mechanisms in Section 3. We choose the complete information environment to test the theoretical predictions, especially those on Nash equilibrium outcomes. While incomplete information environments might be more realistic than the complete information environments in the school choice context, it has proven useful to attack the problem one piece at a time.¹⁹ In the closely related area of implementation theory, “understanding implementation in the complete information setting has helped significantly in developing characterizations of implementation in Bayesian settings” (Jackson 2001).

A $3(\text{mechanisms}) \times 2(\text{environments})$ factorial design is implemented to evaluate the performance of the three mechanisms $\{\text{BOS}, \text{SH}, \text{DA}\}$ in two different environments, a simple 4-school environment and a more complex 6-school environment. The environments are designed to capture the key aspects of the school choice problem and to simulate the

¹⁸Intuitively, the reason why the ranking depends only on the number of parallel choices of the initial round is because manipulations that happen in subsequent rounds can always be “translated” to the initial round by including the target school among the parallel choices of the initial round. Consequently, the number of choices of in subsequent rounds do not matter for manipulability.

¹⁹In a follow-up study, we test the same set of mechanisms under both the complete and incomplete information settings in the college admissions context (Chen, Jiang and Kesten 2012).

complexity inherent in potential applications of the mechanisms.

4.1 The 4-School Environment

The first environment, which we call the **4-school environment**, has four students, $i \in \{1, 2, 3, 4\}$, and four schools, $s \in \{a, b, c, d\}$. Each school has one slot, which is allocated to one participant. We choose the parameters of this environment to satisfy several criteria: (1) no one lives in the district of her top or bottom choices; (2) the first choice accommodation index, i.e., the proportion of first choices an environment can accommodate, is $1/2$; (3) there is a small number of Nash equilibrium outcomes, which reduces the complexity of the games.

The payoffs for each student are presented in Table 1. The square brackets, [], indicate the resident of each school district, who has higher priority in that school than other applicants. Payoffs range from 16 points for a first-choice school to 5 points for a last-choice school. Each student resides in her second-choice school.

Table 1: Payoff Table for the 4-School Environment

| | a | b | c | d |
|------------------|------|------|------|------|
| Payoff to Type 1 | [11] | 7 | 5 | 16 |
| Payoff to Type 2 | 5 | [11] | 7 | 16 |
| Payoff to Type 3 | 7 | 16 | [11] | 5 |
| Payoff to Type 4 | 5 | 16 | 7 | [11] |

For each session in the 4-school environment, there are 12 participants of four different types. Participants are randomly assigned types at the beginning of each session. At the beginning of each period, they are randomly re-matched into groups of four, each of which contains one of each of the four different types. Four schools are available for each group. In each period, each participant ranks the schools. After all participants have submitted their rankings, the server allocates the schools in each group and informs each person of his school allocation and respective payoff. The experiment consists of 20 periods to facilitate learning. Furthermore, we change the priority queue every five periods to investigate

whether participant strategies are conditional on their priority.²⁰

For each of the 4 different queues, we compute the Nash equilibrium outcomes under the Boston and Shanghai mechanisms (which are the same) as well as under the DA. For all four blocks, Boston and Shanghai each have a unique Nash equilibrium outcome, where each student is assigned to her district school. This college/student-optimal matching, $\mu^{C/S}$, is Pareto inefficient, with the sum of ranks of 8 and an aggregate payoff of 44:

$$\mu^{C/S} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & b & c & d \end{pmatrix}$$

For all four blocks, the matching $\mu^{C/S}$ is also a Nash equilibrium outcome under the DA. However, the DA has exactly one more Nash equilibrium outcome for all four cases, which is the following Pareto efficient matching μ^* , with the sum of ranks of 6 and an aggregate payoff of 54:

$$\mu^* = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & d & c & b \end{pmatrix}.$$

The Nash equilibrium profile that sustains outcome μ^* is the following (asterisks are arbitrary): $P_1 = (a, *, *, *)$, $P_2 = (d, b, *, *)$, $P_3 = (c, *, *, *)$, and $P_4 = (b, d, *, *)$. This is an equilibrium profile regardless of the priority order.²¹ Note that, in this equilibrium profile, types 1 and 3 misrepresent their first choices by reporting their district school as their first choices, while types 2 and 4 report their true top choices.²²

We now analyze participant incentives to reveal their true preferences in this environment. We observe that, in blocks 1 and 3, while truth-telling is a Nash equilibrium strategy under the Shanghai mechanism, it is not a Nash equilibrium under Boston. Furthermore, under truth-telling, Shanghai and the DA yield the same Pareto inefficient outcome. Recall that Corollary 2 implies that, if truth-telling is a Nash equilibrium under Boston, then it is

²⁰The priority queues for each five-period block are 1-2-3-4, 4-1-2-3, 3-4-1-2 and 2-3-4-1, respectively. Appendix D has detailed experimental instructions.

²¹This is a Nash equilibrium because, for example, if student 1 (or 3) submits a profile where she lists school d (resp. b) as her first choice, then she may kick out student 2 (resp. 4) in the first step but 2 (resp. 4) would then apply to b (resp. d) and kick out 4 (resp. 2) who would in turn apply to d (resp. b) and kick out 1 (resp. 3). Hence student 1 (or 3), even though she may have higher priority than 2 (resp. 4), she cannot secure a seat at b (resp. d) under DA.

²²Note that types 1 and 3's manipulation benefits types 2 and 4, thus it does not violate truth-telling as a weakly dominant strategy, since type 1 (resp. 3) is indifferent between truth-telling and lying. If type 1 (resp. 3) reverts to truth-telling, she will then cause a rejection chain which gives everyone their district school, including herself. Therefore, she is not better off by deviating from the efficient but unstable Nash equilibrium strategy.

also a Nash equilibrium under the Shanghai mechanism, but the converse is not necessarily true. Blocks 1 and 3 are examples of the latter.

Table 2: Truthtelling and Nash Equilibrium Outcomes in the 4-School Environment

| | Truthful Preference Revelation | | | Nash Equilibrium Outcomes | | |
|---------|--------------------------------|--------|-------------------|---------------------------|-------------|------------------------|
| | BOS | SH | DA | BOS | SH | DA |
| Block 1 | not NE | NE | dominant strategy | | | |
| Block 2 | not NE | not NE | dominant strategy | $\mu^{C/S}$ | $\mu^{C/S}$ | $\{\mu^{C/S}, \mu^*\}$ |
| Block 3 | not NE | NE | dominant strategy | | | |
| Block 4 | not NE | not NE | dominant strategy | | | |

In comparison, for blocks 2 and 4, truth-telling is not a Nash equilibrium strategy under either Shanghai or Boston. Under truthtelling, Boston, Shanghai and the DA each yield different outcomes. While the outcome under Shanghai is Pareto efficient, those under the DA is not. Table 2 summarizes our analysis on truthtelling and Nash equilibrium outcomes.

4.2 The 6-School Environment

While the 4-school environment is designed to compare the mechanisms in a simple context, we now test the mechanisms in a more complex environment where student preferences are generated by school proximity and quality.

In this **6-school environment**, each group consists of six students, $i \in \{1, 2, \dots, 6\}$, and six schools $s \in \{a, b, \dots, f\}$. Each school has one slot. Following Chen and Sönmez (2006), each student's ranking of the schools is generated by a utility function, which depends on school quality, school proximity and a random factor. There are two types of students: for notation purposes, odd labelled students are gifted in sciences while even labelled students are gifted in the arts. Schools a and b are higher quality schools, while c - f are lower quality schools. School a is stronger in the arts and b is stronger in sciences: a is a first tier school in the arts and second tier in sciences, while b is a second tier school in the arts and first tier in sciences; c - f are each third tier in both arts and sciences. The utility function of each student has three components:

$$u^i(s) = u_p^i(s) + u_q^i(s) + u_r^i(s), \quad (1)$$

where the first component, $u_p^i(s)$, represents the proximity utility for student i for school s . We designate this as 10 if student i lives within the walk zone of School s and 0 otherwise. The second component, $u_q^i(s)$, represents the quality utility for student i at school s . For odd labelled students, $u_q^i(a) = 20$, $u_q^i(b) = 40$, and $u_q^i(s) = 10$ for $s = c - f$. For even labelled students, $u_q^i(a) = 40$, $u_q^i(b) = 20$, and $u_q^i(s) = 10$ for $s = c - f$. Finally, the third component, $u_r^i(s)$, represents a random utility (uniform in the range 0-40) which includes diversity in tastes. Based on this utility function, we randomly generate 20 environments. We choose an environment which again satisfies several criteria: (1) no one lives within the district of her top or bottom choices; and (2) the first choice accommodation index is 1/3, a more competitive scenario than the 4-school environment.

We use Equation (1) to generate payoffs. We then normalize the payoffs such that the payoff from the first to last choice schools spans $\{16, 13, 11, 9, 7, 5\}$, the same payoff range as in the 4-school environment. The normalized payoff table is reported in Table 3.

Table 3: Payoff Table for the 6-School Environment

| | a | b | c | d | e | f |
|------------------|------------|-------------|------------|-------------|------------|------------|
| Payoff to Type 1 | [9] | 16 | 11 | 13 | 7 | 5 |
| Payoff to Type 2 | 16 | [11] | 5 | 13 | 9 | 7 |
| Payoff to Type 3 | 9 | 16 | [7] | 11 | 5 | 13 |
| Payoff to Type 4 | 16 | 7 | 9 | [13] | 5 | 11 |
| Payoff to Type 5 | 16 | 13 | 11 | 7 | [9] | 5 |
| Payoff to Type 6 | 16 | 13 | 11 | 5 | 7 | [9] |

For each session in the 6-school environment, we include 18 participants of six different types. Participants are randomly assigned types at the beginning of each session. The experiment consists of 30 periods, with random re-matching into three groups of six in each period. Again, we change the priority queue every five periods.

Compared with the 4-school environment, the 6-school environment has a much larger set of Nash equilibrium outcomes. Furthermore, there are more equilibrium strategy profiles under Shanghai than under Boston. We examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. The list of Nash equilibrium outcomes for each block is included in Appendix C.

Table 4: Features of Experimental Sessions

| Treatment | Mechanism | Environment | # Subjects \times # sessions | Total # of subjects |
|------------------|---------------------|-------------|--------------------------------|---------------------|
| BOS ₄ | Boston | 4-school | 12 \times 4 | 48 |
| SH ₄ | Shanghai | 4-school | 12 \times 4 | 48 |
| DA ₄ | Deferred Acceptance | 4-school | 12 \times 4 | 48 |
| BOS ₆ | Boston | 6-school | 18 \times 4 | 72 |
| SH ₆ | Shanghai | 6-school | 18 \times 4 | 72 |
| DA ₆ | Deferred Acceptance | 6-school | 18 \times 4 | 72 |

Lastly, we present the efficiency analysis for the 6-school environment. The allocations that maximizes the sum of payoffs are the following ones, each leading to the sum of ranks of 13 with an aggregate payoff of 78.

$$\mu_1^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & d & f & a & e & c \end{pmatrix} \text{ or } \mu_2^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & f & d & e & c \end{pmatrix}.$$

In comparison, the No Choice benchmark, where each student is assigned to her district school, generates the sum of ranks of 22 with an aggregate payoff of 58.

4.3 Experimental Procedures

In each experimental session, each participant is randomly assigned an ID number and is seated in front of a terminal in the laboratory. The experimenter then reads the instructions aloud. Subjects have the opportunity to ask questions, which are answered in public. Subjects are then given 10 minutes to read the instructions at their own pace and to finish the review questions. After everyone finishes the review questions, the experimenter distributes the answers and goes over the answers in public. Afterwards, participants go through 20 (respectively 30) periods of a school choice experiment in the 4-school (respectively 6-school) environment. At the end of the experiment, each participant fills out a demographics and strategy survey on the computer. Each participant is paid in private at the end of the experiment. The experiment is programmed in z-Tree (Fischbacher 2007).

Table 4 summarizes the features of the experimental sessions. For each mechanism in each environment, we conduct four independent sessions between May 2009 and April 2012 at the Behavioral Economics and Cognition Experimental Lab at the University of

Michigan.²³ The subjects are students from the University of Michigan. This gives us a total of 24 independent sessions and 360 participants (354 unique subjects).²⁴ Each 4-school session consists of 20 periods. These sessions last approximately 60 minutes. In comparison, each 6-school session consists of 30 periods. These sessions last approximately 90 minutes. The first 20-30 minutes in each session are used for instructions. The conversion rate is \$1 = 20 points for all treatments. Each subject also receives a participation fee of \$5, and up to \$3.5 for answering the Review Questions correctly. The average earning (including participation fee) is \$19.08 for the 4-school treatments, and \$25.42 for the 6-school treatments. Experimental instructions are included in Appendix D. The data are available from the authors upon request.

5 Experimental Results

In examining our experimental results, we first explore individual behavior and equilibrium selection, and then report our aggregate performance measures, including first choice accommodation, efficiency and stability of the three mechanisms. We also investigate the sensitivity of our results to environment changes.

In presenting the results, we introduce several shorthand notations. First, let $x > y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y at the 5% significance level or less. Second, let $x \geq y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y , but the comparison is not statistically significant at the 5% level.

5.1 Individual Behavior

We first examine the extent to which individuals reveal their preferences truthfully, and the pattern of any preference manipulation under each of the three mechanisms. Theorem 1 suggests that the Shanghai mechanism is less manipulable than the Boston mechanism.

²³All Boston and DA sessions were conducted between May 2009 and July 2010. However, we found a z-Tree coding error for the BOS₆ treatment during our data analysis. Thus, four additional sessions were conducted in July 2011 for this treatment, to replace the corresponding sessions. Sessions for the Shanghai mechanism were conducted in March and April 2012.

²⁴Despite our explicit announcement in the advertisement that subjects should not participate in the school choice experiment more than once and our screening before each session, six subjects participated twice.

Furthermore, under the DA, truth-telling is a weakly dominant strategy. This leads to our first hypothesis.

Hypothesis 1 (Truth-telling) (a) *There will be a higher proportion of truth-telling under Shanghai than under Boston.* (b) *Under the DA, participants will be more likely to reveal their preferences truthfully than under Boston.* (c) *Under the DA, participants will be more likely to reveal their preferences truthfully than under Shanghai.*

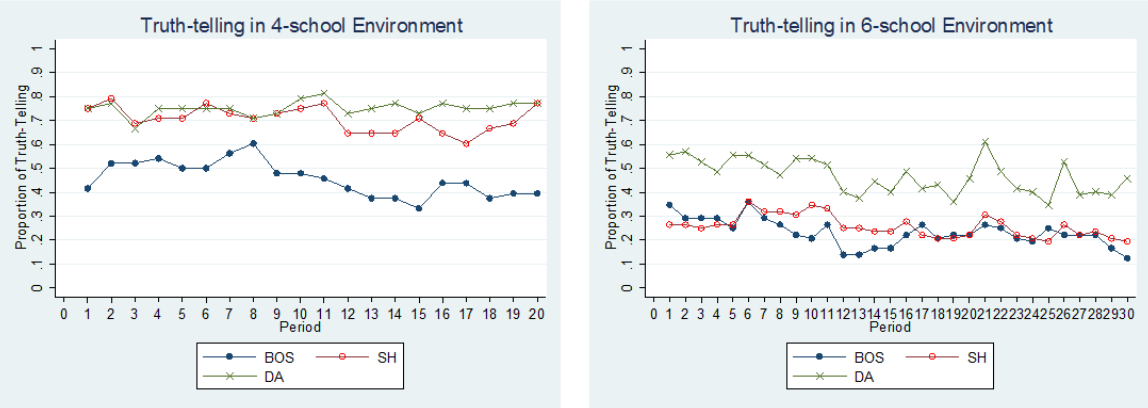


Figure 1: Proportion of Truth-Telling in Each Environment

Figure 1 presents the proportion of truth-telling in the 4- and 6-school environments under each mechanism. Note that, under the Boston and Shanghai mechanisms, truthful preference revelation requires that the entire reported ranking is identical to a participant’s true preference ranking.²⁵ However, under the DA, truthful preference revelation requires that the reported ranking be identical to the true preference ranking from the first choice through the participant’s district school. The remaining rankings, from the district school to the last choice, are irrelevant under the DA. While the DA has a robustly higher proportion of truth-telling than Boston, we find that Shanghai has more truth-telling behavior than Boston. Further, under each mechanism, the proportion of truth-telling is higher under the 4-school than under the 6-school environment, especially under the DA, which indicates that it is easier to figure out the dominant strategy in a simpler environment.

²⁵The only exception is when a participant’s district school is her top choice. In this case, truthful preference revelation entails stating the top choice. However, by design, this case never arises in our experiment, as no one’s district school is her first choice.

Result 1 (Truth-telling) : *In both environments, the proportion of truthful preference revelation under the DA is significantly higher than that under Boston over all periods, whereas it is significantly (weakly) higher than that under Shanghai in the 6-school (4-school) environment. The proportion of truthful preference revelation under Shanghai is significantly (weakly) higher than that under Boston in the 4-school (6-school) environment.*

Table 5: Proportions of Truthful Preference Revelation and Misrepresentations

| All Periods | Truthful Preference Revelation | | | District School Bias | | |
|------------------|--------------------------------|-----------|-------------|----------------------|-----------|-------------|
| | Proportion | H_a | p-value | Proportion | H_a | p-value |
| BOS ₄ | 0.456 | BOS < SH: | $p = 0.014$ | 0.478 | BOS > SH: | $p = 0.014$ |
| SH ₄ | 0.706 | SH < DA: | $p = 0.200$ | 0.147 | SH > DA: | $p = 0.100$ |
| DA ₄ | 0.751 | BOS < DA: | $p = 0.014$ | 0.107 | BOS > DA: | $p = 0.014$ |
| BOS ₆ | 0.232 | BOS < SH: | $p = 0.271$ | 0.549 | BOS > SH: | $p = 0.343$ |
| SH ₆ | 0.258 | SH < DA: | $p = 0.014$ | 0.526 | SH > DA: | $p = 0.014$ |
| DA ₆ | 0.468 | BOS < DA: | $p = 0.014$ | 0.144 | BOS > DA: | $p = 0.014$ |

SUPPORT: Table 5 presents the proportion of truthful preference revelation, as well as the proportion of district school bias, a prevalent form of misrepresentation, for each treatment. P-values are computed from one-sided permutation tests, treating each session as an observation. ■

By Result 1, we reject the null in favor of Hypothesis 1(a) that the Shanghai mechanism is less manipulable than the Boston mechanism at the 5% level in the 4-school environment. Furthermore, we reject the null in favor of Hypothesis 1(b) that the DA is less manipulable than the Boston mechanism. Lastly, we reject the null in favor of Hypothesis 1(c) that the DA is less manipulable than the Shanghai mechanism in the 6-school environment. The result is similar for inexperienced participants (first period). While the ranking of truth-telling between Boston and the DA is consistent with Chen and Sönmez (2006), manipulability of the Shanghai mechanism is reported for the first time. Even though truth-telling is not a dominant strategy under the Shanghai mechanism, the extent of manipulation is significantly less under the Shanghai mechanism than under the Boston mechanism in our simple 4-school environment. The same ranking holds in the more complex 6-school environment but it is only significant at the 10% level.

While we do not observe 100% truthtelling under the DA, it is less manipulable than the Boston mechanism in both environments and the Shanghai mechanism in the 6-school environment. Furthermore, we observe that the proportion of truthtelling in the DA is significantly higher in the 4-school environment than in the 6-school environment ($p = 0.014$, one-sided permutation test). We interpret this as due to the relative simplicity of the environment.

Note that subjects are *not* told that truthtelling is a dominant strategy under the DA in the experimental instructions (Appendix D). Following the convention in the experimental mechanism design literature, we describe each algorithm without prompting the subjects to behave in one way or another. Thus, results in this section summarize participant behavior without prompting from the experimenter. In practice, however, the market designer can educate the students when truthtelling is a dominant strategy. In fact, the Boston Public Schools, after switching to the DA, advise the students to “list your school choices in your true order of preference” and that “there is no need to “strategize.”²⁶ If parents follow the advice, we expect the DA to achieve close to 100% truthtelling in practice, further enlarging the gap between the DA and the other mechanisms reported in Result 1. Table 7 in Appendix E presents probit regressions investigating factors affecting truthtelling. We find a significant lottery position effect on truthtelling, namely, a better lottery position significantly increases the likelihood of truthtelling. Additionally, we also observe a small but significant effect of learning to manipulate.

A main critique of the Boston mechanism is centered around the fact that the mechanism puts a lot of pressure on manipulation of first choices. The Shanghai mechanism alleviates this pressure. We now examine the likelihood that participants reveal their first choices truthfully under each mechanism.

Hypothesis 2 (Truthful First Choice) *A higher proportion of reported first choices will be true first choices under the Shanghai than under the Boston mechanism.*

Result 2 (Truthful First Choice) : *The proportion of truthful first choices under the Shanghai mechanism is significantly higher than that under the Boston mechanism in both environments.*

²⁶Source: A Publication of the Boston Public Schools (2009), retrieved on January 7, 2013.

SUPPORT: In the 4-school (6-school) environment, the proportion of truthful first choices is 78% (55%) under the DA, 78% (48%) under the Shanghai, and 49% (37%) under the Boston. Using each session as an observation, one-sided permutation tests for pairwise comparisons of the proportion of truthful first choices yield $DA > BOS$ ($p = 0.014$), $DA \geq SH$ ($p = 0.529$), and $SH > BOS$ ($p = 0.014$) for the 4-school environment. For the 6-school environment, using the same tests, we obtain $DA > BOS$ ($p = 0.014$), $DA > SH$ ($p = 0.057$), and $SH > BOS$ ($p = 0.029$). ■

By Result 2, we reject the null in favor of Hypothesis 2 that the Shanghai mechanism generates a higher proportion of truthful first choices than the Boston mechanism. In particular, the Shanghai mechanism is virtually identical to the DA in the proportion of truthful first choices in the 4-school environment. Regardless of the environment, participants are more likely to submit true first choices under the Shanghai mechanism than under the Boston mechanism.

We next examine our results regarding District School Bias, a prevalent form of manipulation where a participant puts her district school into a higher position than that in the true preference order. This type of preference manipulation has been reported in previous experimental studies of the Boston mechanism (Chen and Sönmez 2006, Calsamiglia, Haeringer and Klijn 2010, Klijn, Pais and Vorsatz 2010).

Result 3 (District School Bias) : *The proportion of participants who exhibits District School Bias is significantly (weakly) higher under Boston than under Shanghai in the 4-school (6-school) environment, which is then followed by the DA.*

SUPPORT: See columns under “District School Bias” in Table 5. ■

The proceeding analysis of individual behavior has implications for Nash equilibrium outcomes. Generically, there are multiple Nash equilibria in the application-rejection family of mechanisms. Thus, from both the theoretical and practical implementation perspectives, it is important to investigate which equilibrium outcomes are more likely to arise. To our knowledge, equilibrium selection in school choice mechanisms has not been studied before.

Our 4-school environment is particularly well suited to study equilibrium selection. Recall that in our 4-school environment, the student-optimal Nash equilibrium outcome, $\mu^{C/S}$, is the unique Nash equilibrium outcome under the Boston and the Shanghai mechanisms, while there are two Nash equilibrium outcomes under the DA, $\mu^{C/S}$ and μ^* , where the latter

Pareto dominates the former. Thus, it will be interesting to examine which of the two equilibrium outcomes arises more frequently under the DA. While the Pareto criterion predicts that the Pareto optimal unstable Nash equilibrium should be selected, experimental results from secure implementation suggest that the dominant strategy equilibrium, when coinciding with the Nash equilibrium, is more likely to be chosen (Cason, Saijo, Sjöström and Yamato 2006). This predicts that the student-optimal Nash equilibrium outcome is more likely to arise.

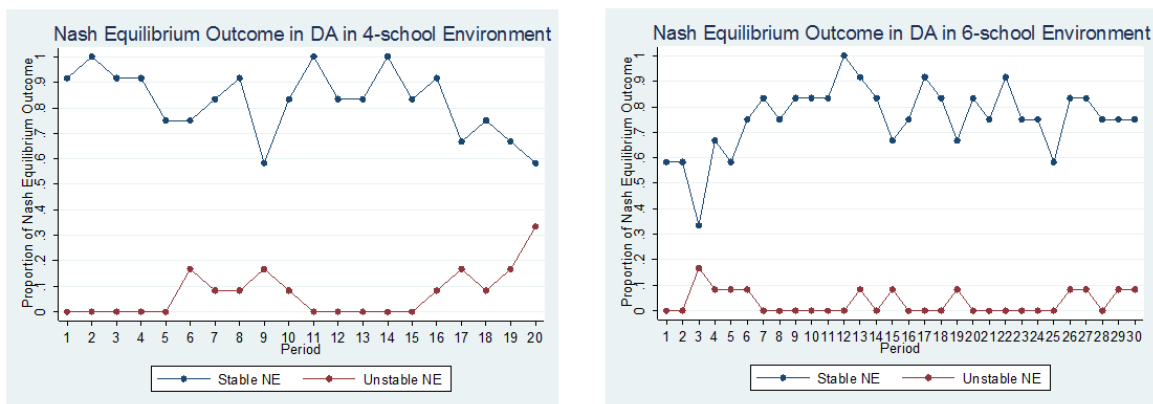


Figure 2: Proportion of Stable and Unstable Nash Equilibrium Outcomes under DA

Figure 2 reports the proportion of the stable and unstable equilibrium outcomes over time under the DA in the 4-school (left panel) and 6-school (right panel) environments, while Table 8 in Appendix E reports session-level statistics for each mechanism and pairwise comparisons between mechanisms and outcomes.

Result 4 (Nash Equilibrium Outcomes) : *Under the DA, the proportion of the inefficient but stable Nash equilibrium outcome (82.5%) is weakly higher than that of the efficient but unstable Nash equilibrium outcome (8.9%) in the 4-school environment.*

SUPPORT: The last column in Table 8 in Appendix E presents the p-values for permutation tests comparing the proportion of equilibrium outcomes under different mechanisms. The null of equal proportion against the H_a of $DA(\mu^*) < DA(\mu^{C/S})$ yields $p = 0.063$ (paired permutation test, one-sided). ■

We conjecture that the stable Nash equilibrium outcome ($\mu^{C/S}$) is observed more often despite being Pareto dominated by μ^* , because the former requires truthful preference revelation, the weakly dominant strategy adopted by about 75% of the participants under the DA, while the latter requires coordinated manipulation of top choices by players 1 and 3. However, we also note an increase of the unstable but efficient Nash equilibrium outcome, μ^* , in the last block in Figure 2 (left panel), indicating that players 1 and 3 learn to coordinate their manipulation towards the end of the game. This increase has direct implications for the efficiency comparisons in Result 6.

In comparison to the 4-school environment, the 6-school environment generates many Nash equilibrium outcomes. Because of this multitude of Nash equilibria, without strategy-proofness, on average, 3% and 20% of the outcomes are Nash equilibrium outcomes under the Boston and Shanghai mechanisms, respectively. In contrast, 79% of the outcomes under the DA are Nash equilibrium outcomes. The proportion of this Nash equilibrium outcome follows $DA > BOS$ ($p = 0.014$), $DA > SH$ ($p = 0.014$), and $SH > BOS$ ($p = 0.014$). If we break down the Nash equilibrium outcomes under the DA into stable and unstable equilibria, we again observe that the stable outcomes arise weakly more frequently than the unstable ones ($p = 0.063$, paired permutation test, one-sided).

In sum, Result 4 and our analysis of the 6-school data indicate that the stable Nash equilibrium outcome is more likely to arise than the unstable Nash equilibrium outcomes under the DA. To our knowledge, this is the first empirical result on equilibrium selection under the DA.

5.2 Aggregate Performance

Having presented the individual behavior and equilibrium outcomes, we now evaluate the aggregate performance of the mechanisms using three measures: the proportion of participants receiving their reported and true first choices, the efficiency achieved, and the stability under each mechanism.

In the education literature, the performance of a school choice mechanism is often evaluated through the proportion of students who receive their reported top choices. Thus, we compare the proportion of participants receiving their reported top choices, as well as the proportion who actually receive their true top choices. Theorem 3 suggests the following

hypothesis.

Hypothesis 3 (First Choice Accommodation) *The proportion of participants receiving their reported top choices will be the highest under Boston, followed by Shanghai, and then the DA.*

Table 6: First Choice Accommodation: Reported versus True First Choices

| 4-school | Proportion Receiving <i>Reported</i> First Choice | | | | | Proportion Receiving <i>True</i> First Choice | | | | |
|-----------|---|-------|-------|----------|---------|---|-------|-------|---------------|---------|
| | BOS | SH | DA | H_a | p-value | BOS | SH | DA | H_a | p-value |
| Session 1 | 0.596 | 0.217 | 0.138 | BOS > SH | 0.014 | 0.088 | 0.033 | 0.017 | BOS \neq SH | 0.114 |
| Session 2 | 0.617 | 0.221 | 0.271 | BOS > DA | 0.014 | 0.113 | 0.063 | 0.121 | BOS \neq DA | 0.114 |
| Session 3 | 0.583 | 0.158 | 0.192 | SH > DA | 0.257 | 0.121 | 0.067 | 0.071 | SH \neq DA | 0.943 |
| Session 4 | 0.608 | 0.304 | 0.183 | | | 0.138 | 0.125 | 0.075 | | |
| 6-school | BOS | SH | DA | H_a | p-value | BOS | SH | DA | H_a | p-value |
| Session 1 | 0.717 | 0.400 | 0.196 | BOS > SH | 0.014 | 0.217 | 0.157 | 0.109 | BOS \neq SH | 0.057 |
| Session 2 | 0.665 | 0.344 | 0.270 | BOS > DA | 0.014 | 0.230 | 0.139 | 0.111 | BOS \neq DA | 0.029 |
| Session 3 | 0.667 | 0.441 | 0.231 | SH > DA | 0.014 | 0.178 | 0.157 | 0.085 | SH \neq DA | 0.029 |
| Session 4 | 0.706 | 0.398 | 0.241 | | | 0.202 | 0.181 | 0.120 | | |

Table 6 reports the proportion of participants receiving their reported (left panel) and true first choices (right panel) in each session in each treatment. Note that the alternative hypotheses comparing mechanisms accommodating true first choices are two-sided, as neither the Boston nor the Shanghai mechanism is strategy-proof. P-values of permutation tests are reported in the last column. The results are summarized below.

Result 5 (First Choice Accommodation) : *In both environments, the proportion of subjects receiving their reported first choice is significantly higher under Boston than under either the Shanghai or the DA mechanisms. Furthermore, the proportion receiving their reported first choice is significantly higher under Shanghai than under the DA in the 6-school environment. However, for the proportion receiving their true first choices, the Boston and Shanghai mechanisms are not significantly different, but each significantly outperforms the DA in the 6-school environment.*

SUPPORT: Treating each session as an observation, p-values from the corresponding permutation tests are reported in Table 6. ■

By Result 5, we reject the null in favor of Hypothesis 3 for reported first choices. However, looking at the accommodation of true first choices, we find that reported top choices are not a good measure of performance when the incentive properties under each mechanism are different. In the 4-school environment, the three mechanisms are not significantly different from each other, while in the 6-school environment, Boston and Shanghai are not significantly different from each other, but each outperforms the DA.

We next compare the efficiency of the mechanisms in each environment. As our theoretical benchmarks are based ordinal preferences, we present a corresponding efficiency measure using ordinal ranking of assignments.²⁷ We define a normalized efficiency measure as

$$\text{Normalized Efficiency} = \frac{\text{maximum group rank} - \text{actual group rank}}{\text{maximum group rank} - \text{minimum group rank}}, \quad (2)$$

where the *minimum group rank* is the sum of ranks for all group members for the Pareto efficient allocation(s), which equals 6 (resp. 13) for for the 4-school (resp. 6-school) environment. Likewise, the *maximum group rank* is the sum of ranks for the worst allocation, which equals 14 (resp. 33) for the 4-school (resp. 6-school) environment. Because of this normalization, this measure always lies between zero and one, inclusive.

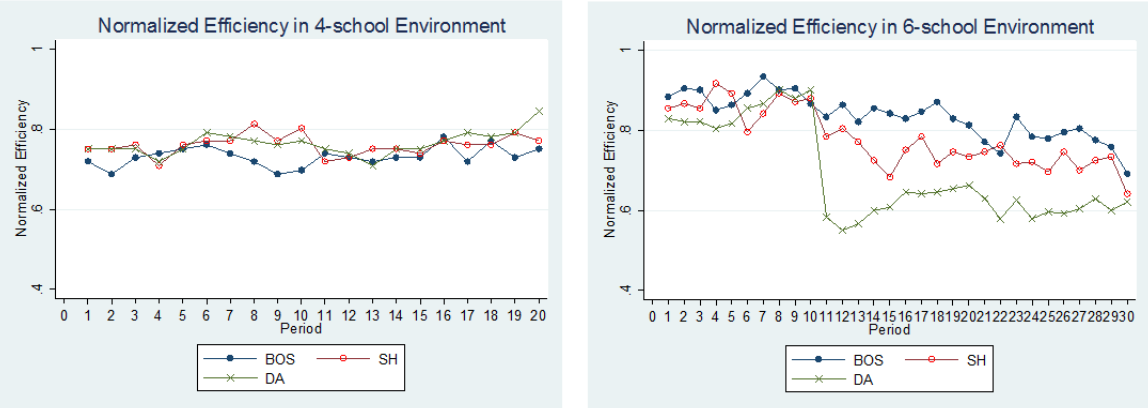


Figure 3: Normalized Efficiency in the 4- and 6-School Environments

Figure 3 presents the normalized efficiency under each mechanism in the 4-school and

²⁷For robustness check, we have also completed a parallel set of efficiency analysis based on the sum of payoffs, which yields similar results.

6-school environments. Session-level normalized efficiency for the first and last blocks, as well as the average efficiency over all periods, is reported in Table 9 in Appendix E.

Result 6 (Efficiency) : *While the DA is significantly more efficient than Boston and weakly more efficient than Shanghai in the 4-school environment, Boston is more efficient than Shanghai, which in turn is more efficient than the DA in the 6-school environment.*

SUPPORT: Using one-sided permutation tests with each session as an observation, we find that

- (1) First block: $BOS_6 > DA_6$ ($p = 0.029$), $SH_6 > DA_6$ ($p = 0.029$), while none of the pairwise efficiency comparisons in the 4-school environment is significant.
- (2) Last block: $DA_4 > BOS_4$ ($p = 0.029$); $BOS_6 > DA_6$ ($p = 0.014$); $SH_6 > DA_6$ ($p = 0.043$); $BOS_6 \geq SH_6$ ($p = 0.057$).
- (3) All periods: $DA_4 > BOS_4$ ($p = 0.014$); $DA_4 \geq SH_4$ ($p = 0.071$); $BOS_6 > SH_6$ ($p = 0.043$); $BOS_6 > DA_6$ ($p = 0.014$); $SH_6 > DA_6$ ($p = 0.014$). ■

Result 6 is consistent with Theorem 3 in that there is no systematic efficiency ranking within the class of the Chinese parallel mechanisms. It also contributes to our understanding of the empirical performance of the school choice mechanisms. First, it indicates efficiency comparison is environment sensitive. While no single mechanism is more efficient in both environments, the Shanghai mechanism is never the worst. Second, while a first-period pairwise efficiency comparison is not significant in either environment, separation of performance occurs with learning, so that the last block ranking is significant. Our first period results are consistent with Calsamiglia, Haeringer and Klijn (2011). Our results point to the importance of allowing subjects to learn in school choice experiments. Lastly, our finding that the DA is more efficient than Boston in the last block is driven by the rise of the unstable but efficient Nash equilibrium outcome observed in Figure 2 (left panel).

Finally, we evaluate the stability achieved under each mechanism. Corollary 3 suggests the following ranking:

Hypothesis 4 (Stability) *The DA is more stable than Shanghai, which in turn is more stable than Boston.*

Figure 4 presents the proportion of stable allocations under each mechanism in the 4-school (left panel) and 6-school (right panel) environments. An allocation is marked as

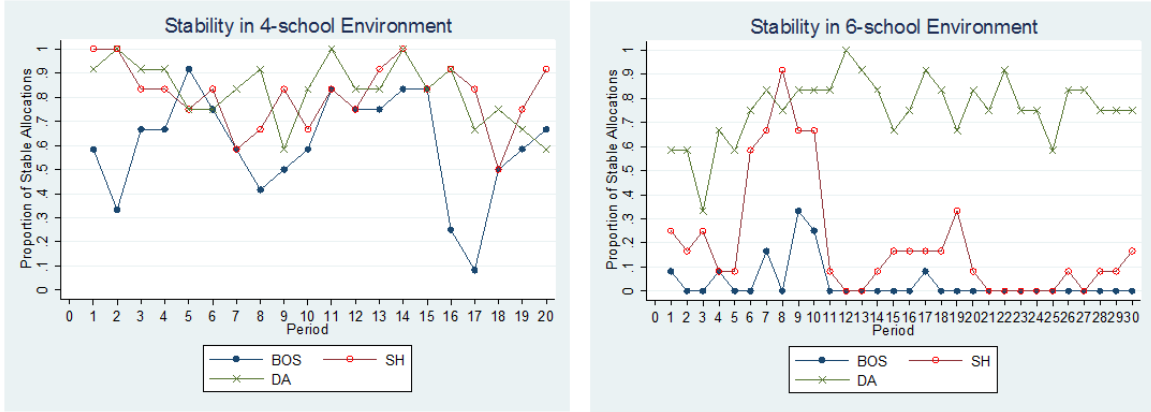


Figure 4: Proportion of Stable Allocations in the 4- and 6-School Environments

unstable if any student in a group of four (resp. six) is justifiably envious of another student in the group.

Result 7 (Stability) : *The DA and Shanghai mechanisms are each significantly more stable than the Boston mechanism in both environments. the DA mechanism is significantly more stable than the Shanghai mechanism in the 6-school environment.*

SUPPORT: Table 10 in Appendix E reports the proportion of stable allocations among all allocations in the first and last block, and averaged over all periods in each session. Using one-sided permutation tests with each session as an observation, we find that (1) $DA_4 \geq SH_4$ ($p = 0.457$), $DA_4 > BOS_4$ ($p = 0.014$), $SH_4 > BOS_4$ ($p = 0.029$); (2) $DA_6 > BOS_6$ ($p = 0.014$), $DA_6 > SH_6$ ($p = 0.014$), and $SH_6 > BOS_6$ ($p = 0.014$). ■

By Result 7, we reject the null in favor of Hypothesis 4. Thus, consistent with Corollary 3, in both environments, the DA and Shanghai mechanisms each achieve a significantly higher proportion of stable allocations than the Boston mechanism. In the 6-school environment, the DA also achieves a higher proportion of stable outcomes than the Shanghai mechanisms. However, in the 4-school environment, the proportion of stable outcomes is indistinguishable between the DA and Shanghai mechanisms. While our empirical stability ranking between the DA and Boston is consistent with Calsamiglia et al. (2010), the stability evaluation of the Shanghai mechanism is new.

In sum, our experimental study has several new findings. First, we evaluate the performance of the Shanghai mechanism, and find that its manipulability, reported first-choice

accommodation, efficiency and stability measures are robustly sandwiched in between the Boston and the DA mechanisms. Second, compared to the one-shot implementation of previous experiments on school choice except Featherstone and Niederle (2008),²⁸ our experimental design with repeated random re-matching enables us to compare the performance of the mechanisms with experienced participants. In doing so, we find that learning separates the performance of the mechanisms in terms of efficiency. Lastly, we report equilibrium selection under the DA for the first time, which reveals that stable Nash equilibrium outcomes are more likely to arise than the unstable ones even when the latter Pareto dominates the former.

6 Conclusions

School choice and college admissions have profound implications for the education and labor market outcomes of the students involved in these processes worldwide. Whereas much of the debate on school choice in the literature exclusively focused on the Boston vs. DA comparison, in this paper we synthesize these well-known mechanisms with those used for college admissions in China, and characterize them as members of a family of application-rejection mechanisms, with the Boston, the Chinese parallel, and the DA being special cases. A key insight is that the Chinese parallel mechanism used for both high school admissions in Shanghai and for college admissions in many provinces in China bridges the well studied Boston and the DA mechanisms.

Our theoretical analysis indicates a systematic change in the incentive, stability, and welfare properties of this family of mechanisms as one goes from one extreme member to the other. We also see that the Nash equilibrium strategies corresponding to the induced preference revelation games associated with members of the application-rejection family are nested. Although the DA has been shown to dominate the equilibria of the Boston under complete information, no such conclusion holds relative to the Shanghai mechanism.

In practice, parallel mechanisms such as the Shanghai may alleviate the pressure parents face under the Boston mechanism by giving them the guarantee to maintain priority at their

²⁸Featherstone and Niederle (2008) investigate the performance of the Boston and DA mechanisms under incomplete information, whereas we study the family of mechanisms under complete information. While their experiment is implemented under a random re-matching protocol, they do not explicitly analyze the effects of learning.

safety schools while keeping more desirable options within their target range. Unlike with the DA, such insurance does not entail any *ex ante* welfare cost since the Shanghai also allows students to communicate their preference intensities more efficiently relative to the DA.

To test our theoretical predictions and to search for behavioral regularities where theory is silent, we conduct laboratory experiments in two environments differentiated by their complexity. We find that the proportion of truth-telling follows the order of $DA > \text{Shanghai} > \text{Boston}$, while the proportion of District School Bias follows the reverse order. While the manipulability ranking of the DA and the Boston is consistent with both theory and prior experimental findings, the manipulability of the Shanghai mechanism is reported for the first time. Whereas theory is silent about equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than unstable ones. On the stability front, consistent with theory, the DA achieves a significantly higher proportion of stable outcomes than either the Shanghai or the Boston in both environments, while the Shanghai is more stable than Boston. However, the efficiency comparison is sensitive to the environment. In our 4-school environment, the DA is weakly more efficient than Boston, while the Shanghai mechanism is not significantly different from either. In comparison, in our 6-school environment, the Boston achieves significantly higher efficiency than Shanghai, which, in turn, achieves higher efficiency than the DA.

Our study represents the first systematic theoretical and experimental investigation of the Chinese parallel mechanisms. The analysis yields valuable insights which enable us to treat this class of mechanisms as a family, and systematically study their properties and performance. More importantly, our results have policy implications for school choice and college admissions. As the Shanghai mechanism is less manipulable than the Boston mechanism, and its achieved efficiency is robustly sandwiched between the two extremes whose efficiency varies with the environment, it might be a less radical replacement for the Boston mechanism compared to the DA.

Like school choice in the United States, college admissions reform is among the most intensively discussed public policies in China. While variants of the parallel mechanism have been implemented in various provinces to replace the sequential mechanism since 2003, the choice of the number of parallel colleges (e) seems arbitrary. Our study provides the first

theoretical analysis and experimental data on the effects of the number of parallel colleges on the incentives and aggregate performance of these mechanisms.

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Appendix A: Evolution of the Chinese College Admissions Mechanisms (For Online Publication)

In this Appendix, we present the evolution of the Chinese College Admissions mechanisms from 1949 to 2012. In summarizing its main variations, we rely primarily on several books written by educators, policy-makers and historians. In particular, Yang (2006) provides the historical and political contexts of Chinese college admissions from 1949 to 1999. Liu (2009) reports the policy debates surrounding college admissions reforms up to 2009, including survey data around some major policy reforms. In comparison, Qiu and Zhao (2011) offer practical advice for high school seniors and their parents on recent admission statistics of each university, the admissions mechanisms, and application strategies. While Chinese college admissions have been traditionally studied by educators, Chinese economists recently started analyze their game-theoretic properties. We reference most of the latter in the main text of this paper. As matching mechanisms in historical documents are not described in game theoretic language, we provide the translation of the relevant paragraphs and our own interpretation in game-theoretic terms.

For more up to date information on college admissions rules and policies in various provinces, we refer the reader to the official Ministry of Education website on college admissions, <http://gaokao.chsi.com.cn/>²⁹

6.1 From Decentralized to Centralized Examinations and Admissions (1952 - 1957)

After the establishment of the People's Republic of China in 1949, Chinese universities continued to admit students via decentralized mechanisms, i.e., each university administered its own entrance exams and admissions processes. In 1950, there were 227 universities and colleges, with 134,000 students (Yang 2006, p. 5).³⁰ Historians identified two major problems with decentralized admissions during this time period. From the perspectives of the universities, as each student could be admitted into multiple universities, the enrollment to admissions ratio was low, ranging from 20% for some ordinary universities to 75% among the best universities in 1949 (Yang 2006, p. 6). Therefore, many ordinary universities could

²⁹This website has remained stable at least since 2006. We last accessed it on January 7, 2013.

³⁰In reporting statistics, we exclude universities in Taiwan, Hong Kong and Macau. Also note that Chinese sources prior to 1977 typically report statistics in units of ten thousand (*wàn*).

not fill their first-year classes. From the students' perspectives, however, after being rejected by the best universities, some qualified students missed the application and examination deadlines of ordinary universities and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances, with centralized admissions within each alliance. This experiment achieved an improved average enrollment to admissions ratio of 50% for an ordinary university (Yang 2006, p. 7).

Based on the success of the alliances, the Ministry of Education decided to transition to centralized matching in 1952 by implementing the first National College Entrance Examination, also known as *gaokao*, in August 1952.³¹ The exam consisted of eight subjects (math, physics, chemistry, biology, foreign language, history and geography, politics, and Chinese), and lasted for three consecutive days, a format that more or less persisted to 2012, with various adjustments on the content of the exam. The enrollment to admissions ratio for an ordinary university in 1952 was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

Between 1952 and 1957, the Ministry of Education made several adjustments to the centralized admissions process. First, minority-serving institutions, fine arts and music institutions were allowed to include institution-specific admissions processes in addition to *gaokao*, such as interviews, auditions and portfolio presentations. Second, the single-track *gaokao* evolved into two tracks in 1954, and three tracks in 1955. The three tracks included the science/engineering track, the medicine/biology/agriculture track, and the humanities/social sciences track. The first two tracks were recombined into a single track in 1964, forming the present-day two-track exam system. Lastly, *key universities*, such as Beijing, Tsinghua, and Jiaotong, were allowed to recruit nationwide, while *ordinary universities* were restricted to recruit within their respective province, which created the *tier system* among universities.

From a game-theoretic perspective, the centralized admissions mechanism used during this time period, “Exam-Score Based Admissions” (*fēn jī lǜ qǔ*), resembled a serial dictatorship mechanism. “[Admissions] should proceed in decreasing exam scores, starting with the highest score, and proceeding to the next score after [the admission of the student with] the highest score is finished. For each student, proceed based on the student’s preference

³¹Using a national examination to select talent for various government positions had been a long tradition in China, dating back to 605 A.D. (Liu 2009, p. 2).

ranking. That is, send the student's application to his first choice. If that university decides to admit the student, it keeps his application file and marks 'Admitted' in the Admission Results column. If the university decides not to admit the student or if its quota is full, it should mark 'Not Admitted' on the student's application, and pass his file to his second-choice university (with the same process as described above). And so on." (Yang 2006, p. 76-77)

The transition from decentralized to centralized matching was designed to alleviate coordination failure and excess demand. In 1956, for example, universities had a target of admitting 165,500 students, whereas 156,000 students graduated from high school that year (Yang 2006, p. 40). By encouraging cadres from workplaces to apply for colleges,³² the situation changed in 1957, with a target of admitting 120,000 students and 199,000 applicants (Yang 2006, p. 45). After a nation-wide debate of whether to go back to the decentralized admissions processes, used in the Soviet Union at the time, the Ministry of Education decided to continue the centralized admissions processes, mainly based on its advantages of better coordination and lower transaction costs, i.e., students did not have to participate in multiple exams administered by different universities. It appears that, after a national exam, separate admissions processes within each province was established after the 1957 debate.

6.2 The Leftists' Attacks on College Admissions (1958 - 1965)

Since 1958, gaokao had been scrutinized and attacked by the leftists in the Communist Party, on its intellectual focus and its lack of communist ideology. In response, the Ministry of Education stepped up the screening of student political backgrounds in the admissions process, and implemented the Guaranteed Admissions of cadres from proletariat families who went through the Crash Training Schools for Workers and Farmers. Prior to 1958, the cadres were required to take gaokao and go through the same admissions process after bonus points were added to their scores. In contrast, they were exempt from gaokao since 1958 (Yang 2006, p. 91). The admissions rate was a staggering 97% in 1958 (Yang 2006, p. 139).

To our knowledge, the first documented tiered admissions appeared in 1959. "Admissions of new students should proceed in tiers. National key universities admit students first." The second tier included provincial and ministry-level key universities, whereas the third tier

³²Affirmative action, in the form of adding 10-15 points per subject (out of a 100-point scale), was implemented in 1954 to increase the number of cadres in universities (Yang 2006, p. 55).

included all other universities and colleges (Yang 2006, p. 104).

After the Great Leap Forward (1958 - 1961) ended in a disastrous famine, in 1962, college admissions rate reached its lowest point prior to the Cultural Revolution, 24%, with 107,000 students admitted among 440,000 applicants.

In 1963, it appeared that the college admissions mechanism transitioned from a serial dictatorship into a hybrid of serial dictatorship and priority matching mechanism, “Exam-Score Interval Based Admissions” (*fēn duàn lù qǔ*). Average exam scores were chunked into (typically) five-point intervals (*duàn*), e.g., [80, 100], [75, 79], [70, 74], [65, 69], etc. Admissions proceeded sequentially from the highest interval downward, clearing one interval before starting the next (*duàn duàn qīng*). Within an interval, admissions proceeded in the order of student preference ranking of universities and exam scores (Yang 2006, p. 135-136). Under this mechanism, each student could apply for five national key universities. Within each university, he could apply for three different departments. Admission decisions were made by each university. This mechanism was designed to reduce the disparity of student qualities between different departments within a university (Yang 2006, p. 150).

Meanwhile, because of the increased competitiveness, some students considered that “gaokao is a battle that determines your fate: one point [difference] in gaokao can determine whether you go to heaven [i.e., universities] or hell [i.e., becoming a farmer]” (Yang 2006, p. 171), which underscores the importance of gaokao in labor market outcomes. Until recently, labor market mobility had been constrained by the Household Registration (*hù kǒu*) system. For millions of youths from rural areas, gaokao offered the only way of breaking away from a life time on the farms.

6.3 Demise of Gaokao During the Cultural Revolution (1966-1976)

The year 1966 marked the start of the ten-year Cultural Revolution, and the abolition of gaokao. In its place, farmers, workers and soldiers who had the equivalence of a high school education could be recommended to go to universities. The political turmoil dictated that none of the universities recruited new students for the subsequent six years. From 1972 to 1976, university education resumed based on a recommendation system. Students had to have completed at least two years of real-life work experience, i.e., having worked on farms, in factories or served in the armed forces, to be eligible. The recommendation system

opened the door for rampant corruption in college admissions during this time period.

6.4 College Admissions Reform (1977 - 2012)

With the end of the Cultural Revolution in October 1976, gaokao resumed in 1977. As a result, 5.7 million applicants participated in gaokao, including many from the ten-year backlog of high school graduates together with the class of 1977, with 4.8% of all applicants admitted into universities. In 1977, each province wrote its own exams and administered its own admissions process. Starting 1978, gaokao again became a national exam, written by the Ministry of Education. A record 6.1 million students participated in the 1978 gaokao, with admissions rate again at 4.8%. To further curb corruption, every applicant's score was publicly posted.³³ Compared with gaokao before the Cultural Revolution, where the average admissions rate was 55.92%, the average admissions rate between 1977 and 1982 was 6.05% (Yang 2006, p. 278), indicating a much more competitive process.

While the hybrid serial dictatorship and priority matching mechanism, "Exam-Score Interval Based Admissions," continued to be used till 1984, to grant more autonomy to individual universities, starting from 1985, it was gradually replaced by a priority matching mechanism, which resembled the Boston mechanism with tiers (Yang 2006, p 314-315; Liu 2009, p. 41). Using this mechanism, based on the distribution of gaokao exam scores, the number of applicants who list it as their first choice, and its quota, each university determines a minimum threshold. It then receives applications that list it as the applicants' first choice. After admitting first-choice applicants in the order of high to low exam scores up to its quota, the first round allocations are finalized and the first round is closed. After the first round, universities which have not fulfilled their quotas each review applicants who list it as their second choice; etc. This mechanism is called the *sequential mechanism* (*shùn xù zhì yuàn*), which prioritizes students' preference orderings over their score rankings (*zhì yuàn yōu xiān*).

The sequential mechanism places huge strategic importance on an applicant's first choice. Among those admitted into a key university in 2010, more than 95% of them list it as his or her first choice, whereas 80% of those admitted into an ordinary university list it as his or her first choice (Qiu and Zhao 2011, p. 243). Therefore, Qiu and Zhao (2011) warn the

³³In comparison, individual gaokao scores were kept secret before the Cultural Revolution (Yang 2006, p. 269-270).

applicants that if their first- and second-choice universities are too close in quality, they might not get into any university in the first tier (p. 243). An obvious problem is that some students with very high scores do not get into any university in the first tier simply because they miss their first choice, leading to the popular saying that “a good score in the college entrance exam is worth less than a good strategy in the preference ranking of universities” (Nie 2007b).

To remedy the strategic manipulation inherent in the sequential mechanism, the *parallel mechanism* (*píng xíng zhì yuàn*) used in Shanghai high school admissions was first introduced into college admissions in Hunan Province in 2003. Jiangsu and Zhejiang adopted the mechanism in 2005 and 2007, respectively (Liu 2009, p. 382). The main innovation of the parallel mechanism is that students can put several “parallel” universities for each choice. For example, a student’s first choice can contain four universities, A, B, C and D, in decreasing desirability. Among matching theorists, there are two interpretations of the parallel mechanism, which are theoretically equivalent in the college admissions context. One interpretation is that it is serial dictatorship with tiers (Wei 2009). Applicants are ranked by exam scores. Starting from the applicant with the highest score to the one with the lowest score, each applicant applies for the parallel universities in the order of her preference ranking, from A to D. She gets into the first university with unfulfilled quota. After every applicant has applied to his first choice universities, the first round is closed. Those who are not admitted in the first round start the same process in the second round, and so on. The second interpretation is that it is a modified deferred acceptance mechanism as we formalize in our paper. Our interpretation has a broader set of applications as it can also be applied to the school choice context where priorities are not unique.

In addition to the matching mechanisms, many other important components of the college admissions process underwent changes in the 1990s and the early 21 century. While these components are not the focus of our paper, we include five of them below to illustrate the scope of the reform. First, the content of the exam, i.e., subjects that should be covered and the number of tracks, changed several times. For example, in 1999, “3 + X” system was implemented, where 3 refers to the three exams required for every applicant, math, Chinese, and foreign language, and X refers to any number of exams taken from physics, chemistry, biology, geography, history, politics. Second, a controversial institutionalized feature started

in the 1990s is the guaranteed admissions for up to 5% of the high school graduates, each recommended by his high school. Third, standardized test techniques, such as an increase in multiple choice problems and machine grading, were gradually implemented in the late 80s and 90s. Fourth, computerized admissions process was first implemented in Guangxi and Tianjin in 1996. By 2001, nation-wide computerized admissions through the Internet was completed (Liu 2009, p. 41). Lastly, as of 1985, Shanghai has been implementing its own exams. By 2006, 16 provinces each implemented its own exams.

Compared to the historical accounts and qualitative analysis of Chinese college admissions, game-theoretic analysis of Chinese college admissions mechanisms has been relatively new. The latter focuses on two issues, the timing of preference ranking submissions and the matching mechanisms themselves. We discuss both aspects in the main text of our paper.

6.5 Shanghai Mechanism: Online Q&A

We translate the following question and answer from an online Q&A forum about the parallel choices in Shanghai high school admissions, posted in May 2003.

Question: If a student lists a school as his first choice or second choice, what difference does it make in the admission process?³⁴

Answer: Middle school admission principles are: based on the student exam scores and school preference ranking, place the applications accordingly, while also considering their moral, intellectual and physical aspects, choose the best from high to low scores. For each individual student, the Middle School Admissions Office will submit his application in the order of his preference ranking. Only when he cannot get into his first choice, will his second choice be considered. In the admissions process of the entire district, each school has only one threshold. If a student's score is above the school threshold, whether he lists it as his first or second choice, he should be admitted.

For example, if student A's first choice is Luwan Middle School, and student B's second choice is Luwan. If A and B's scores are both above the Luwan

³⁴Translated from <http://edu.sina.com.cn/1/2003-05-15/42912.html>, accessed on January 7, 2013.

minimum threshold, then both should be admitted into Luwan. However, if student B is already admitted by his first choice, it is impossible for him to get into Luwan. On the other hand, if the two students have different scores, e.g., A's score is low and below the Luwan threshold, while B (whose second choice is Luwan) has a high score, which is above the Luwan threshold, then A (whose first choice is Luwan) cannot be admitted into Luwan because his score is below the threshold; whereas B (whose second choice is Luwan), if not admitted by his first choice, should be admitted by Luwan, even though he listed Luwan as his second choice.

The complication which we abstract away is the minimum threshold used by each school, which is endogenously determined based on the distribution of student exam scores.

6.6 College admissions in Hong Kong: JUPAS

College admissions in Hong Kong use a centralized system called the Joint University Programmes Admissions System (JUPAS). Under JUPAS, each student submits preferences for up to 25 programmes. These 25 choices are further divided into 5 bands, A, B, C, D, and E. For each student, the first three choices are band A choices, next three are band B, etc. Each programme ranks its applicants with an objective formula (based on academic performance and other considerations) to form a base priority ranking. Each programme is informed of the band it is placed by a student, but not the precise ranking. Most programmes use this information to adjust the base priority ranking. Finally given student preferences and adjusted priority rankings, the outcome is obtained via the student-proposing deferred acceptance algorithm (Liu and Chiu 2011).

The flexibility for colleges to modify priorities under JUPAS has led some colleges to strategically choose their priority construction formula in response to the formulas chosen by more popular colleges. “The rating criterion is independently determined by each programme: although some would adopt Boston-like criterion which assigns band A student highest priority, some may also rate students only by their eligibility [based on their academic performances, interview outcomes and extracurricular activities]. Some unpopular programmes tend to employ the latter strategy if they find most excellent students listed it as band B or C choices rather than band A.” (p. 4 and 5, Liu and Chiu 2011)

While the parallel mechanisms in CCA and JUPAS have some similarities, they also have important differences. Specifically, to determine the priority order under JUPAS, each college uses a combination of the student academic performances together with the band the student places the college in his preferences. It is important to note that each college has its own formula for doing this, which is captured by the α_c parameter in Liu and Chiu (2011). This means that, depending on the formula of the college, a student who places a school in band B may still have higher priority than another student who places it in band A if the former has a sufficiently higher exam score. This situation can never happen under the parallel mechanism in CCA. Under CCA, the priority construction is lexicographic, first based on the band, and second based on the exam score (for those in the same band). Moreover, this is the same for each college, i.e. there is no college-specific formula.

Appendix B: Proofs and Examples (For Online Publication)

Proof of Proposition 1: (Part i). It is easy to see that the Boston mechanism is Pareto efficient. Now consider the following problem with four students and four schools each with one seat. Priority orders and student preferences are as follows.

| | | | | | | | |
|---------------|---------------|---------------|---------------|-----------|-----------|-----------|-----------|
| \succ_{s_1} | \succ_{s_2} | \succ_{s_3} | \succ_{s_4} | P_{i_1} | P_{i_2} | P_{i_3} | P_{i_4} |
| i_4 | i_2 | \vdots | \vdots | s_1 | s_1 | s_2 | s_2 |
| i_2 | i_3 | | | s_4 | s_2 | s_3 | s_1 |
| i_1 | i_4 | | | \vdots | \vdots | \vdots | \vdots |
| i_3 | i_1 | | | | | | |

The outcome of the application-rejection mechanism (e) for all $e \geq 2$ is the following Pareto inefficient matching

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}.$$

(Parts ii & iii). Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \dots, i_{e+2}\}$ and $S = \{s_1, s_2, \dots, s_{e+2}\}$, where each school has a quota of one. Each $i_k \in I$ with $k \in \{1, 2, \dots, e\}$ ranks school s_k first and each $i_k \in I$ with $k \in \{1, 2, \dots, e+1\}$ has the

highest priority for school s_k . The preferences of student i_{e+1} are as follows: $s_1 P_{e+1} s_2 P_{e+1} \dots s_{e+1} P_{e+1} s_{e+2}$. And student i_{e+2} ranks school s_{e+1} first. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+1} . It is easy to see that he applies to school s_{e+1} in step $e + 1$ of the algorithm when a lower priority student is already permanently assigned to it in round 0. Hence he is rejected from school s_{e+1} and his final assignment is necessarily worse than s_{e+1} . Then the outcome of the application-rejection (e) mechanism for this problem is clearly unstable. Moreover, student i_{e+1} can secure a seat at school s_{e+1} when he submits an alternative preference list in which he ranks school s_{e+1} first. ■

Example 1a. (The Boston mechanism is manipulable whenever the Shanghai mechanism is) Consider the following example with five students and four schools. Schools s_1 , s_2 , and s_4 each have a quota of one, while school s_3 has a quota of two.

| \succ_{s_1} | \succ_{s_2} | \succ_{s_3} | \succ_{s_4} | P_{i_1} | P'_{i_1} | P_{i_2} | P_{i_3} | P_{i_4} | P'_{i_4} | P_{i_5} |
|---------------|---------------|---------------|---------------|-----------|------------|-----------|-----------|-----------|------------|-----------|
| i_4 | i_1 | \vdots | i_5 | s_1 | s_2 | s_1 | s_2 | s_2 | s_1 | s_4 |
| i_1 | i_3 | | i_1 | s_4 | \vdots | s_3 | s_3 | s_1 | \vdots | \vdots |
| i_2 | i_4 | | \vdots | s_2 | \vdots | s_2 | s_1 | s_3 | | |
| \vdots | \vdots | | | s_3 | | s_4 | s_4 | s_4 | | |

The following two tables illustrate the steps of the Shanghai mechanism applied to the problem (\succ, P) . A student tentatively placed at a school at a particular step is outlined in a box.

| Round 0 | $s_1 (q_{s_1}^{r=1} = 1)$ | $s_2 (q_{s_2}^{r=1} = 1)$ | $s_3 (q_{s_3}^{r=1} = 2)$ | $s_4 (q_{s_4}^{r=1} = 1)$ |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Step 1 | $\boxed{\ell_1}, i_2$ | $\boxed{\ell_3}, i_4$ | | $\boxed{\ell_5}$ |
| Step 2 | $\boxed{\ell_4}, i_1$ | $\boxed{\ell_3}$ | $\boxed{\ell_2}$ | |
| Step 3 | | | | $\boxed{\ell_5}, i_1$ |
| Round 1 | $s_1 (q_{s_1}^{r=2} = 0)$ | $s_2 (q_{s_2}^{r=2} = 0)$ | $s_3 (q_{s_3}^{r=2} = 1)$ | $s_4 (q_{s_4}^{r=1} = 0)$ |
| Step 4 | | i_1 | | |
| Step 5 | \vdots | | $\boxed{\ell_1}$ | |

In the above tables, observe that student i_1 ends up at his last choice at problem (\succ, P) . Now consider the following two tables that illustrate the steps of the Shanghai mechanism when student i_1 reports P'_{i_1} , as opposed to P_{i_1} .

| Round 0 | $s_1 (q_{s_1}^{r=1} = 1)$ | $s_2 (q_{s_2}^{r=1} = 1)$ | $s_3 (q_{s_3}^{r=1} = 2)$ | $s_4 (q_{s_4}^{r=1} = 1)$ |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Step 1 | $\boxed{i_2}$ | $\boxed{i_1, i_3, i_4}$ | | $\boxed{i_5}$ |
| Step 2 | $\boxed{i_4, i_2}$ | $\boxed{i_1}$ | $\boxed{i_3}$ | |
| Step 3 | | | $\boxed{i_2, i_3}$ | |

In this case, student i_1 is assigned to school s_2 . Thus, the Shanghai mechanism is manipulable by student i_1 at problem (\succ, P) . Next, let us apply the Boston mechanism to problem (\succ, P) . The specifications are illustrated in the following tables.

| Round 0 | $s_1 (q_{s_1}^{r=1} = 1)$ | $s_2 (q_{s_2}^{r=1} = 1)$ | $s_3 (q_{s_3}^{r=1} = 2)$ | $s_4 (q_{s_4}^{r=1} = 1)$ |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Step 1 | $\boxed{i_1, i_2}$ | $\boxed{i_3, i_4}$ | | $\boxed{i_5}$ |
| Round 1 | $s_1 (q_{s_1}^{r=2} = 0)$ | $s_2 (q_{s_2}^{r=2} = 0)$ | $s_3 (q_{s_3}^{r=2} = 1)$ | $s_4 (q_{s_4}^{r=1} = 0)$ |
| Step 2 | i_4 | | $\boxed{i_2}$ | |
| Step 3 | \vdots | | $\boxed{i_2, i_4}$ | |

Observe that student i_1 ends up at s_1 (his first choice), and thus cannot gain by a misreport, but student i_4 ends up at s_3 (his third choice) at problem (\succ, P) . Next consider the following tables that illustrate the steps of the Boston mechanism when student i_4 reports P'_{i_4} , as opposed to P_{i_4} .

| Round 0 | $s_1 (q_{s_1}^{r=1} = 1)$ | $s_2 (q_{s_2}^{r=1} = 1)$ | $s_3 (q_{s_3}^{r=1} = 2)$ | $s_4 (q_{s_4}^{r=1} = 1)$ |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Step 1 | $\boxed{i_4, i_1, i_2}$ | $\boxed{i_3}$ | | $\boxed{i_5}$ |
| Round 1 | $s_1 (q_{s_1}^{r=2} = 0)$ | $s_2 (q_{s_2}^{r=2} = 0)$ | $s_3 (q_{s_3}^{r=2} = 2)$ | $s_4 (q_{s_4}^{r=1} = 0)$ |
| Step 2 | | | $\boxed{i_2}$ | i_1 |
| Round 2 | $s_1 (q_{s_1}^{r=2} = 0)$ | $s_2 (q_{s_2}^{r=2} = 0)$ | $s_3 (q_{s_3}^{r=2} = 1)$ | $s_4 (q_{s_4}^{r=1} = 0)$ |
| Step 3 | | | $\boxed{i_1}$ | |

Now student i_4 ends up at school s_1 . Thus, the Boston mechanism is also manipulable at problem (\succ, P) .

Example 1b. (Shanghai mechanism is not manipulable when the Boston mechanism is) Consider the following example with the given priority structure and the profile of preferences. Each school, s_1 , s_2 , and s_3 , has a quota of one.

| | | | | | | |
|---------------|---------------|---------------|-----------|-----------|------------|-----------|
| \succ_{s_1} | \succ_{s_2} | \succ_{s_3} | P_{i_1} | P_{i_2} | P'_{i_2} | P_{i_3} |
| i_1 | i_2 | \vdots | s_1 | s_1 | s_2 | s_2 |
| i_2 | i_3 | | \vdots | s_2 | \vdots | s_3 |
| \vdots | \vdots | | | s_3 | | \vdots |

Clearly, at problem (\succ, P) under the Boston mechanism, student i_2 can obtain a seat at s_2 by submitting P'_{i_2} as opposed to P_{i_2} which places him at s_3 . Note, however, that under the Shanghai mechanism no student can ever gain by a misreport at problem (\succ, P) .

Proof of Theorem 1 (Manipulability):

We start with a useful definition. Given a preference relation P_i of a student i , let $rank_i(a)$ denote the rank of school a in student i 's preferences.

Definition: Given a preference profile P , student i ranks school a at a higher e -class than student j iff

$$\left\lceil \frac{rank_i(a)}{e} \right\rceil < \left\lceil \frac{rank_j(a)}{e} \right\rceil.$$

Intuitively, a student who lists a school among his first e choices ranks that school at a higher e -class than those who do not list it as one of their first e choices; a student who lists a school among his first $e + 1$ through $2e$ choices ranks that school at a higher e -class than those who do not list it as one of their first $2e$ choices; etc. The following construction will be instrumental in the proof of Theorem 1 as well as some of the subsequent proofs.

For a given problem (\succ, P) , the corresponding e -augmented priority profile $\tilde{\succ}$ is constructed as follows. For each $a \in S$, and all $i, j \in I$, we have $i \tilde{\succ}_a j$ if and only if either

- (1) i ranks school a at a higher e -class than j , or
- (2) i and j both rank school a in the same e -class and $i \succ_a j$.

Lemma 1: Given a problem (\succ, P) and the corresponding e -augmented priority profile $\tilde{\succ}$, $\varphi^e(\succ, P) = \varphi^\infty(\tilde{\succ}, P)$.

Proof of Lemma 1: Let J^r denote the set of students who are permanently assigned to some school at the end of round r of φ^e at problem (\succ, P) . We first argue that the students in J^0 receive the same assignments under the DA at problem $(\tilde{\succ}, P)$. First observe that by the construction of the e -augmented priority profile $\tilde{\succ}$, a student who ranks a school in a higher e -class than some other student can never be rejected by that school under the DA at $(\tilde{\succ}, P)$ because of the application of that other student. Then since round 0 of φ^e is equivalent to applying the DA algorithm to the first e choices of all students and the assignments are made permanent at the end of round 0 of φ^e , the assignments of students in J^0 under φ^e at problem (\succ, P) has to coincide with their assignments under the DA at problem $(\tilde{\succ}, P)$. Decreasing each school's quota under φ^e before round 1 and applying the same reasoning to this round the students in J^1 must receive the same assignments under the DA at problem $(\tilde{\succ}, P)$. Iterating this reasoning for the next rounds in turn we conclude that $\varphi^e(\succ, P) = \varphi^\infty(\tilde{\succ}, P)$.

■

Given $i \in I$ and $x \in S$, let P_i^x denote a preference relation where student i ranks school x as his first choice.

Lemma 2: Given a problem (\succ, P) , let $\varphi_i^E(\succ, P) = x$. Then $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$ where $e < E$.

Proof of Lemma 2: By Lemma 1, $\varphi_i^E(\succ, P) = \varphi_i^\infty(\tilde{\succ}, P) = x$ where $\tilde{\succ}$ is the E -augmented priority profile corresponding to (\succ, P) . By the strategy-proofness of the DA, $\varphi_i^\infty(\tilde{\succ}, P) = \varphi_i^\infty(\tilde{\succ}, P_i^x, P_{-i})$. Hence, we have

$$\varphi_i^E(\succ, P) = \varphi_i^\infty(\tilde{\succ}, P_i^x, P_{-i}). \quad (1)$$

On the other hand, by Lemma 1,

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^\infty(\hat{\succ}, P_i^x, P_{-i}) \quad (2)$$

where $\hat{\succ}$ is the E -augmented priority profile corresponding to (\succ, P_i^x, P_{-i}) . Note that $\hat{\succ}_{-x}$ and $\tilde{\succ}_{-x}$ agree on all students' relative priority orderings but i and $\hat{\succ}_x$ (weakly) improves the priority of student i for school x in comparison to $\tilde{\succ}_x$. Then it follows from the

working of the DA algorithm that

$$\varphi_i^\infty(\widehat{\succ}, P_i^x, P_{-i}) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i}). \quad (3)$$

Last we claim that

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^e(\succ, P_i^x, P_{-i}). \quad (4)$$

To see this note that when applied to (\succ, P_i^x, P_{-i}) , the set of students who apply to school x in round 0 of φ^E is weakly larger than that in round 0 of φ^e and since student i is not rejected from school x after applying to it in the first step under φ^E , he cannot be rejected from it under φ^e either.

Combining (1), (2), (3), and (4), we obtain $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$. \blacksquare

Now we are ready to prove Theorem 1. Let (\succ, P) be a problem such that there exists $i \in I$ and preferences P'_i where $\varphi_i^{e'}(\succ, P'_i, P_{-i}) P_i \varphi_i^{e'}(\succ, P)$. We show that there exists $j \in I$ and preferences P'_j such that $\varphi_j^e(\succ, P'_j, P_{-j}) P_j \varphi_j^e(\succ, P)$ where $e < e'$. Let $\varphi_i^{e'}(\succ, P'_i, P_{-i}) = x$. We consider two cases.

Case 1. $x P_i \varphi_i^e(\succ, P)$: Since $\varphi_i^{e'}(\succ, P'_i, P_{-i}) = x$, by Lemma 2 $\varphi_i^e(\succ, P_i^x, P_{-i}) = x$. Thus, i manipulates φ^e at (\succ, P) .

Case 2. $\varphi_i^e(\succ, P) R_i x$: We claim that for all $k \in I$, $\varphi_k^e(\succ, P) R_k \varphi_k^{e'}(\succ, P)$. Suppose not. Then, there exists $j \in I$ such that $\varphi_j^{e'}(\succ, P) P_j \varphi_j^e(\succ, P)$. By Lemma 2, $\varphi_j^e(\succ, P_j^{\varphi_j^{e'}(\succ, P)}, P_{-j}) = \varphi_j^{e'}(\succ, P)$ and thus j manipulates φ^e at (\succ, P) . Hence the claim is true. Moreover, since $\varphi_i^e(\succ, P) R_i x$ and $x P_i \varphi_i^{e'}(\succ, P)$, by transitivity we have $\varphi_i^e(\succ, P) P_i \varphi_i^{e'}(\succ, P)$. This together with the preceding claim implies that $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$.

Next, consider the rounds of $\varphi^{e'}$ when applied to problem (\succ, P) . Let $y = \varphi_i^{e'}(\succ, P)$. Let also r be the round at the end of which student i is (permanently) assigned to school y . We claim that $r \geq 1$. Suppose for a contradiction that $r = 0$. Then since $\varphi_i^{e'}(\succ, P'_i, P_{-i}) = x P_i y = \varphi_i^{e'}(\succ, P)$, student i ranks school x at the same (and the highest) e' -class at both (P'_i, P_{-i}) and P . Let $\widetilde{\succ}$ and $\widehat{\succ}$ be the e' -augmented priority profiles corresponding to (\succ, P'_i, P_{-i}) and (\succ, P) respectively. Thus, by Lemma 1, $\varphi_i^\infty(\widetilde{\succ}, P'_i, P_{-i}) = x$ and $\varphi_i^\infty(\widehat{\succ}, P) = y$. Let P_i^{xy} be a relation where i ranks x first and y second. By

the strategy-proofness of the DA, $\varphi_i^\infty(\tilde{\succ}_i, P_i^{xy}, P_{-i}) = x$. Note that since student i ranks school x at the same (and the highest) e' -class at both (P'_i, P_{-i}) and P , $\tilde{\succ}_x = \tilde{\succ}_x$. Thus, $\varphi_i^\infty(\tilde{\succ}_x, \tilde{\succ}_{-x}, P_i^{xy}, P_{-i}) = x$. Recall that $\varphi_i^\infty(\tilde{\succ}_i, P) = y$. By the strategy-proofness of the DA, $\varphi_i^\infty(\tilde{\succ}_i, P_i^{xy}, P_{-i}) = y$. But then, at both $(\tilde{\succ}_x, \tilde{\succ}_{-x}, P_i^{xy}, P_{-i})$ and $(\tilde{\succ}_i, P_i^{xy}, P_{-i})$ the preference profiles are the same and student i lists school x as first choice. Since the priority order for x is also identical at both problems, the DA should give i the same assignment for both problems. A contradiction. Thus, $r \geq 1$ as claimed.

Let $z_0 = \varphi_i^e(\succ, P)$. Since $\varphi_i^e(\succ, P) P_i \varphi_i^{e'}(\succ, P)$ and $\varphi_i^{e'}$ is nonwasteful, there exists $j_1 \in \varphi^{e'}(\succ, P)(z_0) \setminus \varphi^e(\succ, P)(z_0)$. Since $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$, we must have $\varphi_{j_1}^e(\succ, P) P_{j_1} \varphi_{j_1}^{e'}(\succ, P)$. Letting $z_1 = \varphi_{j_1}^e(\succ, P) \neq z_0$, there exists $j_2 \in \varphi^{e'}(\succ, P)(z_1) \setminus \varphi^e(\succ, P)(z_0)$. Since I is finite, iterating this reasoning we obtain a set $J = \{i, j_1, \dots, j_k\}$ of students with $k \geq 1$ each of whom is assigned to a distinct school from the set $A = \{z_0, z_1, \dots, z_k = y\}$ at $\varphi^e(\succ, P)$. Reconsidering the $\varphi^{e'}$ algorithm when applied to problem (\succ, P) , each student in J must then be assigned to the corresponding school in A in the same round. For otherwise, the school from the set A that admits a student at a later round will still have a vacant position in all previous rounds which contradicts the fact that the student from the set J assigned to it at $\varphi^e(\succ, P)$ is better off compared to $\varphi_i^{e'}(\succ, P)$. In other words, all Pareto improving assignment exchanges from $\varphi^{e'}(\succ, P)$ to $\varphi^e(\succ, P)$ must involve students who receive their (permanent) assignments in the same round. Hence, each student in J are (permanently) assigned to the corresponding school in A in round $r \geq 1$.

Consider round r of the $\varphi^{e'}$ algorithm when applied to problem (\succ, P) . Let $J^r \supset J$ be the set of students such that (1) they each receive their (permanent) assignments at the end of round r , and (2) they each are better off at $\varphi^e(\succ, P)$ compared to $\varphi^{e'}(\succ, P)$.³⁵ Let $j^* \in J^r$ be the last student in J^* to apply to his assignment at $\varphi^e(\succ, P)$ in that round and let $z^* = \varphi_{j^*}^* e'(\succ, P)$. Let k^* be the student who is kicked out from z^* at that step. Note that k^* necessarily exists since a student from J^r has already been kicked out from z^* at a previous step in that round. Thus, $z^* P_{k^*}^* \varphi_{k^*}^* e'(\succ, P)$. Moreover, by the choice of j^* , $k^* \notin J^r$. If student k^* receives his (permanent) assignment at the end of round r , then $\varphi_{k^*}^* e(\succ, P) = \varphi_{k^*}^* e'(\succ, P)$. Otherwise, student k^* receives his (permanent) assignment at a later round than r and by the argument in the preceding paragraph pertaining to students

³⁵Note that the set J^r is well-defined by the argument made in the previous paragraph.

who are better off at $\varphi^e(\succ, P)$, $z^* P_k^* \varphi_k^* e(\succ, P)$.

Finally, since school z^* has a vacancy before round $r \geq 1$, it follows that $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^*$. Then by Lemma 2, $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = \varphi_{k^*}^e(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^* P_{k^*}^* \varphi_{k^*}^e(\succ, P)$. Hence, student k^* manipulates φ^e at (\succ, P) .

We next prove that $\varphi^{e'}$ may not be manipulable when φ^e is. Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \dots, i_{e+2}\}$ and $S = \{s_1, s_2, \dots, s_{e+1}\}$ where each school has a quota of one. Each student $i \in I$ has the following preferences: $s_1 P_i s_2 P_i \dots s_e P_i s_{e+1} P_i \emptyset$. There is a single priority order for each school given as follows: for each $s \in S$, suppose $i_k \succ_s i_{k'}$ whenever $k < k'$, i.e., i_1 has the highest priority, i_2 has the second highest priority and so on. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+2} . It is easy to see that he is unassigned in round 0 and is assigned to his last choice (i.e., the null school) at step $e + 2$ of round 1 after being rejected from school s_{e+1} . If student i_{e+2} were to report school s_{e+1} as his first choice, he would clearly be assigned to it in round 0. Hence, φ^e is manipulable by student i_{e+1} at this problem. It is easy to see that no student can manipulate $\varphi^{e'}$ via a preference misreport at this problem. ■

Proof of Theorem 2:

Part (i). Proposition 1 implies that Boston is more efficient than all φ^e with $e > 1$. To show the second statement let $1 \neq e < e'$. First consider the following problem (\succ, P) . Let $I = \{i_1, i_2, \dots, i_{e+1}\}$ and $S = \{s_1, s_2, \dots, s_e\}$, where $q_s = 1$ for all $s \in S$. Each $i_j \in I \setminus \{i_e, i_{e+1}\}$ top-ranks school s_j and has the highest priority for it. Let $P_{i_e} : s_e, s_1, \emptyset$ and $P_{i_{e+1}} : s_1, s_2, \dots, s_e, \emptyset$. Also let $i_e \succ_{s_1} 1 \succ_{s_1} i_{e+1}$ and $i_{e+1} \succ_{s_e} i_e$. We then have for each $i_j \in I \setminus \{1, i_e, i_{e+1}\}$, $\varphi_{i_j}^e(\succ, P) = \varphi_{i_j}^{e'}(\succ, P) = s_j$, $\varphi_{i_e}^e(\succ, P) = \varphi_{i_e}^{e'}(\succ, P) = \emptyset$, $\varphi_{i_{e+1}}^e(\succ, P) = \varphi_{i_{e+1}}^{e'}(\succ, P) = s_e$, and $\varphi_1^e(\succ, P) = \varphi_1^{e'}(\succ, P) = s_1$. Note that $\varphi^e(\succ, P)$ is Pareto efficient but $\varphi^{e'}(\succ, P)$ is not since students i_e and i_{e+1} can be better off by swapping their assignments.

Next consider the following problem (\succ', P') . Let $I = \{i_1, i_2, \dots, i_{e+5}\}$ and $S = \{s_1, s_2, \dots, s_{e+2}\}$ where $q_s = 1$ for all $s \in S$. Each $i_j \in \{i_1, i_2, \dots, i_e\}$ top-ranks school s_j and has the highest priority for it. Let $P'_{i_{e+1}} : s_{e+1}, s_{e+2}, \emptyset$; $P'_{i_{e+2}} : s_{e+2}, s_{e+1}, \emptyset$; $P'_{i_{e+3}} : s_{e+1}, \emptyset$; and $P'_{i_{e+4}} : s_{e+2}, \emptyset$. Also let $i_{e+5} \succ'_{s_{e+1}} i_{e+2} \succ'_{s_{e+1}} i_{e+3} \succ'_{s_{e+1}} i_{e+1}$ and $i_{e+1} \succ'_{s_{e+2}} i_{e+4} \succ'_{s_{e+2}} i_{e+2}$. We then have for each $i_j \in \{i_1, i_2, \dots, i_e\}$, $\varphi_{i_j}^e(\succ', P') = \varphi_{i_j}^{e'}(\succ', P') = s_j$, for each $k \in \{i_{e+3}, i_{e+4}, i_{e+5}\}$ and each $l \in \{i_{e+2}, i_{e+3}, i_{e+4}\}$, $\varphi_k^e(\succ', P') = \varphi_l^{e'}(\succ', P') =$

\emptyset , $\varphi_{i_{e+1}}^e(\succ', P') = \varphi_{i_{e+1}}^{e'}(\succ', P') = s_{e+2}$, and $\varphi_{i_{e+2}}^e(\succ', P') = \varphi_{i_{e+2}}^{e'}(\succ', P') = s_{e+1}$. Note that $\varphi^{e'}(\succ', P')$ is Pareto efficient but $\varphi^e(\succ', P')$ is not since students i_{e+1} and i_{e+2} can be better off by swapping their assignments.

Part (ii). Fix a problem (\succ, P) . Take any two mechanisms φ^e and $\varphi^{e'}$ with $e' > e$. We contrast round 0 of φ^e with that of $\varphi^{e'}$. For any school $s \in S$, the set of students who apply to s in round 0 of $\varphi^{e'}$ is weakly larger than the set of students who apply to s in round 0 of φ^e . This implies that any student who is assigned to his first choice at the end of round 0 of $\varphi^{e'}$ is also assigned to his first choice at the end of round 0 of φ^e but not vice versa. In other words, a student who is assigned to his first choice under φ^e , may be rejected from that school under $\varphi^{e'}$ due to the application a higher priority student who ranks it as one of his $e + 1$ through e' choices.

Part (iii). Fix a problem. Suppose $e' < e$. Consider any student-say i - who is assigned to one of his first e choices-say s - under $\varphi^{e'}$ but not under φ^e . Since assignments under φ^e are final after the first e choices have been considered (or alternatively, since the equivalent the DA algorithm constructed in Lemma 1 prioritizes the first e choices), student i 's slot at s is filled by another student who also ranks s as one of his first e choices. Thus, the number of students who receive one of their first e choices cannot decrease under φ^e .

Suppose $e' > e$. Take any student-say j - who is assigned to one of his first e choices under $\varphi^{e'}$. Note that the corresponding e -augmented priority profile for this problem gives (weakly) higher priority to student j for all his first e choices than the corresponding e' -augmented priority profile. Then by Lemma 1 and the stability of the DA, student j must be assigned to one of his first e choices under φ^e as well. ■

Proof of Theorem 3:

(Part i). Let $e' = ke$. If $k = \infty$, Proposition 1 implies that the DA is more stable than φ^e for any $e < \infty$. So let $k \in \mathbb{N}$. We show that if $\varphi^{e'}$ is unstable at a problem, then so is φ^e . We prove the contrapositive of this statement. Let (\succ, P) be a problem at which $\varphi^e(\succ, P)$ is stable. We show that $\varphi^e(\succ, P) = \varphi^{e'}(\succ, P)$.

Consider mechanism φ^e when applied to problem (\succ, P) . Since $\varphi^e(\succ, P)$ is stable, any unassigned student of round 0 (who was rejected from all his first e -choices) must have lower priority at his first e -choice schools than every student who obtained a seat at any such school in round 0. Similarly, since $\varphi^{e'}(\succ, P)$ is stable, any unassigned student of

round 1 (who was rejected from all his first $2e$ -choices) must have lower priority at his first $2e$ -choice schools than every student who obtained a seat at any such school in round 0 or round 1. In general, any unassigned student of round $k - 1$ must have lower priority at his first ke -choice schools than every student who obtained a seat at any such school in round $k - 1$ or any previous round. But this implies that any student who is unassigned at the end of round $k - 1$ of φ^e is also unassigned at the end of round 0 of $\varphi^{e'}$ as he applies to and gets rejected from the same set of schools in the same order under both mechanisms. Similarly, any student who is assigned to some school s in some round $t \leq k - 1$ of φ^e is also assigned to school s in round 0 of $\varphi^{e'}$ as he cannot be rejected by a student who does not list school s among his first $(t + 1)e$ -choices. Then the students who participate in rounds k through $2k - 1$ of φ^e are the same as those who participate in round 1 of $\varphi^{e'}$ and by the same argument they apply to and get rejected from the same set of schools in the same order under both mechanisms. Iterating this reasoning, we conclude that $\varphi^{e'}(\succ, P) = \varphi^e(\succ, P)$.

The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not.

(Part ii). Since $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, there exists $t \in \mathbb{N}$ such that $te < e' < (t + 1)e$. Consider the following problem (\succ, P) . Let $I = \{i_1, i_2, \dots, i_{te+e'+3}\}$ and $S = \{s_1, s_2, \dots, s_{te+e'+2}\}$ where $q_s = 1$ for all $s \in S$. Each $i_j \in I \setminus \{i_{te+1}, i_{te+e'+3}\}$ top-ranks school s_j and has the highest priority for it. The remaining two students' preferences are as follows. $P_{i_{te+1}} : s_1, s_2, \dots, s_{te+1}, \emptyset$ and $P_{i_{te+e'+3}} : s_{te+2}, s_{te+3}, \dots, s_{te+2+e'}, s_{te+1}, \emptyset$. Let $i_{te+e'+3} \succ_{s_{te+1}} i_{te+1}$.

It is not difficult to calculate that for each $i_j \in I \setminus \{i_{te+1}, i_{te+e'+3}\}$, $\varphi_{i_j}^e(\succ, P) = \varphi_{i_j}^{e'}(\succ, P) = s_j$, $\varphi_{i_{te+1}}^e(\succ, P) = \varphi_{i_{te+e'+3}}^{e'}(\succ, P) = \emptyset$, and $\varphi_{i_{te+1}}^{e'}(\succ, P) = \varphi_{i_{te+e'+3}}^e(\succ, P) = s_{te+1}$. Clearly, $\varphi^e(\succ, P)$ is stable whereas $\varphi^{e'}(\succ, P)$ is not. The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not. ■

Proof of Proposition 2: Part (i) is established in Theorem 1 of Ergin and Sönmez (2006). We prove part (ii). Let $I = \{i_1, i_2, i_3\}$ and $S = \{s_1, s_2, s_3\}$, where each school has a quota of one. Consider the following priority profile \succ and true preferences $P = (P_1, P_2, P_3)$ of students.

| | | |
|---------------|---------------|---------------|
| \succ_{s_1} | \succ_{s_2} | \succ_{s_3} |
| i_3 | i_2 | i_2 |
| i_2 | \vdots | i_1 |
| i_1 | | i_3 |

| | | |
|-----------|-----------|-----------|
| P_{i_1} | P_{i_2} | P_{i_3} |
| s_1 | s_1 | s_3 |
| s_3 | s_2 | s_1 |
| s_2 | s_3 | s_2 |

the DA outcome for problem (\succ, P) is the following matching

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_3 & s_2 & s_1 \end{pmatrix}.$$

Consider a strategy profile $Q = (Q_1, Q_2, Q_3)$ where $Q_1 = P_{i_1}$, $Q_3 = P_{i_3}$, and Q_2 is a strategy in which student i_2 ranks school s_2 as his first choice. For problem (\succ, Q) the outcome of the application-rejection mechanism (e) , for any $e \geq 2$, is the unstable matching

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix},$$

where μ' Pareto dominates μ . To see that Q is indeed an equilibrium profile, it suffices to consider possible deviations by student i_2 . For any preferences in which he ranks s_1 first, he gets rejected from s_1 at the third step. If he ranks s_2 first, clearly his assignment does not change. If she ranks s_3 first, he is assigned to s_3 . ■

Proof of Proposition 3: Let (\succ, P) be the problem where P is the list of true student preferences. By Proposition 2, μ is stable under (\succ, P) . Let P' be a preference profile where each $i \in I$ lists $\mu(i)$ as his e -th choice and such that for any $s \in S$, $s P'_i \mu(i)$ implies $s P_i \mu(i)$. We show that for each $i \in I$, $\varphi_i^e(\succ, P) R_i \mu(i)$ for any e . Suppose to the contrary that student i remains unassigned at the end of round 0. This means that school $\mu(i)$ is full at the end of round 0, and in particular, there is $j \neq i$ such that $\varphi_j^e(\succ, P) = \mu(i) \neq \mu(j)$ and $j \succ_{\mu(i)} i$. Then, since $\mu(i) P_j \mu(j)$ and $j \succ_{\mu(i)} i$, μ is not stable under (\succ, P) . ■

Proof of Proposition 4:

Part (i). We start by adopting the ACY model. Let $S = \{s_0, s_1, \dots, s_m\}$ with $m \geq 1$ be the set of schools (without the outside option). Each student privately draws vNM utility values $v = (v_0, \dots, v_m)$ from a finite set $\mathcal{V} = \{(v_0, \dots, v_m) \in [0, 1]^m | v_0 > v_1 \dots > v_m\}$ with probability $f(\mathbf{v})$, which is common knowledge. Without loss of generality, we assume

that $\sum_{s \in S} q_s = n = |I|$. Let Π be the set of all ordinal preferences over S , and $\Delta(\Pi)$ the set of probability distributions over Π . A symmetric Bayesian strategy is a mapping $\sigma : \mathcal{V} \rightarrow \Delta(\Pi)$.

In showing the dominance of Shanghai over the DA, we use exactly the same proof strategy as ACY. Following ACY, the probability that any student is assigned to school $s \in S$ is given by

$$P_s^{theDA} = \frac{q_s}{n}.$$

For any equilibrium strategy $\sigma \in \{\sigma^*(v)\}_{v \in \mathcal{V}}$, let $P_s^{SHA}(\sigma)$ be the probability that a student is assigned to school s if he plays σ when all other students play σ^* . Then, in equilibrium, for each $s \in S$,

$$\sum_{v \in \mathcal{V}} n P_s^{SHA}(\sigma^*(v)) f(v) = q_s.$$

Suppose a type $\tilde{v} \in \mathcal{V}$ student chooses to play $\sigma^*(v)$ with probability $f(v)$. Denote that strategy by $\tilde{\sigma}$. Then he is assigned to $s \in S$ with probability

$$P_s^{SHA}(\tilde{\sigma}) = \sum_{v \in \mathcal{V}} P_s^{SHA}(\sigma^*(v)) f(v) = \frac{q_s}{n} = P_s^{theDA}.$$

That is, by playing $\tilde{\sigma}$, which is not necessarily an equilibrium strategy, a student can guarantee himself the same random assignment as that he would get under the DA.

Part (ii). We start by showing that the specified strategies for the complete information example given in the text indeed constitute the unique equilibrium of Shanghai. Let $u_i(s)$ denote the vNM utility of student i for school s and σ_i denote a (pure) strategy of student i . Suppose students 1 and 2 are of the low type, student 3 and 4 are respectively of the medium and high types. Let $EU_i^{SHA}(\sigma^*)$ be the expected utility of student i at the specified strategy profile, i.e., when $\sigma_i^* = s_0 s_1 s_2 s_3$ for $i = 1, 2, 3$ and $\sigma_4^* = s_1 s_2 s_0 s_3$. Then we have $EU_i^{SHA} = \frac{1}{3}u_i(s_0) + \frac{1}{6}u_i(s_1) + \frac{1}{6}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA} = \frac{1}{2}u_4(s_1) + \frac{1}{2}u_4(s_2) = .32$.

Clearly, for any student, ranking s_3 at any position but the bottom is dominated. Moreover, σ_1^* and σ_2^* are dominant strategies. We first claim that σ_3^* is a best response to σ_1^* and σ_2^* regardless of what 4 does. To show this, we fix σ_1^* and σ_2^* , and consider three possibilities

for σ_4^* .

1. $\sigma_4^* = s_0 s_1 s_2 s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1 s_2 s_0 s_3) = .24 > EU_3^{SHA}(\sigma_3 = s_0 s_2 s_1 s_3) = .23$.³⁶

2. $\sigma_4^* = s_1 s_2 s_0 s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_0 s_2 s_1 s_3) = .22 > EU_3^{SHA}(\sigma_3 = s_1 s_2 s_0 s_3) = .21$.

3. $\sigma_4^* = s_0 s_2 s_1 s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1 s_2 s_0 s_3) = .23 > EU_3^{SHA}(\sigma_3 = s_0 s_2 s_1 s_3) = .19$.

Last, we claim that σ_4^* is a best response to σ_1^* , σ_2^* , and σ_3^* . Indeed, $EU_4^{SHA}(\sigma_4^*) = .32 > EU_4^{SHA}(\sigma_4 = s_0 s_2 s_1 s_3) = .31 > EU_4^{SHA}(\sigma_4 = s_0 s_1 s_2 s_3) = .25$. Thus, we have confirmed that profile σ^* constitutes the unique equilibrium of Shanghai.

We next prove part (ii) of Proposition 3 building on the example given in the main text. Let $I = \{1, 2, 3, 4\}$, $S = \{s_0, s_1, s_2, s_3\}$, and $\mathcal{V} = \{\mathbf{v}_L, \mathbf{v}_M, \mathbf{v}_H\}$ (as in the example) with probabilities $p_L = \frac{3}{4} - \frac{\varepsilon}{2}$, $p_M = \frac{1}{4} - \frac{\varepsilon}{2}$, and $p_H = \varepsilon$, where $\varepsilon > 0$ can be chosen arbitrarily close to zero. Consider the following strategies under Boston: $\sigma^{BOS}(\mathbf{v}_L) = s_0 s_1 s_2 s_3$, $\sigma^{BOS}(\mathbf{v}_M) = s_1 s_0 s_2 s_3$, and $\sigma^{BOS}(\mathbf{v}_H) = s_2 s_0 s_1 s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrium for a sufficiently small ε .

Since an exact analysis would be unnecessarily lengthy and cumbersome, we provide only rough arguments. For a low type student it is still a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, the following table provides possible realizations of the types of the remaining three students and a corresponding best response of a high type student to the particular realization in each case. With an abuse of notation, let $|\mathbf{v}_x|$ denote the number of students of type \mathbf{v}_x . Note that we do not display those realizations involving a high type student as they will have no affect on equilibrium verification when ε is chosen to be sufficiently close to zero.

| Realization | Probability | Best response | Payoff loss from $\sigma^{BOS}(\mathbf{v}_H)$ | Minimum gain from $\sigma^{BOS}(\mathbf{v}_H)$ |
|--|-------------|------------------------------|---|--|
| $ \mathbf{v}_L = 3$ | .42 | $\sigma = s_1$ | -.06 | - |
| $ \mathbf{v}_L = 2, \mathbf{v}_M = 1$ | .42 | $\sigma^{BOS}(\mathbf{v}_H)$ | - | .11 |
| $ \mathbf{v}_L = 1, \mathbf{v}_M = 2$ | .14 | $\sigma^{BOS}(\mathbf{v}_H)$ | - | .11 |
| $ \mathbf{v}_M = 3$ | .02 | $\sigma^{BOS} = s_0$ | -.07 | - |

³⁶Upon fixing σ_1^* and σ_2^* , we calculate that $EU_i^{SHA}(\sigma_3 = s_1 s_2 s_0 s_3, \sigma_4^* = s_0 s_1 s_2 s_3) = \frac{1}{4}u_i(s_0) + \frac{1}{3}u_i(s_1) + \frac{1}{12}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA}(\sigma_3 = s_1 s_2 s_0 s_3, \sigma_4^* = s_0 s_1 s_2 s_3) = \frac{1}{4}u_4(s_1) + \frac{3}{4}u_4(s_2)$.

For example, the first row of the table represents the case when all three students are of low type, which occurs with probability $p_L^3 \cong .42$. In this case, a high type maximizes his payoff by ranking s_1 first, by which he receives a payoff of .35. But since $\sigma^{BOS}(\mathbf{v}_H)$ is not a best response to this realization, a high type receives only .29 by playing $\sigma^{BOS}(\mathbf{v}_H)$. The second row represents the case when two students are of low type and one of medium type, which occurs with probability $3p_L^2p_M \cong .42$. In this case, $\sigma^{BOS}(\mathbf{v}_H)$ is a best response of a high type to this realization, by which he receives a payoff of .29. The next-best action of a high type to this realization is playing $\sigma = s_1$, by which he receives $\frac{.35}{2} \cong .18$. Hence playing $\sigma^{BOS}(\mathbf{v}_H)$ gives him an extra payoff of at least .11 over any other strategy. The rest of the table is filled in similarly. It follows from the table that expected utility loss of a high type due to playing $\sigma^{BOS}(\mathbf{v}_H)$ when it is not a best response, is more than offset by his gain from playing $\sigma^{BOS}(\mathbf{v}_H)$ when it is a best response.

Consider a medium type student. Fixing the strategies of the other students as above, the following table provides possible realizations for the types of the remaining three students and the corresponding best responses of a medium type student to the particular realization in each case. Once again, we do not display those realizations involving a high type student.

| Realization | Probability | Best response | Payoff loss from $\sigma^{BOS}(\mathbf{v}_M)$ | Minimum gain from $\sigma^{BOS}(\mathbf{v}_M)$ |
|--|-------------|------------------------------|---|--|
| $ \mathbf{v}_L = 3$ | .42 | $\sigma^{BOS}(\mathbf{v}_M)$ | - | .12 |
| $ \mathbf{v}_L = 2, \mathbf{v}_M = 1$ | .42 | $\sigma^{BOS}(\mathbf{v}_M)$ | - | .04 |
| $ \mathbf{v}_L = 1, \mathbf{v}_M = 2$ | .14 | $\sigma = s_0$ | -.15 | - |
| $ \mathbf{v}_M = 3$ | .02 | $\sigma = s_0$ | -.44 | - |

It follows from the table that expected utility loss of a medium type due to playing $\sigma^{BOS}(\mathbf{v}_M)$ when it is not a best response, is more than offset by his gain from playing $\sigma^{BOS}(\mathbf{v}_M)$ when it is a best response. Thus, $(\sigma^{BOS}(\mathbf{v}_L), \sigma^{BOS}(\mathbf{v}_M), \sigma^{BOS}(\mathbf{v}_H))$ is a Bayesian equilibrium under Boston. In particular, $EU_{\mathbf{v}_H}^{BOS} \cong .29$.

Next consider the following strategies under Shanghai: $\sigma^{SHA}(\mathbf{v}_L) = \sigma^{SHA}(\mathbf{v}_M) = s_0s_1s_2s_3$ and $\sigma^{SHA}(\mathbf{v}_H) = s_1s_2s_0s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrium for a sufficiently small ε . For a low type student, it is a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, for any particular realization (that does not involve a high type), a high type student faces three students that are playing $\sigma^{SHA}(\mathbf{v}_L)$, and as calculated above for the example with complete information, it is then a best response for him

to play $\sigma^{SHA}(\mathbf{v}_H)$. Similarly, for a medium type student, it is also a best response for him to play $\sigma^{SHA}(\mathbf{v}_H)$ for any particular realization (that does not involve a high type). Thus, $(\sigma^{SHA}(\mathbf{v}_L), \sigma^{SHA}(\mathbf{v}_M), \sigma^{SHA}(\mathbf{v}_H))$ is a Bayesian equilibrium under Shanghai. In particular, $EU_{\mathbf{v}_H}^{SHA} \cong .32 > EU_{\mathbf{v}_H}^{BOS}$. ■

Proof of Theorem 4: Clearly, Theorem 1 shows this result for the special case when all the terms in a choice sequence are identical. It is fairly straightforward to check that the proof of Theorem 1 depends only on the number of choices that are considered in round 0 and not on the number of choices considered in any subsequent round of the application-rejection algorithm. Hence, the same proof still applies once Lemmas 1 and 2 are appropriately modified for the extended class. For brevity, we omit these details. ■

Appendix C: Nash Equilibrium Outcomes in the 6-School Environment (For Online Publication)

We first rewrite Table 3 as a preference profile, where, for each student, the underlined school is her district school:

| P_1 | P_2 | P_3 | P_4 | P_5 | P_6 |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| b | a | b | a | a | a |
| d | d | f | <u>d</u> | b | b |
| c | <u>b</u> | d | f | c | c |
| <u>a</u> | e | a | c | <u>e</u> | <u>f</u> |
| e | f | <u>c</u> | b | d | e |
| f | c | e | e | f | d |

We now examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. Since the outcomes are stable, the analysis is simplified by first computing the student optimal the DA outcome μ^S and the college optimal μ^C and checking if there are any stable allocations in between the two in case they are different. Note that since school e is worse for each student than his district school, student 5 always gets matched to school e in all stable matchings. An allocation below μ^C is always the same regardless of the priority order since it simply assigns each student to his district school.

Every stable matching (with respect to the given profile and the corresponding priority order) is a Nash equilibrium outcome of the DA. That is, the Nash equilibrium outcomes of the DA is a superset of the stable set. This means any Nash equilibrium we compute for Boston (or Shanghai) is also a Nash equilibrium of the DA. But there may be other unstable Nash equilibrium outcomes. In what follows, we present the Nash equilibrium outcomes for each block.

Block 1: $f = 1 - 2 - 3 - 4 - 5 - 6$.

There are two Nash equilibrium outcomes that are stable:

$$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix} \text{ and } \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are three unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}.$$

Block 2: $f = 6 - 1 - 2 - 3 - 4 - 5$

There are three Nash equilibrium outcomes that are stable:

$$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & f & d & e & a \end{pmatrix}, \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}, \text{ and } \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are three other unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}.$$

Block 3: $f = 5 - 6 - 1 - 2 - 3 - 4$

There is one stable Nash equilibrium outcome:

$$\mu^S = \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are four other unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}.$$

Block 4: $f = 4 - 5 - 6 - 1 - 2 - 3$.

There are two stable Nash equilibrium outcomes:

$$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix} \text{ and } \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are three other unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}.$$

Block 5: $f = 3 - 4 - 5 - 6 - 1 - 2.$

There is one stable Nash equilibrium outcome:

$$\mu^S = \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are three other unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}.$$

Block 6: $f = 2 - 3 - 4 - 5 - 6 - 1$

There is one stable Nash equilibrium outcome:

$$\mu^S = \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

There are four other unstable Nash equilibrium outcomes:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}.$$

Appendix D: Experimental Instructions (For Online Publication)

Instructions for the SH₄ treatment (Type 1) is presented first. Instructions for the BOS₄ and the DA₄ treatments are identical except for the subsection, “The allocation of schools . . .,” and the work sheet for Review Question #1. Thus, only this subsection is presented. Instructions for the 6-school treatments are identical except for the number of schools and players. Hence they are omitted, but are available from the authors upon request.

D.1: Instructions for the Shanghai Mechanism (SH₄, Type 1)

Instructions - Mechanism SH₄

(Please turn off your cell phone. Thank you.)

This is an experiment in the economics of decision making. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. The amount of money you earn will depend upon the decisions you make and on the decisions other people make. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you. At the end of the instructions, you will be asked to provide answers to a series of review questions. Once everyone has finished the review questions, we will go through the answers together.

Procedure

- There are 12 participants of four different types in this experiment. You are type 1. Your type remains the same throughout the experiment.
- You will be randomly matched into groups of four at the beginning of each period. Each group contains one of each of the four different types.
- In this experiment, four schools are available for each group. Each school has one slot. These schools differ in geographic location, specialty, and quality of instruction in each specialty. Each school slot is allocated to one participant.

- **Your payoff** amount depends on the school you are assigned to at the end of each period. Payoff amounts are outlined in the following table. These amounts reflect the desirability of the school in terms of location, specialty and quality of instruction.

| Slot received at School: | A | B | C | D |
|--------------------------|------|---|---|----|
| Payoff to Type 1 | [11] | 7 | 5 | 16 |

The table is explained as follows:

- You will be paid 11 points if you hold a slot of School A at the end of a period.
- You will be paid 7 points if you hold a slot of School B at the end of a period.
- You will be paid 5 points if you hold a slot of School C at the end of a period.
- You will be paid 16 points if you hold a slot of School D at the end of a period.

- ***NOTE* different types have different payoff tables. This is a complete payoff table for each of the four types:**

| | A | B | C | D |
|------------------|------|------|------|------|
| Payoff to Type 1 | [11] | 7 | 5 | 16 |
| Payoff to Type 2 | 5 | [11] | 7 | 16 |
| Payoff to Type 3 | 7 | 16 | [11] | 5 |
| Payoff to Type 4 | 5 | 16 | 7 | [11] |

The square brackets, [], indicate the resident of each school district, who has higher priority in that school than other applicants. We will explain this in more detail in the next section.

- In this experiment, participants are defined as belonging to the following school districts:
 - Participant Type 1 lives within the school district of school A,
 - Participant Type 2 lives within the school district of school B,
 - Participant Type 3 lives within the school district of school C,
 - Participant Type 4 lives within the school district of school D.

- The experiment consists of 20 periods. In each period, you will be randomly matched with 3 other people in the room to form a group of four, which has one of each type. Your earnings for each period depend on your choices as well as the choices of the three other people you are matched with.
- Every period, each participant will rank the schools. Note that you need to rank all four schools in order to indicate your preferences.
- After all participants have submitted their rankings, the server will allocate the schools in each group and inform each person of his/her school allocation and respective payoff. Note that your allocation in each period is independent of your allocations in the previous periods.
- Your total payoff equals the sum of your payoffs in all 20 periods. Your earnings are given in points. At the end of the experiment you will be paid based on the exchange rate,

\$1 = 20 points.

In addition, you will be paid \$5 for participation, and up to \$3.5 for answering the Review Questions correctly. Everyone will be paid in private and you are under no obligation to tell others how much you earn.

Allocation Method

- **The priority order for each school is separately determined as follows:**

–

- **High Priority Level:** Participant who lives within the school district.
- **Low Priority Level:** Participants who do not live within the school district.

The priority among the Low Priority Students is based on their respective position in a lottery. The lottery is changed every five periods. In the first five periods, your lottery number is the same as your type number. In each subsequent block of five periods, your lottery number increases by one per block. Specifically, the lottery number for each type in each five-period block is tabulated below:

| | Type 1 | Type 2 | Type 3 | Type 4 |
|---------------|--------|--------|--------|--------|
| Periods 1-5 | 1 | 2 | 3 | 4 |
| Periods 6-10 | 2 | 3 | 4 | 1 |
| Periods 11-15 | 3 | 4 | 1 | 2 |
| Periods 16-20 | 4 | 1 | 2 | 3 |

● **The allocation of schools is obtained as follows:**

- An application to the first choice school is sent for each participant.
- Throughout the allocation process, a school can hold no more applications than its capacity.
If a school receives more applications than its capacity, then it *temporarily* retains the student with the highest priority and rejects the remaining students.
- Whenever an applicant is rejected at a school, his/her application is sent to his or her second choice.
- Whenever a school receives new applications, these applications are considered together with the retained application for that school. Among the retained and new applications, the one with the highest priority is retained *temporarily*.
- After each applicant's **first two choices** have been considered by the corresponding schools, each applicant is assigned a school that holds his or her application in that step. These students and their assignments are removed from the system. The remaining students are rejected. Assignments at the end of this step is *final*.
- Students rejected from their first two choices then apply for their third choice.
- The process repeats for the third and fourth choices.
- The allocation process ends when no more applications can be rejected.

Note that the allocation is finalized every two choices.

An Example:

We will go through a simple example to illustrate how the allocation method works. This example has the same number of students and schools as the actual decisions you will make. You will be asked to work out the allocation of this example for Review Question 1.

Feel free to refer to the experimental instructions before you answer any question. Each correct answer is worth 25 cents, and will be added to your total earnings. You can earn up to \$3.5 for the Review Questions.

Students and Schools: In this example, there are four students, 1-4, and four schools, A, B, C and D.

| | |
|-------------------------------|---------------------|
| Student ID Number: 1, 2, 3, 4 | Schools: A, B, C, D |
|-------------------------------|---------------------|

Slots and Residents: There is one slot at each school. Residents of districts are indicated in the table below.

| School | Slot | District Residents |
|--------|--------------------------|--------------------|
| A | <input type="checkbox"/> | 1 |
| B | <input type="checkbox"/> | 2 |
| C | <input type="checkbox"/> | 3 |
| D | <input type="checkbox"/> | 4 |

Lottery: The lottery produces the following order.

| |
|---------------|
| 1 - 2 - 3 - 4 |
|---------------|

Submitted School Rankings: The students submit the following school rankings:

| | 1st Choice | 2nd Choice | 3rd Choice | Last Choice |
|-----------|---------------|---------------|---------------|----------------|
| Student 1 | D | A | C | B |
| Student 2 | D | A | B | C |
| Student 3 | A | B | C | D |
| Student 4 | A | D | B | C |

Priority : School priorities first depend on whether the school is a district school, and next on the lottery order:

| | Resident | Non-Resident |
|----------------------|----------|--------------|
| Priority order at A: | 1 | - 2 - 3 - 4 |
| Priority order at B: | 2 | - 1 - 3 - 4 |
| Priority order at C: | 3 | - 1 - 2 - 4 |
| Priority order at D: | 4 | - 1 - 2 - 3 |

The allocation method consists of the following steps: Please use this sheet to work out the allocation and enter it into the computer for Review Question #1.

Step 1 (temporary): Each student applies to his/her first choice. If a school receives more applications than its capacity, then it holds the application with the highest priority and rejects the remaining students.

| Applicants | School | Accept | Hold | Reject |
|------------|--------|--------|--------------------------|--------|
| 3, 4 → | A | → N/A | <input type="checkbox"/> | |
| → | B | → N/A | <input type="checkbox"/> | |
| → | C | → N/A | <input type="checkbox"/> | |
| 1, 2 → | D | → N/A | <input type="checkbox"/> | |

Step 2 (temporary): Each student rejected in Step 1 applies to his/her second choice. When a school receives new applications, these applications are considered together with the application on hold for that school. Among the new applications and those on hold, the one with the highest priority is temporarily on hold, while the rest are rejected.

| Accepted | Held | New Applicants | School | Accept | Hold | Reject |
|----------|--------------------------|----------------|--------|--------|------|--------------------------|
| | <input type="checkbox"/> | → | A | → | N/A | <input type="checkbox"/> |
| | <input type="checkbox"/> | → | B | → | N/A | <input type="checkbox"/> |
| | <input type="checkbox"/> | → | C | → | N/A | <input type="checkbox"/> |
| | <input type="checkbox"/> | → | D | → | N/A | <input type="checkbox"/> |

Step 3 (final): Each student rejected in Step 2 applies to his/her second choice. When a school receives new applications, these applications are again considered together with the application on hold for that school. Among the new applications and those on hold, the one with the highest priority is accepted, while the rest are rejected. Since every student's top two choices have been considered, the allocation is final at this step.

| Accepted | Held | New Applicants | School | Accept | Hold | Reject |
|----------|--------------------------|----------------|--------|--------|--------------------------|--------|
| | <input type="checkbox"/> | → | A | → | <input type="checkbox"/> | N/A |
| | <input type="checkbox"/> | → | B | → | <input type="checkbox"/> | N/A |
| | <input type="checkbox"/> | → | C | → | <input type="checkbox"/> | N/A |
| | <input type="checkbox"/> | → | D | → | <input type="checkbox"/> | N/A |

Step 4 (temporary): Each student rejected in Step 3 applies to his/her third choice. If a school still has vacancy, it holds the application with the highest priority and rejects the rest. If a school is already full, it rejects all new applications.

| Accepted | Held | New applicants | School | Accepted | Hold | Reject |
|--------------------------|------|----------------|--------|----------|--------------------------|--------|
| <input type="checkbox"/> | | → | A | → | <input type="checkbox"/> | |
| <input type="checkbox"/> | | → | B | → | <input type="checkbox"/> | |
| <input type="checkbox"/> | | → | C | → | <input type="checkbox"/> | |
| <input type="checkbox"/> | | → | D | → | <input type="checkbox"/> | |

Step 5 (final): Each student rejected in Step 4 applies to his/her fourth choice. If the fourth choice has a vacancy, it accepts the application. Furthermore, all applications on hold are accepted in this step.

| Accepted | Held | New Applicants | | School | | Accept | Hold | Reject |
|--------------------------|------|----------------|---|--------|---|--------------------------|------|--------|
| <input type="checkbox"/> | | | → | A | → | <input type="checkbox"/> | N/A | N/A |
| <input type="checkbox"/> | | | → | B | → | <input type="checkbox"/> | N/A | N/A |
| <input type="checkbox"/> | | | → | C | → | <input type="checkbox"/> | N/A | N/A |
| <input type="checkbox"/> | | | → | D | → | <input type="checkbox"/> | N/A | N/A |

The allocation ends at Step 5.

- Please enter your answer to the computer for Review Question 1.
- Afterwards, you will be asked to answer another 10 review questions. When everyone is finished with them, we will go through the answers together.

Review Questions 2 - 11

2. How many participants are there in your group each period?
3. True or false: You will be matched with the same three participants each period.
4. True or false: Participant living in a school district has higher priority than any other applicants for that school.
5. True or false: The priority for non-residents of a school district is determined by a lottery.
6. True or false: The lottery is fixed for the entire 20 periods.
7. True or false: A lottery number of 1 means that I have the highest priority among the other non-resident applicants in a school.
8. True or false: Other things being equal, a low lottery number is better than a high lottery number.
9. True or false: If you are accepted by a school of your choice, the schools ranked below are irrelevant.
10. True or false: If you are not rejected at a step, then you are accepted into that school.
11. True or false: The allocation is final at the end of each step.

You will have 5 minutes to go over the instructions at your own pace. Feel free to earn as much as you can. Are there any questions?

D.2: Instructions for the Boston Mechanism (BOS₄)

.....

- **The allocation of schools is described by the following method:**

Step 1.

–

- a. An application to the first ranked school is sent for each participant.
- b. Each school accepts the student with highest priority in that school. These students and their assignments are removed from the system. The remaining applications for each respective school are rejected.

Step 2.

–

- a. The rejected applications are sent to his/her second choice.
- b. If a school is still vacant, then it accepts the student with the highest priority and rejects the remaining applications.

Step 3.

–

- a. The application of each participant who is rejected by his/her top two choices is sent to his/her third choice.
- b. If a school is still vacant, then it accepts the student with the highest priority and rejects the remaining applications.

Step 4. Each remaining participant is assigned a slot at his/her last choice.

Note that the allocation is final in each step.

.....

D.3: Instructions for the Deferred Acceptance Mechanism (DA₄)

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The allocation of schools is described by the following method:

- An application to the first ranked school is sent for each participant.
- Throughout the allocation process, a school can hold no more applications than its capacity.
If a school receives more applications than its capacity, then it temporarily retains the student with the highest priority and rejects the remaining students.
- Whenever an applicant is rejected at a school, his or her application is sent to the next highest ranked school.
- Whenever a school receives new applications, these applications are considered together with the retained application for that school. Among the retained and new applications, the one with the highest priority is temporarily on hold.
- The allocation is finalized when no more applications can be rejected.
Each participant is assigned a slot at the school that holds his/her application at the end of the process.

Note that the allocation is temporary in each step until the last step.

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Appendix E: Additional Tables and Analysis (For Online Publication)

This appendix contains additional tables and data analysis, including the probit analysis of factors affecting truthtelling, Nash equilibrium outcomes, as well as session-level efficiency and stability results.

Table 7: Probit: Truthful Preference Revelation

| Dependent Variable: Truthtelling | | | | | | |
|----------------------------------|----------------------|----------------------|-------------------|----------------------|----------------------|----------------------|
| Environments: | 4-School Environment | | | 6-School Environment | | |
| Specifications: | (1) | (2) | (3) | (4) | (5) | (6) |
| Mechanisms: | BOS | SH | DA | BOS | SH | DA |
| Lottery Position | -0.128*** (0.027) | -0.074*** (0.013) | -0.013 (0.018) | -0.098*** (0.010) | -0.045*** (0.008) | -0.028*** (0.005) |
| Period | -0.009** (0.004) | -0.004 (0.003) | 0.002 (0.003) | -0.004 (0.003) | -0.003*** (0.000) | -0.005** (0.002) |
| Log Likelihood | -619.97 | -564.42 | -538.00 | -986.75 | -1194.58 | -1475.81 |
| Observations | 960 | 960 | 960 | 2160 | 2160 | 2160 |

Notes:

1. Robust standard errors are adjusted for clustering at the session level.
2. Coefficients are probability derivatives.
3. Significant at the: ** 5 percent level; *** 1 percent level.

To investigate factors affecting truthtelling, we use probit regressions for each treatment. In Table 7, we present six probit specifications. The dependent variable is a dummy variable indicating whether a participant reveals her preferences truthfully. The independent variables include lottery position (1 being the best, and 6 being the worst), and a period variable to capture any effects of learning. In the 4-school environment (specifications 1-3), participants are 12.8% (resp. 7.4%) less likely to tell the truth under BOS (resp. SH) for each increase in the lottery position, while such an effect is absent under the DA, where truthtelling is a dominant strategy. We also observe a small but significant effect of learning to manipulate under BOS. In comparison, in the 6-school environment (specifications 4-6), we observe a similar lottery position effect on truthtelling, but for all three mechanisms. The 2.8% marginal effect of lottery position on truthtelling under the DA indicates that some participants might not understand the incentives in the DA in the 6-school environment, consistent with the significantly lower level of truthtelling in this environment

compared to the 4-school environment (Figure 1). Again, we observe a small but significant effects of learning on preference manipulation under SH and the DA.

Table 8: Proportion of Nash Equilibrium Outcomes

| 4-School | BOS ($\mu^{C/S}$) | SH ($\mu^{C/S}$) | DA | DA ($\mu^{C/S}$) | DA (μ^*) | H_a | p-value |
|-----------|---------------------|--------------------|-------|--------------------|----------------|-----------------------------------|---------|
| Session 1 | 0.683 | 0.933 | 0.967 | 0.950 | 0.017 | BOS \neq SH | 0.028 |
| Session 2 | 0.600 | 0.817 | 0.850 | 0.717 | 0.133 | BOS < DA | 0.014 |
| Session 3 | 0.600 | 0.867 | 0.817 | 0.800 | 0.017 | SH < DA | 0.457 |
| Session 4 | 0.533 | 0.633 | 0.950 | 0.833 | 0.117 | DA(μ^*) < DA($\mu^{C/S}$) | 0.063 |
| 6-School | BOS | SH | DA | DA(Stable) | DA(Unstable) | H_a | p-value |
| Session 1 | 0.011 | 0.122 | 0.822 | 0.811 | 0.011 | BOS \neq SH | 0.028 |
| Session 2 | 0.011 | 0.267 | 0.778 | 0.778 | 0.000 | BOS < DA | 0.014 |
| Session 3 | 0.033 | 0.189 | 0.844 | 0.789 | 0.056 | SH < DA | 0.014 |
| Session 4 | 0.078 | 0.222 | 0.711 | 0.644 | 0.067 | DA(unstable) < DA(stable) | 0.063 |

Table 8 reports session-level statistics for each mechanism and pairwise comparisons between mechanisms and outcomes, using each session as an observation.

Table 9: Normalized Efficiency: First Block, Last Block and All Periods

| | First Block (periods 1-5) | | | Last Block | | | All Periods | | |
|-----------|---------------------------|-------|-------|------------|-------|-------|-------------|-------|-------|
| 4-school | BOS | SH | DA | BOS | SH | DA | BOS | SH | DA |
| Session 1 | 0.733 | 0.750 | 0.750 | 0.733 | 0.817 | 0.767 | 0.721 | 0.767 | 0.752 |
| Session 2 | 0.742 | 0.758 | 0.733 | 0.758 | 0.733 | 0.808 | 0.744 | 0.752 | 0.777 |
| Session 3 | 0.742 | 0.758 | 0.750 | 0.742 | 0.775 | 0.775 | 0.733 | 0.775 | 0.748 |
| Session 4 | 0.683 | 0.717 | 0.742 | 0.767 | 0.758 | 0.833 | 0.727 | 0.746 | 0.777 |
| 6-school | BOS | SH | DA | BOS | SH | DA | BOS | SH | DA |
| Session 1 | 0.870 | 0.887 | 0.800 | 0.773 | 0.753 | 0.567 | 0.849 | 0.805 | 0.676 |
| Session 2 | 0.850 | 0.820 | 0.807 | 0.780 | 0.597 | 0.593 | 0.850 | 0.714 | 0.685 |
| Session 3 | 0.910 | 0.907 | 0.850 | 0.740 | 0.710 | 0.560 | 0.810 | 0.801 | 0.679 |
| Session 4 | 0.890 | 0.893 | 0.817 | 0.767 | 0.777 | 0.717 | 0.828 | 0.792 | 0.720 |

Table 9 reports session-level normalized efficiency for the first and last blocks, as well as the average efficiency over all periods.

Table 10 reports the proportion of stable allocations among all allocations in the first and last block, and averaged over all periods in each session.

Table 10: Stability: First Block, Last Block and All Periods

| 4-school | First Block (periods 1-5) | | | Last Block | | | All Periods | | |
|-----------|---------------------------|-------|-------|------------|-------|-------|-------------|-------|-------|
| | BOS | SH | DA | BOS | SH | DA | BOS | SH | DA |
| Session 1 | 0.733 | 1.000 | 1.000 | 0.533 | 0.733 | 0.867 | 0.683 | 0.933 | 0.950 |
| Session 2 | 0.533 | 0.867 | 0.733 | 0.333 | 0.867 | 0.733 | 0.600 | 0.817 | 0.717 |
| Session 3 | 0.800 | 0.867 | 0.933 | 0.400 | 0.867 | 0.600 | 0.600 | 0.867 | 0.800 |
| Session 4 | 0.467 | 0.667 | 0.933 | 0.400 | 0.667 | 0.667 | 0.533 | 0.633 | 0.833 |
| 6-school | BOS | SH | DA | BOS | SH | DA | BOS | SH | DA |
| Session 1 | 0.000 | 0.067 | 0.800 | 0.000 | 0.000 | 0.867 | 0.011 | 0.122 | 0.811 |
| Session 2 | 0.000 | 0.200 | 0.600 | 0.000 | 0.200 | 0.867 | 0.011 | 0.267 | 0.778 |
| Session 3 | 0.000 | 0.067 | 0.333 | 0.000 | 0.133 | 0.933 | 0.033 | 0.189 | 0.789 |
| Session 4 | 0.133 | 0.333 | 0.467 | 0.000 | 0.000 | 0.467 | 0.078 | 0.222 | 0.644 |