From Boston to Shanghai to Deferred Acceptance:
Theory and Experiments on A Family of School Choice Mechanisms *

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Abstract

We characterize a family of application-rejection school choice mechanisms, including the Boston, Shanghai, and Deferred Acceptance mechanisms as special cases, and spanning the parallel mechanisms for Chinese college admissions, the largest centralized matching in the world. Moving from one extreme member to the other results in systematic changes in manipulability and nested Nash equilibria. In the laboratory, participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the Shanghai and then the Boston mechanisms. Furthermore, while DA is significantly more stable than Shanghai, which is more stable than Boston, efficiency comparisons vary across environments.

Keywords: school choice, Boston mechanism, Shanghai mechanism, deferred acceptance, experiment

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1 Introduction

School choice has been one of the most important and widely-debated education policies in the past two decades (Hoxby 2003), with game theory playing a major role in the adoption of school choice mechanisms. Some school districts have reexamined their school choice mechanisms after game theoretic analysis (Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006) and experimental evidence (Chen and Sönmez 2006) indicated that one of the most popular school choice mechanisms, the Boston mechanism, is vulnerable to strategic manipulation and thus might not result in efficient allocations. Following intensive policy discussions, in 2003, New York City public schools decided to replace its allocation mechanism with a version of the student-proposing deferred acceptance (DA) mechanism (Gale and Shapley 1962, Abdulkadiroğlu, Pathak and Roth 2005b). Similarly, in 2005, the Boston Public School Committee voted to replace the existing Boston school choice mechanism with the deferred acceptance mechanism (Abdulkadiroğlu, Pathak, Roth and Sönmez 2005a).

Despite the concern regarding potential manipulation, some recent literature on school choice has provided a more optimistic view of the Boston mechanism and highlighted some virtues of the Boston mechanism. Under certain restrictions, Abdulkadiroğlu, Che and Yasuda (2011) and Miralles (2009) emphasize possible ex ante welfare advantages of the Boston mechanism compared to DA. Featherstone and Niederle (2008) confirm these predictions in the laboratory. In a similar vein, Kojima and Ünver (2010) offer axiomatic characterizations of the Boston mechanism, whereas Kesten (2011) shows that, contrary to DA, the Boston mechanism is immune to manipulation attempts by schools through concealing capacity.

In this paper, we strive to better understand the sources of these different points of view regarding the Boston mechanism and its comparison to DA. Specifically, we ask: how do the efficiency-incentive-stability trade-offs change when transitioning from the Boston mechanism to a mechanism such as the DA? A mechanism that we believe could provide key insights to this question is one pioneered in Shanghai for high school admissions,[1] and later adopted by more than half of the provinces in Chinese college admissions. In the latter context, it is called the parallel mechanism.

Like school choice in the United States, college admissions are among the most intensively discussed public policies in the past thirty years in China. Each year more than 10 million high school seniors compete for approximately 6 million seats at various universities in China. To our knowledge, this annual event is the largest centralized matching process in the world. The matching of students to universities has profound implications for the education and labor market outcomes of these students. For matching theorists and experimentalists, the regional variations

[1]This mechanism was adopted in Shanghai for high school admissions in 2003, `http://edu.sina.com.cn/l/2003-05-15/42912.html`, retrieved on February 2, 2011.

of matching mechanisms and their evolution over time provide a wealth of field observations which can enrich our understanding of matching mechanisms. This paper provides the first theoretical characterization and experimental investigation of the major Chinese college admissions (CCA) mechanisms.

The CCA mechanisms are centralized matching processes via standardized tests, with each province implementing an independent matching process. These matching mechanisms fall into three classes: sequential, parallel, and partial parallel. The sequential mechanism is a priority matching mechanism similar to the Boston mechanism, but executed sequentially across tiers in decreasing prestige. In the sequential mechanism, each college belongs to a tier. Within each tier, the Boston mechanism is used. When assignments in the first tier are finalized, the assignment process in the second tier starts, and so on. A common complaint about the sequential mechanism is that "a good score in the college entrance exam is worth less than a good strategy in the ranking of colleges" (Nie 2007). In response to the college admissions reform survey conducted by the Beijing branch of the National Statistics Bureau in 2006, a parent complained:

> My child has been among the best students in his school and school district. He achieved a score of 632 in the entrance exam last year. Unfortunately, he was not accepted by his first choice. After his first choice rejected him, his second and third choices were already full. My child had no choice but to repeat his senior year.[2]

To alleviate the problem of high-scoring students not accepted by any universities and the pressure to manipulate preference rankings under the sequential mechanism, the parallel mechanism has been adopted by more than half of the provinces. In the parallel mechanism, students can place several "parallel" colleges for each choice. For example, a student's first choice can contain four colleges, A, B, C and D, in decreasing desirability. Colleges consider student applications for a pre-determined number of steps, where allocations among the parallel colleges are temporary until the end of the pre-determined steps. Thus, this mechanism lies between the Boston mechanism, where every step is final, and DA, where every step is temporary until all seats are filled.[3] This mechanism is widely perceived to improve allocation outcomes and adopted by an increasing number of provinces. An interview with a parent in Beijing underscores the incentives to manipulate the first choice under the sequential versus the parallel mechanisms:[4]

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[2]Source: `http://www.bjstats.gov.cn/ldcxxt/tjfx/tjbg/200606/t20060626_44830.htm`, retrieved on August 22, 2011.

[3]An alternative interpretation of the parallel mechanism is that it approximates serial dictatorship with tiers (Wei 2009). Note, however, in the college admissions context when colleges have identical preferences, serial dictatorship and DA are equivalent.

[4]Li Li, 2009. "Ten More Provinces Switch to Parallel College Admissions Mechanism This Year." *Beijing Evening News*, June 8.

> My child really wanted to go to Tsinghua University. However, ..., in order not to take any risks, we unwillingly listed a less prestigious university as her first choice. Had Beijing allowed parallel colleges as the first choice, we could at least give [Tsinghua] a try.

While variants of the parallel mechanisms, each of which differs in the number of pre-determined steps, have been implemented in different provinces, to our knowledge, they have not been theoretically studied or tested in the laboratory. In particular, when the number of pre-determined steps varies, how do manipulation incentives, allocation efficiency and stability change? In this paper, we investigate this question both theoretically and experimentally. We call the entire class of parallel mechanisms the *Chinese parallel mechanisms*, the simplest member of this class the *Shanghai mechanism*.

To study the performance of the different mechanisms more formally, we first provide a theoretical analysis and present a family of *application-rejection* mechanisms where each member is characterized by some positive number $e \in \{1, 2, \ldots, \infty\}$ of periodic steps through which the application and rejection process continues before assignments are made permanent. More precisely, the mechanisms work as follows: During steps 1 through e, students apply to schools in order of reported preference from the most preferred to the least, and schools tentatively admit applicants up to their capacity in order of priority going from the highest to the lowest. At the end of step e, students tentatively held at a school are permanently accepted into that school. The remaining students go through a similar application and rejection process from steps $e + 1$ through $2e$. The process continues in this periodic fashion until no student remains unassigned.

It is quite easy to see that as e increases, we go from the familiar Boston mechanism ($e = 1$) to the Chinese parallel mechanisms ($e \in [2, \infty)$) which include the Shanghai mechanism ($e = 2$), and from those to the DA ($e = \infty$). In this framework, we find that, as one moves from one extreme member of this family to the other, the experienced trade-offs are in terms of efficiency, stability and strategic immunity. We provide property-based rankings of the members of this family following the works of Kesten (2006, 2011) and Pathak and Sönmez (2011). Specifically, we show that members of this family can be ranked according to their immunity against strategic manipulations. Under certain restrictions on the problem domain, any given member is *more manipulable* than a member with a higher e number. On the welfare side, a more subtle comparison emerges. The number of students receiving their reported first choices diminishes with an increasing e. As far as stability or Pareto efficiency is concerned, the ranking is ambiguous within the general class of mechanisms. While the Shanghai mechanism is *more stable* than the Boston mechanism, the latter is *more efficient* than the Shanghai mechanism.[5] Furthermore, we show that the set of Nash equilibrium outcomes of the preference revelation game induced by the

[5] More precisely, if the outcome of the Boston mechanism is stable at a given problem, then the outcome of the

Boston mechanism is equal to that induced by the Shanghai mechanism, which is equal to the stable set (under students' true preferences).

Since the theoretical welfare ranking in this family of mechanisms assumes truthtelling, which is a dominant strategy only under DA, it is important to assess the behavioral responses to members of this family. Furthermore, because of the multiplicity of Nash equilibrium outcomes in this family of mechanisms, empirical evaluations of the performance of these mechanisms in controlled laboratory settings will inform policymakers in school choice or college admissions reform.

For these reasons, we evaluate three members of this family in two environments in the laboratory. In both environments, we find that participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the Shanghai and then the Boston mechanisms. Consistent with theory, DA achieves a significantly higher proportion of stable outcomes than Shanghai, which is more stable than Boston. However, the efficiency comparison is sensitive to the environment. While theory is silent on equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than the unstable ones. To our knowledge, our paper presents the first experimental evaluation of the Shanghai mechanism relative to Boston and DA, as well as equilibrium selection in school choice mechanisms.

The rest of this paper is organized as follows. Section 2 formally introduces the school choice problem and the family of mechanisms. Section 3 presents the theoretical results. Section 4 describes the experimental design. Section 5 summarizes the results of the experiments. Section 6 concludes.

2 School choice problem and the three mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) consists of a number of students each of whom is to be assigned a seat at one of a number of schools. Further, each school has a maximum capacity, and the total number of seats in the schools is no less than the number of students. We denote the set of students by $I \equiv \{i_1, i_2, \ldots, i_n\}$. A generic element in I is denoted by i. Likewise, we denote the set of schools by $S \equiv \{s_1, s_2, \ldots, s_m\} \cup \{\emptyset\}$ where $\emptyset$ denotes a student's outside option, or the so-called null school. A generic element in S is denoted by s. Each school has a number of available seats. Let q_s be the number of available seats at school s, or the **quota** of s. Let $q_\emptyset = \infty$. For each school, there is a strict priority order of all students, and each student has strict preferences over all schools. The priority orders are determined according to state or local laws as well as the criteria of school districts. We denote the priority order

Shanghai mechanism is also stable at the same problem, while the converse statement is not necessarily true. As far as Pareto efficiency is concerned, the positions of the two mechanisms in this statement are switched. See Kesten (2006, 2011) for similar problem-wise property comparisons across and within mechanisms for matching problems.

for school s by $\succ_s$, and the preferences of student i by P_i. Let R_i denote the at-least-as-good-as relation associated with P_i. Formally, we assume that R_i is a linear order, i.e., a complete, transitive, and anti-symmetric binary relation on S. That is, for any $s, s' \in S$, $s \; R_i \; s'$ if and only if $s = s'$ or $s \; P_i \; s'$.

A **school choice problem**, or simply a problem, is a pair $(\succ = (\succ_s)_{s \in S}, P = (P_i)_{i \in I})$ consisting of a collection of priority orders and a preference profile. Let $\mathcal{R}$ be the set of all problems. A **matching** μ is a list of assignments such that each student is placed at one school and the number of students placed to a particular school does not exceed the quota of that school. Formally, it is a function $\mu : I \rightarrow S$ such that for each $s \in S$, $|\mu^{-1}(s)| \leq q_s$. Given $i \in I$, $\mu(i)$ denotes the assignment of student i at μ and given $s \in S$, $\mu^{-1}(s)$ denotes the set of students assigned to school s at μ. Let $\mathcal{M}$ be the set of all matchings. A matching μ is **non-wasteful** if no student prefers a school with unfilled quota to her assignment. Formally, for all $i \in I$, $s \; P_i \; \mu(i)$ implies $|\mu^{-1}(s)| = q_s$. A matching μ is **Pareto efficient** if there is no other matching which makes all students at least as well off and at least one student better off. Formally, there is no $\alpha \in \mathcal{M}$ such that $\alpha(i) \; R_i \; \mu(i)$ for all $i \in I$ and $\alpha(j) \; P_j \; \mu(j)$ for some $j \in I$.

A closely related problem to the school choice problem is the *college admissions problem* (Gale and Shapley 1962). In the college admissions problem, schools have preferences over students whereas in a school choice problem, schools are merely objects to be consumed. A key concept in college admissions is "stability," i.e., there is no unmatched student-school pair (i, s) such that student i prefers school s to the school he is assigned to, and school s either has not filled its quota or prefers student i to at least one student who is assigned to it. The natural counterpart of stability in our context is defined by Balinski and Sönmez (1999). The **priority of student i for school s is violated** at a given matching μ (or alternatively, **student i justifiably envies student j for school s**) if i would rather be assigned to s to which some student j is assigned who has lower $s-$priority than i, i.e., $s \; P_i \; \mu(i)$ and $i \succ_s j$ for some $j \in I$. A matching is **stable** if it is non-wasteful and no student's priority for any school is violated.

A **school choice mechanism**, or simply a mechanism φ, is a systematic procedure that chooses a matching for each problem. Formally, it is a function $\varphi : \mathcal{R} \rightarrow \mathcal{M}$. Let $\varphi(\succ, P)$ denote the matching chosen by φ for the problem $(\succ, P)$ and let $\varphi_i(\succ, P)$ denote the assignment of student i at this matching. A mechanism is Pareto efficient (stable) if it always selects Pareto efficient matchings. A mechanism φ is **strategy-proof** if it is a dominant strategy for each student to truthfully report her preferences. Formally, for every problem $(\succ, P)$, every student $i \in I$, and every report P_i', $\varphi_i(\succ, P) \; R_i \; \varphi_i(\succ, P_i', P_{-i})$.

We now describe the three mechanisms that are central to our study. The first two are the familiar Boston and DA mechanisms, while the third one is a stylized version of the simplest parallel mechanism.

2.1 Boston Mechanism (BOS)

Our first mechanism is the most common school choice mechanism observed in practice. Its outcome can be calculated via the following algorithm for a given problem:

Step 1: For each school s, consider only those students admissable to it who have listed it as their first acceptable choice. Up to q_s students among them with the highest priority for school s (all students if fewer than q_s) are placed to school s.

Step k, $k \geq 2$: Consider the remaining students. For each school s with q_s^k available seats, consider only those students admissable to it who have listed it as their k-th acceptable choice. Those q_s^k students among them with the highest priority for school s (all students if fewer than q_s^k) are placed to school s.

The algorithm terminates when there are no students or schools left. Any student who is not placed to any school acceptable to her remains unassigned. Note that the assignments in each step are final. An important critique of the Boston mechanism highlighted in the literature is that it gives students strong incentives to misrepresent their preferences. Because a student who has high priority for a school may lose her advantage for that school if she does not list it as her first choice, the Boston mechanism forces students to make hard and risky strategic choices (see e.g., Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006, and Chen and Sönmez 2006).

2.2 Deferred Acceptance Mechanism (DA)

A second matching mechanism is the student-optimal stable mechanism (Gale and Shapley 1962), which finds the stable (*stable* in our context) allocation that is most favorable to each student for any given two-sided matching problem (*school choice problem* in our context). Its outcome can be calculated via the following *deferred acceptance algorithm* for a given problem:

Step 1: Each student applies to her favorite school. For each school s, up to q_s applicants admissable to it who have the highest priority for school s are tentatively placed to school s. The remaining applicants are rejected.

Step k, $k \geq 2$: Each student rejected from a school at step $k-1$ applies to her next favorite school. For each school s, up to q_s students admissable to it who have the highest priority for school s among the new applicants, and those tentatively on hold from an earlier step, are tentatively placed at school s. The remaining applicants are rejected.

The algorithm terminates when each student is either tentatively placed to a school, or has been rejected by every school acceptable to him. Note that, in DA, assignments in each step are temporary until the last step. DA has several desirable theoretical properties, most notably in terms of incentives and stability. Under DA, it is a dominant strategy for students to state their true preferences (Roth 1982, Dubins and Freedman 1981). Furthermore, it is stable. Although it is not Pareto efficient, it is the most efficient among the stable school choice mechanisms.

In practice, DA has been the leading mechanism for school choice reforms. For example, DA has been adopted by New York City and Boston public school systems, which had suffered from congestion and incentive problems from their previous assignment systems, respectively (Abdulkadiroğlu et al. 2005a, Abdulkadiroğlu et al. 2005b).

2.3 Shanghai Mechanism (SH)

The Shanghai mechanism was first implemented as a high school admissions mechanism in Shanghai in 2003. From 2003 to 2009, variants of the mechanism have been adopted by 19 provinces as the parallel (or partial parallel) college admissions mechanisms to replace the sequential mechanisms (Wu and Zhong 2011), which corresponds to the Boston mechanism with tiers.[6]

The Chinese college admissions mechanisms (CCA) are centralized matching processes via standardized tests. Each summer, every high school senior who wishes to enter college takes the National College Entrance Examination, which is the dominant determinant for admissions to all Chinese colleges and universities (hereafter colleges). Since each college has a different quota and admissions threshhold for students from different provinces, students from a given province compete only with each other for admission to a particular college. Effectively, each province implements an independent matching process.

While there are many regional variations in CCA, from a game theoretic perspective, however, they differ in two main dimensions which impact the students' strategic decisions during the application process. The first dimension is the timing of submitting the rank order list, including before the exam (9 provinces), after the exam but *before* knowing the exam scores (1 province), and *after* knowing the exam scores (21 provinces). The second dimension is the actual matching mechanisms used in each province. The sequential mechanism used to be the only college admissions mechanism used in China. In 2003, the parallel mechanism was introduced. In 2009, the sequential mechanism was still used in 8 provinces, the parallel mechanism in 19 provinces, while the rest uses the hybrid partial parallel mechanism.

[6]The college admissions process is executed sequentially by tiers. Within each tier, universities are again divided into subtiers. Here is a typical 4-tier division: Tier 0 includes art institutes, military academies, normal universities, and Hong Kong universities. Tier 1 consists of four-year universities and colleges, which are in turn divided into subtiers, usually by the prestige of the colleges. Tier 2 consists of arts junior colleges, while Tier 3 consists of other junior colleges.

In this study, we use the following stylized version of the parallel mechanisms, adapted for the school choice context.

- An application to the first ranked school is sent for each student at the first step.
- Throughout the allocation process, a school can hold no more applications than its capacity.

  If a school receives more applications than its capacity, it retains the students with the highest priority up to its capacity and rejects the remaining students.
- Whenever a student is rejected at a school, her application is sent to the next highest-ranked school at the next step.
- Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the ones with the highest priority up to its capacity are retained.
- The allocation is finalized every e steps. That is, in steps e, $2e$ and $3e$ etc., each retained applicant is placed to the school that holds her application in that step. These students and their assignments are removed from the system.

The allocation process ends when no more applications can be rejected. The parallel mechanisms are a hybrid between the Boston mechanism where every step is final, and DA where every step is temporary until the last step. In the next section, we offer a formal definition of the parallel mechanisms and characterize the theoretical properties of this family of matching mechanisms.

3 Theoretical Analysis

In this section, we characterize the theoretical properties of the family of application-rejection mechanisms. All proofs and examples are relegated to Appendix A. Given (strict) student preferences, (strict) school priorities, and fixed school quotas, consider the following general *application-rejection algorithm* that indexes each member of the family by a *permanency-execution period* e:

Round $\mathbf{r = 1}$:

- *Step 1:* Each student applies to her first choice. Each school s then considers its applicants. Those students with the highest s-priority are *tentatively* assigned to school s up to the quota of school s. The rest are rejected.

$$\vdots$$

- *Step k, $k < e$:* Each student rejected in the previous step applies to her next choice school. Each school s considers its applicants in this round together with those students who have been tentatively assigned to it in an earlier round (if any). Those students with the highest s-priority are *tentatively* assigned to school s up to the quota of school s. The rest are rejected.

$$\vdots$$

- *Step e:* Each student rejected in the previous step applies to her next choice school. Each school s considers its applicants in this round together with those students who have been tentatively assigned to it in an earlier round (if any). Those students with the highest s-priority are *permanently* assigned to school s up to the quota of school s. The rest are rejected. Each school's quota is reduced by the number of students who are permanently assigned to it.

$$\vdots$$

In general,

Round r $\geq$ 2:

- *Step $(r-1)e+1$:* Each student unassigned in the previous round applies to her next choice school. Each school s considers its applicants in this step. Those students with the highest s-priority are *tentatively* assigned to school s up to the quota of school s. The rest are rejected.

$$\vdots$$

- *Step k, with $k \neq 0\ (mod\ e)$:* Each student rejected in the previous step applies to her next choice school. Each school s considers its applicants in this step together with those students who have been tentatively assigned to it in an earlier step (if any). Those students with the highest s-priority are *tentatively* assigned to school s up to the quota of school s. The rest are rejected.

$$\vdots$$

- *Step k, with $k \equiv 0\ (mod\ e)$:* Each student rejected in the previous step applies to her next choice school. Each school s considers its applicants in this step together with those students who have been tentatively assigned to it in an earlier step (if any). Those students with the highest s-priority are *permanently* assigned to school s up to the quota of school s. The rest are rejected. Each school's quota is reduced by the number of students who are permanently assigned to it.

The algorithm terminates at the earliest step when each student is assigned to some school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* (e) and denoted by φ^e. This family of mechanisms nest the Boston and DA mechanisms as extreme cases, and the Chinese parallel mechanisms as intermediate cases.

Remark 1 *The application-rejection mechanism* (e) *coincides with*

(i) the Boston mechanism when $e = 1$,

(ii) the Shanghai mechanism when $e = 2$,

(iii) the Chinese parallel mechanism when $2 \leq e < \infty$, *and*

(iv) the DA mechanism when $e = \infty$.

Remark 2 *It is easy to see that all members of the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$, *are non-wasteful. Hence, the outcome of an application-rejection mechanism is stable for a given problem if and only if it does not result in a priority violation.*

Next is our first observation about the properties of this family mechanisms.

Lemma 1 *Within the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$,

(i) there is exactly one member that is Pareto efficient. This is the Boston mechanism;

(ii) there is exactly one member that is strategy-proof. This is the DA mechanism; and

(iii) there is exactly one member that is stable. This is the DA mechanism.

Notice that part (i) of Lemma 1 refers to Pareto efficiency with respect to reported preferences. Since the Boston mechanism is not strategy-proof, we expect that efficiency with respect to reported and true preferences will be different. Furthermore, following the convention in the matching literature, our notion of Pareto efficiency is with respect to ordinal rather than cardinal preferences.

3.1 Manipulation Strategies for the Application-Rejection Mechanisms

As Lemma 1 shows, an application-rejection (e) mechanism is manipulable if $e < \infty$. Hence students need to make careful judgments to determine their optimal strategies. Since tentative assignments are made permanent at the end of each round for each member of the family with $e < \infty$, ultimately what matters for a student is where she is assigned at the end of step e, $2e$, $3e$ etc. In particular, an optimal strategy in practice is one that ensures that a student is assigned to her "target school" at the end of step e of the first round.

In general, listing a school high on a preference list does not necessarily increase a student's chance of being assigned to that school. This is because an early application to a school may in fact initiate a rejection chain that may cause the early applicant to be rejected from that school before step e (see Example 1a in Appendix A). On the other hand, if a student is unassigned at the end of the first round, then she has already lost her priority for the schools she has not yet applied to (to students who did so in the first round). Therefore, a good strategy needs to optimally trade off the advantage from listing a school high in the preference list against the disadvantage of being rejected from that school at step e. To elaborate on these points, we present two ways an application-rejection mechanism with $e < \infty$ may be manipulated in Examples 1a (within-round manipulation) and 1b (across-round manipulation) in Appendix A.

We next provide an incentive-based ranking of the family of application-rejection mechanisms. First, we introduce two additional domains of school choice problems due to their relevance for Chinese college admissions: one larger than $\mathcal{R}$ and the other smaller. The first domain extends the strategy space to allow students to leave some ranks on their preference lists unfilled. Submissions of this kind of preference lists are currently observed in many provinces, where student admission to colleges follows a tier-based system (Section 2.3). Under such a system, when student preferences are not aligned with college tiers, submission of preference lists with unfilled ranks may prove to be a successful manipulation strategy.[7] Let us denote $\mathcal{R}^* \supset \mathcal{R}$ as the set of all school choice problems in which a student has the option of not listing any school at certain preference ranks of her choice.[8]

We also consider a second domain, which is a restriction of $\mathcal{R}$ to problems at which the priority order for each school is the same. This is motivated by an important feature of the Chinese college admissions, namely, student priorities are determined through a standardized college entrance examination in every province.[9] Let us denote $\mathcal{R}^{**} \subset \mathcal{R}$ as the set of all school choice problems in which there is a single priority order that applies to all schools.

Kesten (2006, 2011) offers two kinds of problem-wise comparisons of school choice mechanisms. These are "property-specific" and "cross-property" comparisons. For example, Kesten (2006) shows that at any problem for which the celebrated *top trading cycles (TTC)* mechanism (Abdulkadiroğlu and Sönmez 2003) satisfies a *consistency*[10] property, then DA also satisfies the same property at the same problem while the converse is not necessarily true.[11] Recently, Pathak

[7]For example, for the initial problem given in Example 1a, consider an alternative preference list of student i_1 in which she leaves her first two choices unfilled and lists school s_1 as her third choice. By submitting such a preference list, student i_1 would remain unassigned for the first two steps and ensure herself, in the third step, a better assignment than her assignment when she submits P_{i_1}, i.e., school s_1 instead of school s_4.

[8]To be more precise, while applying the application-rejection algorithm on $\mathcal{R}^*$, whenever a student has an unfilled position in her preference list as her next option, we assume that she remains unassigned during that step and at the next step she applies to the next school on her preference list.

[9]Other considerations, such as affirmative action for minority students, are incorporated into the exam score by adding up to 20 points to the eligible student's score.

[10]*Consistency* requires that upon the departure of a group of agents with their assignments, the recommendation of a mechanism to the remaining agents for the reduced problem should agree with the initial one.

[11]A cross-property comparison is based on the idea of problem-wise comparisons of different properties. This

and Sönmez (2011) offer notions of "manipulability" to compare mechanisms based on a similar idea of making problem-wise comparisons of student incentives. In our subsequent theoretical analysis, following the works by Kesten (2006, 2011)[12] and Pathak and Sönmez (2011), we will show that it may be possible to obtain a ranking among the members of the application-rejection family based on a specific property.

We next define a notion of "manipulability" following Pathak and Sönmez (2011). A mechanism ϕ is *manipulable by student* j at problem $(\succ, P)$ if there exists P'_j such that $\phi_j(\succ, P'_j, P_{-j})$ P_j $\phi_j(\succ, P)$. Thus, mechanism ϕ is said to be *manipulable* at a problem $(\succ, P)$ if there exists some student j such that ϕ is manipulable by student j at $(\succ, P)$. Mechanism φ is **more manipulable** than mechanism ϕ if (i) at any problem ϕ is manipulable, φ is also manipulable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is manipulable but ϕ is not.[13]

Theorem 1 (Manipulability) *(i) On $\mathcal{R}$, the Boston mechanism is more manipulable than the Shanghai mechanism, which, in turn, is more manipulable than DA.*
(ii) On $\mathcal{R}^*$ and $\mathcal{R}^{**}$, an application-rejection mechanism (e') is more manipulable than an application-rejection mechanism (e) with $e > e'$.

Part (i) of Theorem 1 pertains to the comparison between the three special members of the family and applies to problems in the domain $\mathcal{R}$. However, because of the more sophisticated strategic environment that emerges when $e > 2$, part (ii) of Theorem 1 holds only on $\mathcal{R}^*$ and $\mathcal{R}^{**}$. To provide some insight into this result, we present Examples 2a and 2b in Appendix A.

The next result enables us to see the efficiency-incentive tension within the class of mechanisms. When true preferences are not observable, a plausible metric for evaluating student welfare under an assignment mechanism is based on the number of students assigned to their reported first choices. For example, in evaluating the outcome of the Boston mechanism, Cookson Jr. (1994) reports that 75% of all students entering the Cambridge public school system at the K-8 levels gained admission to the school of their first choice. Similarly, the analysis of the

kind of a comparison may be particularly helpful when there is an inherent tension between certain properties. Two such properties in the school choice context are stability and Pareto efficiency. DA is stable but not Pareto efficient whereas TTC is Pareto efficient but not stable. Kesten (2006) shows that at any problem for which the DA outcome violates Pareto efficiency, the TTC outcome violates stability while a converse statement does not necessarily hold.

[12] Kesten (2011) deals with a problem-wise comparison of DA with itself in terms of *manipulation via capacities* under different restrictions in a two-sided school choice context. In particular, Kesten (2011) shows that if a school manipulates DA via capacities at a problem by underreporting its capacity, then the same school can manipulate DA via capacities at the same problem when a smaller minimum capacity is imposed. However, the converse is not necessarily true.

[13] Note that this definition allows the two mechanisms to be manipulated by different students for a given problem. Pathak and Sönmez (2011) also consider a stronger notion which is based on the requirement that the manipulating student be the same for both mechanisms.

Boston and NYC school district data by Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) and Abdulkadiroğlu, Pathak and Roth (2009) also report the number of first choices of students.

As a way to assess student welfare, we next present a ranking of the mechanisms within the family based on the number of (reported) first choices they assign. It turns out that the Boston mechanism is the most generous in terms of first choice accommodation, whereas the DA is the least. Hence, the next result suggests that, within the family of application-rejection mechanisms, the decrease in the possibility of manipulation with an increasing e parameter may come at the cost of a diminishing number of first choice assignments. In the remainder of this section, we assume that $\mathcal{R}$ is the default problem domain unless otherwise specified.

Proposition 1 (First Choice Accommodation) *Given any school choice problem, the application-rejection* (e) *mechanism assigns a (weakly) higher number of students to their (reported) first choices than the application-rejection* (e') *mechanism with* $e' > e$.

Nonetheless, one needs to be cautious when interpreting Proposition 1. Since all members of the family with the exception of DA violate strategy-proofness, student preference submission strategies may also vary across mechanisms.

We now turn to investigate a possible ranking of the members of the family based on stability and Pareto efficiency. Mechanism φ is **more stable (more efficient)** than mechanism ϕ if (i) at any problem ϕ is stable (Pareto efficient), φ is also stable (Pareto efficient); and (ii) the converse is not always true, i.e., there is at least one problem at which φ is stable (Pareto efficient) but ϕ is not.

Proposition 2 (Stability and Pareto Efficiency) *Consider any school choice problem.*
(i) DA is more stable than Shanghai, which in turn is more stable than Boston, whereas Boston is more efficient than Shanghai.
(ii) an application-rejection (e) *mechanism with* $2 < e < \infty$ *is not necessarily more stable than an application-rejection mechanism* (e') *with* $e' < e$.
(iii) an application-rejection (e) *mechanism with* $2 \leq e < \infty$ *is not necessarily more efficient than an application-rejection mechanism* (e') *with* $e < e'$.

Proposition 2 indicates that while it is possible to rank the three special members of the family of application-rejection mechanisms, i.e., $e \in \{1, 2, \infty\}$, according to the stability of their outcomes, within the Chinese parallel mechanisms, however, there is no problem-wise systematic ranking in terms of stability or Pareto efficiency .

3.2 The Nash Equilibria of the Induced Preference Revelation Games

Ergin and Sönmez (2006) show that every Nash equilibrium outcome under the preference revelation game induced by the Boston mechanism leads to a stable matching under students' true preferences and that any given stable matching can be sustained as a Nash equlibrium of this game. Hence, if one assumes that all students are strategic and able to coordinate their actions to achieve an equilibrium outcome, then this result has a clear implication. Since DA is strategy-proof and chooses the most favorable stable matching for students, the Boston mechanism can *at best* be as good as DA in terms of the resulting welfare.

To analyze the properties of the equilibrium outcomes of the application-rejection mechanisms, we next study the Nash equilibrium outcomes induced by the preference revelation games under this family of mechanisms. We start with an important implication of Theorem 1: the set of Nash equilibrium strategies corresponding to the preference revelation games associated with members of the application-rejection family has a nested structure.[14]

Corollary 1 (Nested Nash Equilibria) *(i) On $\mathcal{R}$, any Nash equilibrium (of the preference revelation game) under Boston is also a Nash equilibrium under Shanghai; and any Nash equilibrium under Shanghai is also a Nash equilibrium under DA.*
(ii) On $\mathcal{R}^*$ and $\mathcal{R}^{**}$, any Nash equilibrium of the application-rejection (e) mechanism is also a Nash equilibrium of the preference revelation game of the application-rejection (e') mechanism with $e' > e$.

Hence, for any given problem, DA has the largest set of equilibrium profiles within the entire family, whereas Boston has the smallest. Consequently, this observation entails that coordination issues may become more serious for the Chinese parallel mechanisms as e increases. Interestingly, this finding suggests that DA would be subject to a more difficult coordination problem than Boston if not for its strategy-proofness property.

The following result asserts that the link between equilibrium play and the stability of the outcomes pointed out by Ergin and Sönmez (2006) for the Boston mechanism, continues to hold for exactly one more member of the application-rejection family. This is the Shanghai mechanism.

Theorem 2 (Nash Equilibria and Stability) *Consider the preference revelation game induced by the application-rejection (e) mechanism. Let P_I be the list of true student preferences.*
(i) If $e \in \{1, 2\}$, then the set of Nash equilibrium outcomes of this game is equal to the set of stable matchings under the true preferences P_I.

[14]A similar observation is made by Haeringer and Klijn (2008) for the revelation games under the Boston mechanism when the number of school choices a student can make (in her preference list) is limited by a quota.

(ii) If $e \notin \{1, 2\}$, there exist Nash equilibrium outcomes of this game which are unstable under the true preferences P_I.
(iii) If $e \notin \{1, 2, \infty\}$, there exist stable outcomes of this game under the true preferences P_I which correspond to non-equilibrium strategy-profiles.

Remark 3 *On $\mathcal{R}^*$, Lemma 2 (see Appendix A) implies that all Nash equilibrium outcomes of all application-rejection mechanisms except DA are stable. It is easy to see that on $\mathcal{R}^{**}$ the Nash equilibrium outcome of any application-rejection mechanism is the stable DA matching under the true preferences P_I.*

An immediate implication of Theorem 2 is that the Boston mechanism, the Shanghai mechanism and DA are the only members of the family for which a stable matching can arise only as a Nash equilibrium outcome.

Corollary 2 *Let μ be a stable matching (under the true preferences) given by the application-rejection (e) mechanism for a reported preference profile Q. If $e \in \{1, 2, \infty\}$, then Q is a Nash equilibrium profile.*

3.3 Discussion of Theoretical Results

Overall, our theoretical analysis indicates that the incentive and welfare properties vary systematically within the family of application-rejection mechanisms. In particular, the Shanghai mechanism is not as manipulable as the Boston mechanism, while DA is strategy-proof. On the other hand, the set of Nash equilibrium outcomes induced by the Boston mechanism is equal to the set of Nash equilibrium outcomes induced by Shanghai mechanism. Using first choice accommodation as an empirical welfare measure, the Boston mechanism assigns the highest number of reported first choices, followed by Shanghai, which in turn, is followed by DA.

To reconcile these results and to predict the likely outcomes under an application-rejection mechanism, we consider two opposing effects. If we assume that all students are naive and behave truthfully, then Proposition 1 suggests that we will experience welfare losses (i.e., in the sense of decreasing first choice accommodation) as we move from the Boston mechanism to the Shanghai mechanism and from there to DA. We call this first effect the *naive welfare effect*. Clearly, all students are not necessarily honest in their preference reports as observed both in the real world and in the lab. On the other hand, if we assume that all students are strategic and able to best respond to one another in coordination, then Theorem 2 suggests that we will experience welfare losses if we switch from DA to the Boston or the Shanghai mechanisms. We call this second effect the *strategic welfare effect*. In light of Theorem 1, we expect the second effect to be stronger for the Boston mechanism than the Shanghai mechanism.

As the Boston school district was considering a possible transition to DA around 2005, some parent groups showed strong opposition to such an idea. Pathak and Sönmez (2008) argue that this may be because some parents are strategic while others are not and show that strategic parents would in fact prefer to be assigned through the Boston mechanism as opposed to through DA. Relatedly, previous experimental studies also indicate that a significant proportion of the subjects (between 14% and 28% in Chen and Sönmez (2006)) choose to be truthful under the Boston mechanism. In light of these observations, in practice we are likely to see a mixture of student types with varying degrees of strategic sophistication. Therefore, based on our analysis we expect the "observed" welfare outcomes under an application-rejection mechanism to be driven by the two opposite effects working jointly. In particular, since the size of these effects may well vary across problems, we expect the welfare comparison between these mechanisms to be sensitive to the problem choice. To test these predictions we next compare the three mechanisms in an experimental setting.

4 Experimental Design

We design our experiment to compare the performance of the Boston, Shanghai and DA mechanisms based on the theoretical characterization of the family of application-rejection mechanisms in Section 3. We choose the complete information environment to test the theoretical predictions, especially those on Nash equilibrium outcomes.[15] A $3(\text{mechanisms}) \times 2(\text{environments})$ factorial design is implemented to evaluate the performance of the three mechanisms $\{\text{BOS, SH, DA}\}$ in two different environments, a simple 4-school environment and a more complex 6-school environment. The environments are designed to capture the key aspects of the school choice problem and to simulate the complexity inherent in potential applications of the mechanisms.

4.1 The 4-School Environment

The first environment, which we call the **4-school environment**, has four students, $i \in \{1, 2, 3, 4\}$, and four schools, $s \in \{a, b, c, d\}$. Each school has one slot, which is allocated to one participant. We choose the parameters of this environment to satisfy several criteria: (1) no one lives in the district of her top or bottom choices; (2) the first choice accommodation index, i.e., the proportion of first choices an environment can accommodate, is 1/2; (3) there is a small number of Nash equilibrium outcomes, which reduces the complexity of the games.

The payoffs for each student are presented in Table 1. The square brackets, **[ ]**, indicate the resident of each school district, who has higher priority in that school than other applicants.

[15] In real life, school choice is likely to be a game of incomplete information. Thus, in a follow-up study, we test the same set of mechanisms under both the complete and incomplete information settings.

Payoffs range from 16 points for a first-choice school to 5 points for a last-choice school. Each student resides in her second-choice school.

Table 1: Payoff Table for the 4-School Environment

	a	b	c	d
Payoff to Type 1	[**11**]	7	5	16
Payoff to Type 2	5	[**11**]	7	16
Payoff to Type 3	7	16	[**11**]	5
Payoff to Type 4	5	16	7	[**11**]

For each session in the 4-school environment, there are 12 participants of four different types. Participants are randomly assigned types at the beginning of each session. At the beginning of each period, they are randomly re-matched into groups of four, each of which contains one of each of the four different types. Four schools are available for each group. In each period, each participant ranks the schools. After all participants have submitted their rankings, the server allocates the schools in each group and informs each person of his school allocation and respective payoff. The experiment consists of 20 periods to facilitate learning. Furthermore, we change the priority queue every five periods to investigate whether participant strategies are conditional on their priority.[16]

It follows from Theorem 2 that the Nash equilibrium outcomes of Boston and Shanghai mechanisms are the same but the equilibrium strategy profiles leading to those outcomes may differ.

For each of the 4 different queues, we compute the Nash equilibrium outcomes under the Boston and Shanghai mechanisms (which are the same) as well as under DA. For all four blocks, Boston and Shanghai each have a unique Nash equilibrium outcome, where each student is assigned to her district school. This college/student-optimal matching, $\mu^{C/S}$, is Pareto inefficient, with the sum of ranks of 8 and an aggregate payoff of 44:

$$\mu^{C/S} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & b & c & d \end{pmatrix}$$

For all four blocks, the matching $\mu^{C/S}$ is also a Nash equilibrium outcome under DA. However, DA has exactly one more Nash equilibrium outcome for all four cases, which is the following Pareto efficient matching μ^{*}, with the sum of ranks of 6 and an aggregate payoff of 54:

$$\mu^{*} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & d & c & b \end{pmatrix}.$$

[16]The priority queues for each five-period block are 1-2-3-4, 4-1-2-3, 3-4-1-2 and 2-3-4-1, respectively. Appendix C has detailed experimental instructions.

The Nash equilibrium profile that sustains outcome μ^* is the following (asterisks are arbitrary): $P_1 = (a, *, *, *)$, $P_2 = (d, b, *, *)$, $P_3 = (c, *, *, *)$, and $P_4 = (b, d, *, *)$. This is an equilibrium profile regardless of the priority order.[17] Note that, in this equilibrium profile, types 1 and 3 misrepresent their first choices by reporting their district school as their first choices, while types 2 and 4 report their true top choices.[18]

We now analyze participant incentives to reveal their true preferences in this environment. We observe that, in blocks 1 and 3, while truth-telling is a Nash equilibrium strategy under the Shanghai mechanism, it is not a Nash equilibrium under Boston. Furthermore, under truth-telling, Shanghai and DA yield the same Pareto inefficient outcome. Recall that Theorem 1 implies that, if truth-telling is a Nash equilibrium under Boston, then it is also a Nash equilibrium under the Shanghai mechanism, but the converse is not true. Blocks 1 and 3 are examples of the latter.

Table 2: Truthtelling and Nash Equilibrium Outcomes in the 4-School Environment

	Truthful Preference Revelation			Nash Equilibrium Outcomes		
	BOS	SH	DA	BOS	SH	DA
Block 1	not NE	NE	dominant strategy			
Block 2	not NE	not NE	dominant strategy	$\mu^{C/S}$	$\mu^{C/S}$	$\{\mu^{C/S}, \mu^*\}$
Block 3	not NE	NE	dominant strategy			
Block 4	not NE	not NE	dominant strategy			

In comparison, for blocks 2 and 4, truth-telling is not a Nash equilibrium strategy under either Shanghai or Boston. Under truthtelling, Boston, Shanghai and DA each yield different outcomes. While the outcome under Shanghai is Pareto efficient, those under DA is not. Table 2 summarizes our analysis on truthtelling and Nash equilibrium outcomes.

4.2 The 6-School Environment

While the 4-school environment is designed to compare the mechanisms in a simple context, we now test the mechanisms in a more complex environment where student preferences are generated by school proximity and quality.

[17]This is a Nash equilibrium because, for example, if student 1 (or 3) submits a profile where she lists school d (resp. b) as her first choice, then she may kick out student 2 (resp. 4) in the first step but 2 (resp. 4) would then apply to b (resp. d) and kick out 4 (resp. 2) who would in turn apply to d (resp. b) and kick out 1 (resp. 3). Hence student 1 (or 3), even though she may have higher priority than 2 (resp. 4), she cannot secure a seat at b (resp. d) under DA.

[18]Note that types 1 and 3's manipulation benefits types 2 and 4, thus it does not violate truthtelling as a weakly dominant strategy, since type 1 (resp. 3) is indifferent between truthtelling and lying. If type 1 (resp. 3) reverts to truthtelling, she will then cause a rejection chain which gives everyone their district school, including herself. Therefore, she is not better off by deviating from the efficient but unstable Nash equilibrium strategy.

In this **6-school environment**, each group consists of six students, $i \in \{1, 2, \ldots, 6\}$, and six schools $s \in \{a, b, \ldots, f\}$. Each school has one slot. Following Chen and Sönmez (2006), each student's ranking of the schools is generated by a utility function, which depends on school quality, school proximity and a random factor. There are two types of students: for notation purposes, odd labelled students are gifted in sciences while even labelled students are gifted in the arts. Schools *a* and *b* are higher quality schools, while *c*-*f* are lower quality schools. School *a* is stronger in the arts and *b* is stronger in sciences: *a* is a first tier school in the arts and second tier in sciences, while *b* is a second tier school in the arts and first tier in sciences; *c*-*f* are each third tier in both arts and sciences. The utility function of each student has three components:

$$u^i(s) = u^i_p(s) + u^i_q(s) + u^i_r(s), \tag{1}$$

where the first component, $u^i_p(s)$, represents the proximity utility for student i for school s. We designate this as 10 if student i lives within the walk zone of School s and 0 otherwise. The second component, $u^i_q(s)$, represents the quality utility for student i at school s. For odd labelled students, $u^i_q(a) = 20$, $u^i_q(b) = 40$, and $u^i_q(s) = 10$ for $s = c - f$. For even labelled students, $u^i_q(a) = 40$, $u^i_q(b) = 20$, and $u^i_q(s) = 10$ for $s = c - f$. Finally, the third component, $u^i_r(s)$, represents a random utility (uniform in the range 0-40) which includes diversity in tastes. Based on this utility function, we randomly generate 20 environments. We choose an environment which again satisfies several criteria: (1) no one lives within the district of her top or bottom choices; and (2) the first choice accommodation index is 1/3, a more competitive scenario than the 4-school environment.

We use Equation (1) to generate payoffs. We then normalize the payoffs such that the payoff from the first to last choice schools spans {16, 13, 11, 9, 7, 5}, the same payoff range as in the 4-school environment. The normalized payoff table is reported in Table 3.

Table 3: Payoff Table for the 6-School Environment

	a	b	c	d	e	f
Payoff to Type 1	[**9**]	16	11	13	7	5
Payoff to Type 2	16	[**11**]	5	13	9	7
Payoff to Type 3	9	16	[**7**]	11	5	13
Payoff to Type 4	16	7	9	[**13**]	5	11
Payoff to Type 5	16	13	11	7	[**9**]	5
Payoff to Type 6	16	13	11	5	7	[**9**]

For each session in the 6-school environment, we include 18 participants of six different types. Participants are randomly assigned types at the beginning of each session. The experiment consists of 30 periods, with random re-matching into three groups of six in each period. Again, we change the priority queue every five periods.

Compared with the 4-school environment, the 6-school environment has a much larger set of Nash equilibrium outcomes. By Theorem 2, the Nash equilibrium outcomes of Boston and Shanghai are the same, but the equilibrium strategy profiles leading to those outcomes may differ. Furthermore, there are more equilibrium strategy profiles under Shanghai than under Boston. We examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. The list of Nash equilibrium outcomes for each block is included in Appendix B.

Lastly, we present the efficiency analysis for the 6-school environment. The allocations that maximizes the sum of payoffs are the following ones, each leading to the sum of ranks of 13 with an aggregate payoff of 78.

$\mu_1^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & d & f & a & e & c \end{pmatrix}$ or $\mu_2^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & f & d & e & c \end{pmatrix}$.

In comparison, the No Choice benchmark, where each student is assigned to her district school, generates the sum of ranks of 22 with an aggregate payoff of 58.

4.3 Experimental Procedures

In each experimental session, each participant is randomly assigned an ID number and is seated in front of a terminal in the laboratory. The experimenter then reads the instructions aloud. Subjects have the opportunity to ask questions, which are answered in public. Subjects are then given 10 minutes to read the instructions at their own pace and to finish the review questions. After everyone finishes the review questions, the experimenter distributes the answers and goes over the answers in public. Afterwards, participants go through 20 (respectively 30) periods of a school choice experiment in the 4-school (respectively 6-school) environment. At the end of the experiment, each participant fills out a demographics and strategy survey on the computer. Each participant is paid in private at the end of the experiment. The experiment is programmed in z-Tree (Fischbacher 2007).

Table 4 summarizes the features of the experimental sessions. For each mechanism in each environment, we conduct four independent sessions between May 2009 and July 2011 at the Behavioral Economics and Cognition Experimental Lab at the University of Michigan.[19] The subjects are students from the University of Michigan. No one participates in more than one session. This gives us a total of 24 independent sessions and 360 subjects. Each 4-school session

[19]All sessions were conducted between May 2009 and July 2010. However, we found a z-Tree coding error for the BOS_6 treatment during our data analysis. Thus, four additional sessions were conducted in July 2011 for this treatment, to replace the corresponding sessions.

Table 4: Features of Experimental Sessions

Treatment	Mechanism	Environment	# Subjects × # sessions	Total # of subjects
BOS_4	Boston	4-school	12×4	48
SH_4	Shanghai	4-school	12×4	48
DA_4	Deferred Acceptance	4-school	12×4	48
BOS_6	Boston	6-school	18×4	72
SH_6	Shanghai	6-school	18×4	72
DA_6	Deferred Acceptance	6-school	18×4	72

consists of 20 periods. These sessions last approximately 60 minutes. In comparison, each 6-school session consists of 30 periods. These sessions last approximately 90 minutes. The first 20-30 minutes in each session are used for instructions. The conversion rate is $1 = 20 points for all treatments. Each subject also receives a participation fee of $5, and up to $3.5 for answering the Review Questions correctly. The average earning (including participation fee) is $19.23 for the 4-school treatments, and $25.51 for the 6-school treatments. Experimental instructions are included in Appendix C. Data are available from the authors upon request.

5 Experimental Results

In examining our experimental results, we first explore individual behavior and equilibrium selection, and then report our aggregate performance measures, including first choice accommodation, efficiency and stability of the three mechanisms. We also investigate the sensitivity of our results to environment changes.

In presenting the results, we introduce several shorthand notations. First, let $x > y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y at the 5% significance level or less. Second, let $x \geq y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y, but the comparison is not statistically significant at the 5% level.

5.1 Individual Behavior

We first examine the extent to which individuals reveal their preferences truthfully, and the pattern of any preference manipulation under each of the three mechanisms. Theorem 1 suggests that the Shanghai mechanism is less manipulable than the Boston mechanism. Furthermore, under DA, truthtelling is a weakly dominant strategy. This leads to our first hypothesis.

Hypothesis 1 (Truthtelling) *(a) There will be a higher proportion of truthtelling under Shanghai than under Boston. (b) Under DA, participants will be more likely to reveal their preferences truthfully than under either Boston or Shanghai.*

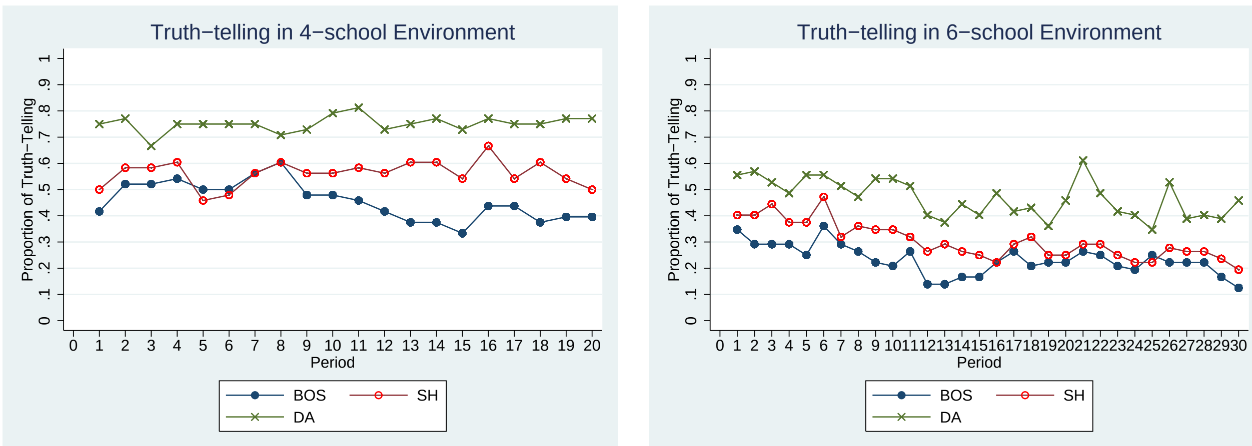

Figure 1: Proportion of Truth-Telling in Each Environment

Figure 1 presents the proportion of truthtelling in the 4- and 6-school environments under each mechanism. Note that, under the Boston and Shanghai mechanisms, truthful preference revelation requires that the entire reported ranking is identical to a participant's true preference ranking.[20] However, under DA, truthful preference revelation requires that the reported ranking be identical to the true preference ranking from the first choice through the participant's district school. The remaining rankings, from the district school to the last choice, are irrelevant under DA. While DA has a robustly higher proportion of truthtelling than the other two mechanisms, we find that Shanghai has more truthtelling behavior than Boston. Further, under each mechanism, the proportion of truthtelling is higher under the 4-school than under the 6-school environment, especially under DA, which indicates that it is easier to figure out the dominant strategy in a simpler environment.

Result 1 (Truthtelling) **:** *In both environments, the proportion of truthful preference revelation under DA is significantly higher than that under Boston or Shanghai over all periods. The proportion of truthful preference revelation under Shanghai is significantly higher than that under Boston in the 4-school environment.*

[20] The only exception is when a participant's district school is her top choice. In this case, truthful preference revelation entails stating the top choice. However, by design, this case never arises in our experiment, as no one's district school is her first choice.

Table 5: Proportions of Truthful Preference Revelation and Misrepresentations

	Truthful Preference Revelation			District School Bias		
All Periods	Proportion	H_a	p-value	Proportion	H_a	p-value
BOS_4	0.456	BOS < SH:	$p = 0.014$	0.478	BOS > SH:	$p = 0.014$
SH_4	0.563	SH < DA:	$p = 0.014$	0.310	SH > DA:	$p = 0.014$
DA_4	0.751	BOS < DA:	$p = 0.014$	0.107	BOS > DA:	$p = 0.014$
BOS_6	0.232	BOS < SH:	$p = 0.129$	0.549	BOS > SH:	$p = 0.186$
SH_6	0.303	SH < DA:	$p = 0.014$	0.459	SH > DA:	$p = 0.014$
DA_6	0.468	BOS < DA:	$p = 0.014$	0.144	BOS > DA:	$p = 0.014$

SUPPORT: Table 5 presents the proportion of truthful preference revelation, as well as the proportion of district school bias, a prevalent form of misrepresentation, for each treatment. P-values are computed from one-sided permutation tests, treating each session as an observation. ∎

By Result 1, we reject the null in favor of Hypothesis 1(a) that the Shanghai mechanism is less manipulable than Boston at the 5% level in the 4-school environment. Furthermore, we reject the null in favor of Hypothesis 1(b) that DA is less manipulable than either the Shanghai or Boston mechanisms. The result is similar for inexperienced participants (first period). While the ranking of truthtelling between Boston and DA is consistent with Chen and Sönmez (2006), manipulability of the Shanghai mechanism is reported for the first time. Even though truthtelling is not a dominant strategy under the Shanghai mechanism, the extent of manipulation is significantly less under the Shanghai mechanism than under Boston in our simple environment. The same ranking holds in the more complex 6-school environment but it is not significant.

While we do not observe 100% truthtelling under DA, it outperforms both the Shanghai and the Boston mechanisms in truthtelling. Furthermore, we observe that the proportion of truthtelling in DA is significantly higher in the 4-school environment than in the 6-school environment ($p = 0.014$, one-sided permutation test). We interpret this as due to the relative simplicity of the environment.

Note that subjects are *not* told that truthtelling is a dominant strategy under DA in the experimental instructions (Appendix C). Following the convention in the experimental mechanism design literature, we describe each algorithm without prompting the subjects to behave in one way or another. Thus, results in this section summarize participant behavior without prompting from the experimenter. In practice, however, the market designer can educate the students when truthtelling is a dominant strategy. In fact, the Boston Public Schools, after switching to DA, advise the students to "list your school choices in your true order of preference" and that "there is no need to "strategize."[21] If parents follow the advice, we expect DA to achieve close to 100% truthtelling in practice, further enlarging the gap between DA and the other mechanisms reported in Result 1. Table 7 in Appendix D presents probit regressions investigating factors affecting truthtelling. We find a significant lottery position effect on truthtelling, namely, a better lottery position significantly increases the likelihood of truthtelling. Additionally, we also observe a small but significant effect of learning to manipulate (resp. tell the truth) under the Boston (resp. Shanghai) mechanism in the 4-school environment.

A main critique of the Boston mechanism is centered around the fact that the mechanism puts a lot of pressure on manipulation of first choices. The Shanghai mechanism alleviates this pressure. We now examine the likelihood that participants reveal their first choices truthfully under each mechanism.

Hypothesis 2 (Truthful First Choice) *A higher proportion of reported first choices will be true first choices under the Shanghai than under the Boston mechanism.*

Result 2 (Truthful First Choice) *: In the 4-school (6-school) environment, the proportion of truthful first choices is 77% (55%) under DA, 62% (46%) under Shanghai, and 49% (37%) under Boston, resulting in statistically significant ranking of DA $>$ Shanghai $>$ Boston for truthful first choices for both environments.*

SUPPORT: Using each session as an observation, pairwise comparisons of the proportion of truthful first choices yield $p = 0.014$ (one-sided permutation tests) for the 4-school environment. For the 6-school environment, using the same tests, we obtain DA $>$ BOS ($p = 0.014$), DA $>$ SH ($p = 0.029$), and SH $>$ BOS ($p = 0.043$). ■

By Result 2, we reject the null in favor of Hypothesis 2 that the Shanghai mechanism generates a higher proportion of truthful first choices than Boston. Thus, the truthful preference revelation ranking of the mechanisms is stronger when comparing first choices. Regardless of the environment, participants are more likely to submit true first choices under the Shanghai mechanism than under Boston.

We next examine our results regarding District School Bias, a prevalent form of manipulation where a participant puts her district school into a higher position than that in the true preference order. This type of preference manipulation has been reported in previous experimental studies of the Boston mechanism (Chen and Sönmez 2006, Calsamiglia, Haeringer and Klijn 2010, Klijn, Pais and Vorsatz 2010).

[21] Source: `http://www.bostonpublicschools.org/choosing-schools`, Boston Public Schools Website, retrieved on June 2, 2011.

Result 3 (District School Bias) *: The proportion of participants who exhibits District School Bias is significantly (weakly) higher under Boston than under Shanghai in the 4-school (6-school) environment, which is then followed by DA.*

SUPPORT: See columns under "District School Bias" in Table 5. ∎

The proceeding analysis of individual behavior has implications for Nash equilibrium outcomes. Generically, there are multiple Nash equilibria in the application-rejection family of mechanisms. Thus, from both the theoretical and practical implementation perspectives, it is important to investigate which equilibrium outcomes are more likely to arise. To our knowledge, equilibrium selection in school choice mechanisms has not been studied before.

Our 4-school environment is particularly well suited to study equilibrium selection. Recall that in our 4-school environment, the student-optimal Nash equilibrium outcome, $\mu^{C/S}$, is the unique Nash equilibrium outcome under the Boston and the Shanghai mechanisms, while there are two Nash equilibrium outcomes under DA, $\mu^{C/S}$ and μ^*, where the latter Pareto dominates the former. Thus, it will be interesting to examine which of the two equilibrium outcomes arises more frequently under DA. While the Pareto criterion predicts that the Pareto optimal unstable Nash equilibrium should be selected, experimental results from secure implementation suggest that the dominant strategy equilibrium, when coinciding with Nash, is more likely to be chosen (Cason, Saijo, Sjöström and Yamato 2006). This predicts that the student-optimal Nash equilibrium outcome is more likely to arise.

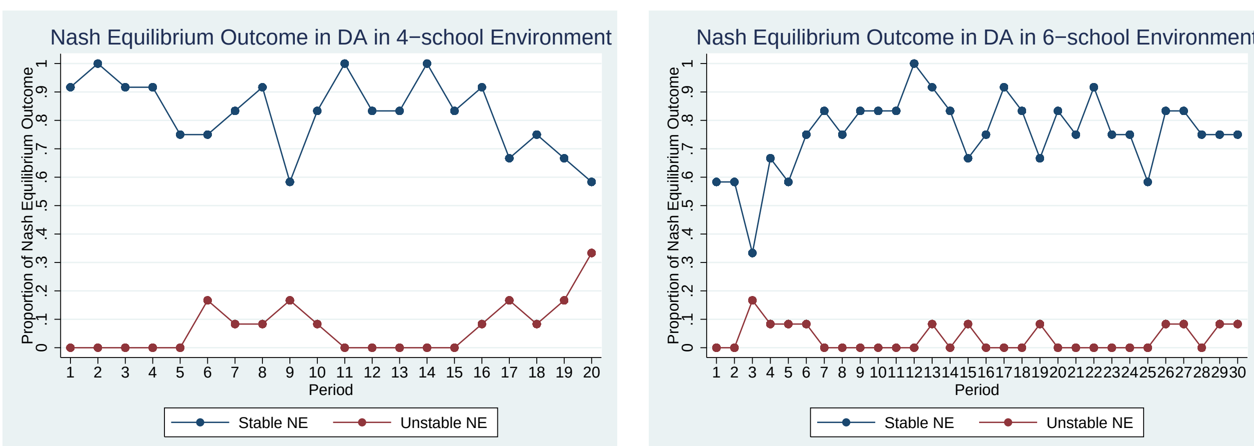

Figure 2: Proportion of Stable and Unstable Nash Equilibrium Outcomes under DA

Figure 2 reports the proportion of the stable and unstable equilibrium outcomes over time under DA in the 4-school (left panel) and 6-school (right panel) environments, while Table 8 in Appendix D reports session-level statistics for each mechanism and pairwise comparisons between mechanisms and outcomes.

Result 4 (Nash Equilibrium Outcomes) *: Under DA, the proportion of the inefficient but stable Nash equilibrium outcome (82.5%) is weakly higher than that of the efficient but unstable Nash equilibrium outcome (8.9%) in the 4-school environment.*

SUPPORT: The last column in Table 8 presents the p-values for permutation tests comparing the proportion of equilibrium outcomes under different mechanisms. The null of equal proportion against the H_a of DA(μ^*) $<$ DA($\mu^{C/S}$) yields $p = 0.063$ (paired permutation test, one-sided). ■

We conjecture that the stable Nash equilibrium outcome ($\mu^{C/S}$) is observed more often despite being Pareto dominated by μ^*, because the former requires truthful preference revelation, the weakly dominant strategy adopted by about 75% of the participants under DA, while the latter requires coordinated manipulation of top choices by players 1 and 3. However, we also note an increase of the unstable but efficient Nash equilibrium outcome, μ^*, in the last block in Figure 2 (left panel), indicating that players 1 and 3 learn to coordinate their manipulation towards the end of the game. This increase has direct implications for the efficiency comparisons in Result 6.

In comparison to the 4-school environment, the 6-school environment generates many Nash equilibrium outcomes. Because of this multitude of Nash equilibria, without strategy-proofness, on average, 10% and 16% of the outcomes are Nash equilibrium outcomes under Boston and Shanghai, respectively. In contrast, 79% of the outcomes under DA are Nash equilibrium outcomes. The proportion of this Nash equilibrium outcome follows DA $>$ BOS ($p = 0.014$), DA $>$ SH ($p = 0.014$), and SH $\geq$ BOS ($p = 0.086$). If we break down the Nash equilibrium outcomes under DA into stable and unstable equilibria, we again observe that the stable outcomes arise weakly more frequently than the unstable ones ($p = 0.063$, paired permutation test, one-sided).

In sum, Result 4 and our analysis of the 6-school data indicate that the stable Nash equilibrium outcome is more likely to arise than the unstable Nash equilibrium outcomes under DA. To our knowledge, this is the first empirical result on equilibrium selection under DA.

5.2 Aggregate Performance

Having presented the individual behavior and equilibrium outcomes, we now evaluate the aggregate performance of the mechanisms using three measures: the proportion of participants receiving their reported and true first choices, the efficiency achieved, and the stability under each mechanism.

In the education literature, the performance of a school choice mechanism is often evaluated through the proportion of students who receive their reported top choices. Thus, we compare the proportion of participants receiving their reported top choices, as well as the proportion who actually receive their true top choices. Proposition 1 suggests the following hypothesis.

Hypothesis 3 (First Choice Accommodation) *The proportion of participants receiving their reported top choices will be the highest under Boston, followed by Shanghai, and then DA.*

Table 6: First Choice Accommodation: Reported versus True First Choices

	Proportion Receiving *Reported* First Choice					Proportion Receiving *True* First Choice				
4-school	BOS	SH	DA	H_a	p-value	BOS	SH	DA	H_a	p-value
Session 1	0.596	0.450	0.138	BOS > SH	0.014	0.088	0.096	0.017	BOS $\neq$ SH	0.257
Session 2	0.617	0.488	0.271	BOS > DA	0.014	0.113	0.100	0.121	BOS $\neq$ DA	0.114
Session 3	0.583	0.463	0.192	SH > DA	0.014	0.121	0.108	0.071	SH $\neq$ DA	0.257
Session 4	0.608	0.325	0.183			0.138	0.096	0.075		
6-school	BOS	SH	DA	H_a	p-value	BOS	SH	DA	H_a	p-value
Session 1	0.717	0.422	0.196	BOS > SH	0.014	0.217	0.133	0.109	BOS $\neq$ SH	0.086
Session 2	0.665	0.402	0.270	BOS > DA	0.014	0.230	0.143	0.111	BOS $\neq$ DA	0.029
Session 3	0.667	0.461	0.231	SH > DA	0.014	0.178	0.207	0.085	SH $\neq$ DA	0.029
Session 4	0.706	0.435	0.241			0.202	0.159	0.120		

Table 6 reports the proportion of participants receiving their reported (left panel) and true first choices (right panel) in each session in each treatment. Note that the alternative hypotheses comparing mechanisms accommodating true first choices are two-sided, as neither the Boston nor the Shanghai mechanism is strategy-proof. P-values of permutation tests are reported in the last column. The results are summarized below.

Result 5 (First Choice Accommodation) *: In both environments, the following ranking of the proportion of participants receiving their reported first choices is significant: BOS > SH > DA. However, for the proportion receiving their true first choices, BOS and SH are not significantly different, but both significantly outperform DA in the 6-school environment.*

SUPPORT: Treating each session as an observation, p-values from the corresponding permutation tests are reported in Table 6. ∎

By Result 5, we reject the null in favor of Hypothesis 3 for reported first choices. However, looking at the accommodation of true first choices, we find that reported top choices are not a good measure of performance when the incentive properties under each mechanism are different. In the 4-school environment, the three mechanisms are not significantly different from each other, while in the 6-school environment, Boston and Shanghai are not significantly different from each other, but both outperform DA.

We next compare the efficiency of the mechanisms in each environment. As our theoretical benchmarks are based ordinal preferences, we present a corresponding efficiency measure using ordinal ranking of assignments.[22] We define a normalized efficiency measure as

[22]For robustness check, we have also completed a parallel set of efficiency analysis based on the sum of payoffs, which yields similar results.

$$(2) \qquad \text{Normalized Efficiency} = \frac{\text{maximum group rank - actual group rank}}{\text{maximum group rank - minimum group rank}},$$

where the *minimum group rank* is the sum of ranks for all group members for the Pareto efficient allocation(s), which equals 6 (resp. 13) for for the 4-school (resp. 6-school) environment. Likewise, the *maximum group rank* is the sum of ranks for the worst allocation, which equals 14 (resp. 33) for the 4-school (resp. 6-school) environment. Because of this normalization, this measure always lies between zero and one, inclusive.

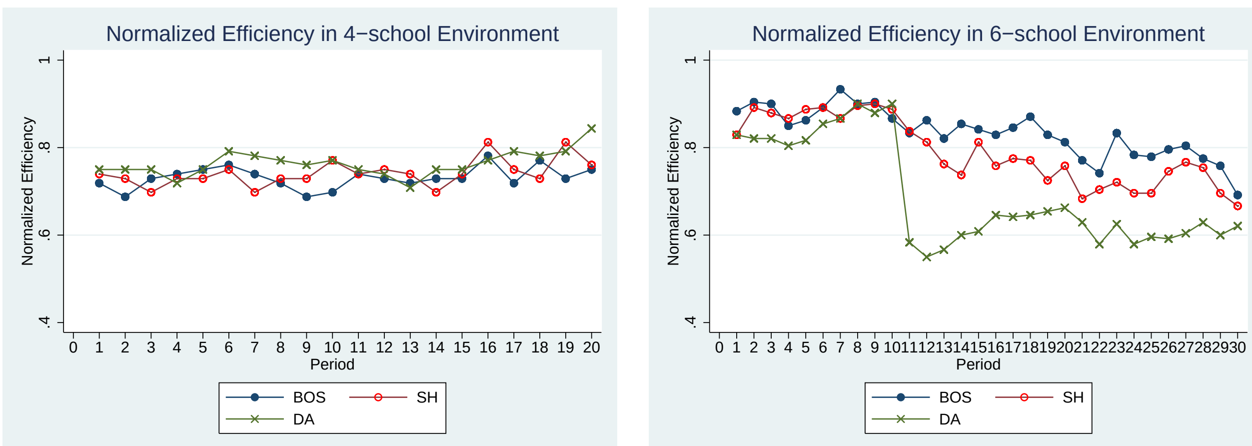

Figure 3: Normalized Efficiency in the 4- and 6-School Environments

Figure 3 presents the normalized efficiency under each mechanism in the 4-school and 6-school environments. Session-level normalized efficiencies for the first and last blocks, as well as the average efficiency over all periods, are reported in Table 9 in Appendix D.

Result 6 (Efficiency) *: While DA is more efficient in the 4-school environment, Boston is more efficient than Shanghai, which in turn is more efficient than DA in the 6-school environment.*

SUPPORT: Using one-sided permutation tests with each session as an observation, we find that (1) First block: $\text{DA}_4 > \text{SH}_4$ ($p = 0.043$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.029$), $\text{SH}_6 > \text{DA}_6$ ($p = 0.029$); (2) Last block: $\text{DA}_4 > \text{BOS}_4$ ($p = 0.029$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.014$); $\text{SH}_6 > \text{DA}_6$ ($p = 0.043$); (3) All periods: $\text{BOS}_6 > \text{SH}_6$ ($p = 0.043$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.014$); $\text{SH}_6 > \text{DA}_6$ ($p = 0.014$), while none of the pairwise efficiency comparisons in the 4-school environment is significant. ∎

Result 6 is consistent with part (iii) of Proposition 2 in that there is no systematic efficiency ranking within the class of the Chinese parallel mechanisms. It also contributes to our understanding of the empirical performance of the school choice mechanisms. First, it indicates efficiency comparison is environment sensitive. While no single mechanism is more efficient in both environments, the Shanghai mechanism is never the worst. Second, while a first-period pairwise efficiency comparison is not significant in either environment, separation of performance occurs with learning, so that the last block ranking is significant. Our first period results are consistent with Calsamiglia, Haeringer and Klijn (2011). Our results point to the importance of allowing subjects to learn in school choice experiments. Lastly, our finding that DA is more efficient than Boston in the last block is driven by the rise of the unstable but efficient Nash equilibrium outcome observed in Figure 2 (left panel).

Finally, we evaluate the stability achieved under each mechanism. Proposition 2 (i) suggests the following ranking:

Hypothesis 4 (Stability) *DA is more stable than Shanghai, which in turn is more stable than Boston.*

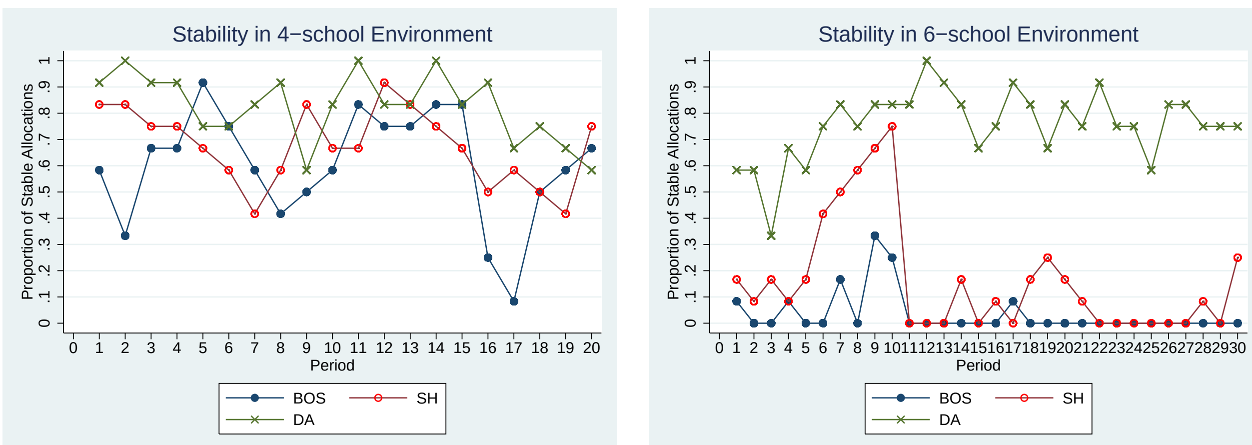

Figure 4: Proportion of Stable Allocations in the 4- and 6-School Environments

Figure 4 presents the proportion of stable allocations under each mechanism in the 4-school (left panel) and 6-school (right panel) environments. An allocation is marked as unstable if any student in a group of four (resp. six) is justifiable envious of another student in the group.

Result 7 (Stability) *: DA is significantly more stable than the Shanghai or the Boston mechanism. The Shanghai mechanism is significantly (resp. weakly) more stable than the Boston mechanism in the 6-school (resp. 4-school) environment.*

SUPPORT: Table 10 in Appendix D reports the proportion of stable allocations among all allocations in the first and last block, and averaged over all periods in each session. Using one-sided permutation tests with each session as an observation, we find that (1) $\text{DA}_4 > \text{SH}_4$ ($p = 0.014$),

$\mathrm{DA}_4 > \mathrm{BOS}_4$ ($p = 0.029$), $\mathrm{SH}_4 > \mathrm{BOS}_4$ ($p = 0.071$); (2) $\mathrm{DA}_6 > \mathrm{BOS}_6$ ($p = 0.014$), $\mathrm{DA}_6 > \mathrm{SH}_6$ ($p = 0.014$), and $\mathrm{SH}_6 > \mathrm{BOS}_6$ ($p = 0.014$). ∎

By Result 7, we reject the null in favor of Hypothesis 4. Thus, consistent with Proposition 2 (i), in both environments, DA achieves a significantly higher proportion of stable allocations than either the Boston or Shanghai mechanisms, while the Shanghai mechanism achieves a higher proportion of stable outcomes than Boston. While our empirical stability ranking between DA and Boston is consistent with Calsamiglia et al. (2010), the stability evaluation of the Shanghai mechanism is new.

In sum, our experimental study has several new findings. First, we evaluate the performance of the Shanghai mechanism, and find that its manipulability, reported first-choice accommodation, efficiency and stability measures are robustly sandwiched in between the Boston and DA mechanisms. Second, compared to the one-shot implementation of previous experiments on school choice except Featherstone and Niederle (2008),[23] our experimental design with repeated random re-matching enables us to compare the performance of the mechanisms with experienced participants. In doing so, we find that learning separates the performance of the mechanisms in terms of efficiency. Lastly, we report equilibrium selection under DA for the first time, which reveals that stable Nash equilibrium outcomes are significantly more likely to arise than the unstable ones even when the latter Pareto dominates the former.

6 Conclusions

School choice and college admissions have profound implications for the education and labor market outcomes of the students involved in these processes worldwide. The actual mechanisms used for the matching of students to schools or colleges differ in their strategic, welfare and stability properties. In this paper, we synthesize well known matching mechanisms for school choice and college admissions, and characterize them as a family of application-rejection mechanisms, with the Boston mechanism, the Shanghai mechanism, and the Deferred Acceptance mechanism as special cases. A key insight is that the Shanghai mechanism used for both high school admissions in Shanghai and for college admissions in many provinces in China bridges the well studied Boston and DA mechanisms. The Boston and DA mechanisms are the two extreme members of this family while variants of the Chinese parallel mechanisms constitute the intermediate members of this family.

Our theoretical analysis indicates a systematic change in the incentive properties of this family of mechanisms as one moves from one extreme member to the other. We also see that the

[23] Featherstone and Niederle (2008) investigate the performance of the Boston and DA mechanisms under incomplete information, whereas we study the family of mechanisms under complete information. While their experiment is implemented under a random re-matching protocol, they do not explicitly analyze the effects of learning.

Nash equilibrium strategies corresponding to the induced preference revelation games associated with members of the application-rejection family are nested. The Boston and Shanghai mechanisms are the only members of the family for which the set of Nash equilibrium outcomes of the preference revelation game is equal to the set of stable matchings under students' true preferences. This finding suggests that the Shanghai mechanism could remedy the incentive problems observed under the Boston mechanism at no welfare cost.

To test our theoretical predictions and to search for behavioral regularities where theory is silent, we conduct laboratory experiments in two environments differentiated by their complexity. We find that the proportion of truthtelling follows the order of DA $>$ Shanghai $>$ Boston, while the proportion of District School Bias follows the reverse order. While the manipulability ranking of DA and Boston is consistent with both theory and prior experimental findings, the manipulability of the Shanghai mechanism is reported for the first time. While theory is silent about equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than unstable ones. On the stability front, consistent with theory, DA achieves a significantly higher proportion of stable outcomes than either Shanghai or Boston in both environments, while Shanghai is more stable than Boston. However, the efficiency comparison is sensitive to the environment. In our 4-school environment, DA is weakly more efficient than Boston, while the Shanghai mechanism is not significantly different from either. In comparison, in our 6-school environment, Boston achieves significantly higher efficiency than Shanghai, which, in turn, achieves higher efficiency than DA.

Our study represents the first theoretical and experimental investigation of the Shanghai mechanism, and more generally, the Chinese parallel mechanisms. The analysis yields valuable insights which enable us to treat this class of mechanisms as a family, and systematically study their properties and performance. More importantly, our results have policy implications for school choice and college admissions. As the Shanghai mechanism is less manipulable than the Boston mechanism, and its achieved efficiency is robustly sandwiched between the two extremes whose efficiency varies with the environment, it might be a less radical replacement for the Boston mechanism compared to DA.

Like school choice in the United States, college admissions reform is among the most intensively discussed public policies in China. Since 2003, variants of the parallel mechanism have been implemented in various provinces to replace the sequential mechanism, i.e., Boston with tiers, to address the latter's incentive problems. However, the choice of the number of parallel options (e) seems arbitrary. Our study provides the first theoretical analysis and experimental data on the effects of the number of parallel options on the incentives and aggregate performance of these mechanisms.

In our ongoing work, we extend this stream of research to evaluate this family of mechanisms in the incomplete information settings, and in the college admissions settings, both of which will help inform education policies in the school choice and college admissions domain.

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Appendix A: Proofs and Examples (For Online Publication)

Proof of Lemma 1: (Part i). It is easy to see that the Boston mechanism is Pareto efficient. Now consider the following problem with four students and four schools each with one seat. Priority orders and student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_2	$\vdots$	$\vdots$
i_2	i_3		
i_1	i_4		
i_3	i_1		

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
s_1	s_1	s_2	s_2
s_4	s_2	s_3	s_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The outcome of the application-rejection mechanism (e) for all $e \geq 2$ is the Pareto inefficient matching $\mu = \binom{i_1 \quad i_2 \quad i_3 \quad i_4}{s_4 \quad s_3 \quad s_2 \quad s_1}$.

(Parts ii & iii). Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{e+2}\}$ and $S = \{s_1, s_2, \ldots, s_{e+2}\}$ where each school has a quota of one. Suppose student i_1 ranks school s_1 first. Suppose also that each $i_k \in I$ with $k \geq 2$ ranks school s_{k-1} first and school s_k second. Suppose further that each $i_k \in I$ has the highest priority for school s_k. Let us apply the application-rejection (e) mechanism with $e < \infty$ to this problem. Consider student i_{e+1}. It is easy to see that she applies to school s_{e+1} in step $e+1$ of the algorithm when a lower student is already permanently assigned to this school at the end of step e. Hence her final assignment is necessarily worse than school s_{e+1}.Then the outcome of the application-rejection (e) mechanism for this problem is clearly unstable. Moreover, student i_{e+1} can secure a seat at school s_{e+1} when she submits an alternative preference list in which she ranks school s_{e+1} first. ∎

Example 1a. (*within-round manipulation*): We first illustrate a *within-round manipulation*. Consider a school choice problem with four students and four schools, each with one seat. Fixed priority orders and possible student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_3	i_2	i_3	$\vdots$
i_1	i_3	i_4	$\vdots$
i_2	$\vdots$	$\vdots$	$\vdots$

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P'_{i_3}	P_{i_4}
s_1	s_3	s_1	s_2	s_1	s_3
s_3	s_1	s_2	s_1	$\vdots$	s_1
s_4	s_4	$\vdots$	s_4		$\vdots$

The following two tables illustrate the steps of the application-rejection mechanism $(e = 3)$ applied to the problem (P_{i_1}, P_{-i_1}). A student tentatively placed to a school at a particular step is indicated with a box.

Round 1	s_1 $(q^{r=1}_{s_1}=1)$	s_2 $(q^{r=1}_{s_2}=1)$	s_3 $(q^{r=1}_{s_3}=1)$	s_4 $(q^{r=1}_{s_4}=1)$
Step 1	$\boxed{i_1}$, i_2	$\boxed{i_3}$	$\boxed{i_4}$	
Step 2	$\boxed{i_1}$	$\boxed{i_2}$, i_3	$\boxed{i_4}$	
Step 3	$\boxed{i_3}$, i_1	$\boxed{i_2}$	$\boxed{i_4}$	
Round 2	s_1 $(q^{r=2}_{s_1}=0)$	s_2 $(q^{r=2}_{s_2}=0)$	s_3 $(q^{r=2}_{s_3}=0)$	s_4 $(q^{r=2}_{s_4}=1)$
Step 4			i_1	
Step 5	$\vdots$			$\boxed{i_1}$
Step 6	$\vdots$	$\vdots$		$\boxed{i_1}$

In the above rounds, at the problem (P_{i_1}, P_{-i_1}), student i_1 applies to school s_1 in the first step but has her placement rescinded at the end of the third step. Her final assignment is school s_4, which is determined in the second round. Now consider the following rounds of the mechanism when student i_1 reports P'_{i_1}.

Round 1	s_1 $(q^{r=1}_{s_1}=1)$	s_2 $(q^{r=1}_{s_2}=1)$	s_3 $(q^{r=1}_{s_3}=1)$	s_4 $(q^{r=1}_{s_4}=1)$
Step 1	$\boxed{i_2}$	$\boxed{i_3}$	$\boxed{i_4}$, i_1	
Step 2	$\boxed{i_1}$, i_2	$\boxed{i_3}$	$\boxed{i_4}$	
Step 3	$\boxed{i_1}$	$\boxed{i_2}$, i_3	$\boxed{i_4}$	
Round 2	s_1 $(q^{r=2}_{s_1}=0)$	s_2 $(q^{r=2}_{s_2}=0)$	s_3 $(q^{r=2}_{s_3}=0)$	s_4 $(q^{r=2}_{s_4}=1)$
Step 4	i_3			
Step 5	$\vdots$			$\boxed{i_3}$
Step 6	$\vdots$	$\vdots$		$\boxed{i_3}$

In this case, at the problem (P'_{i_1}, P_{-i_1}), student i_1 can secure her place at school s_1 by delaying her application to this school to the second step. Note that, at the problem (P_{i_1}, P_{-i_1}) the application of student i_1 in step 1 initiates a rejection chain which eventually causes her to be rejected from this school. On the other hand, this adverse effect is eliminated when she deliberately wastes her first choice at a school (namely, school s_3) where she does not have any chance of being admitted due to her low priority. We refer to such a manipulation as a *within-round manipulation*. In fact, this sort of a strategy is reminiscent of the sniping behavior observed in eBay auctions (Ockenfels and Roth 2002).

In Example 1a, note that the outcome of the application-rejection mechanism ($e = 3$) for the problem (P'_{i_1}, P_{-i_1}) induces a priority violation of student i_3 for school s_1 since she has higher s_1-priority than student i_1. This happens simply because student i_3 gets 'too late' to apply to school s_1 and loses her priority to student i_1. We next illustrate a second kind of manipulation, whereby student i_3 misrepresents her preferences to improve her final allocation. ■

Example 1b. (*across-round manipulation*): Now consider the problem $(P'_{i_1}, P_{i_2}, P'_{i_3}, P_{i_4})$ that results when student i_3 submits school s_1 as her first choice (as opposed to her second). The following table illustrates the resulting application-rejection mechanism ($e = 3$) outcome.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 1)$	s_4 $(q^{r=1}_{s_4} = 1)$
Step 1	$\boxed{i_3}, i_2$		$\boxed{i_4}, i_1$	
Step 2	$\boxed{i_3}, i_1$	$\boxed{i_2}$	$\boxed{i_4}$	
Step 3	$\boxed{i_3}$	$\boxed{i_2}$	$\boxed{i_4}$	$\boxed{i_1}$

By submitting P'_{i_3}, student i_3 improves the ranking of school s_1 and secures a seat at the end of round 1 as opposed to round 2. We refer to such a manipulation as an *across-round manipulation*. This manipulation strategy has been widely observed under the Boston mechanism and diagnosed as one of the main factors behind the gaming issues that arose under the Boston mechanism. ■

Example 2a. (The Boston mechanism is manipulable whenever the Shanghai mechanism is) Consider the following example with the given priority structure and the profile $P = (P_{i_1}, P_{i_2}, P_{i_3}, P_{i_4})$ of true preferences. Schools s_1 and s_2 each have a quota of one, while school s_3 has a quota of two.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_4	i_1	$\vdots$
i_1	i_3	
i_2	i_4	
i_3	i_2	

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P'_{i_4}
s_1	s_2	s_1	s_2	s_2	s_1
s_2	$\vdots$	s_3	s_3	s_1	$\vdots$
s_3	$\vdots$	s_2	s_1	s_3	$\vdots$

The following two tables illustrate the steps of the Shanghai mechanism applied to the problem $(\succ, P)$. A student tentatively placed at a school at a particular step is outlined in a box.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_1}, i_2$	$\boxed{i_3}, i_4$	
Step 2	$\boxed{i_4}, i_1$	$\boxed{i_3}$	$\boxed{i_2}$
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3		i_1	
Step 4	$\vdots$		$\boxed{i_1}$

In the above tables, observe that student i_1 ends up at her last choice at problem $(\succ, P)$. Now consider the following two tables that illustrate the steps of the Shanghai mechanism when student i_1 reports P'_{i_1}, as opposed to P_{i_1}.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_2}$	$\boxed{i_1}, i_3, i_4$	
Step 2	$\boxed{i_4}, i_2$	$\boxed{i_1}$	$\boxed{i_3}$
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3			$\boxed{i_2}$

In this case, student i_1 ends up at school s_2. Thus, the Shanghai mechanism is manipulable by student i_1 at problem $(\succ, P)$. Next, let us apply the Boston mechanism to problem $(\succ, P)$. The specifications are illustrated in the following tables.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_1}, i_2$	$\boxed{i_3}, i_4$	
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 2)$
Step 2	i_4		$\boxed{i_2}$
Round 3	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3			$\boxed{i_4}$

Observe that student i_1 ends up at her first choice at problem $(\succ, P)$, and thus cannot gain by a misreport, but student i_4 ends up at her last choice at problem $(\succ, P)$. Next consider the following tables that illustrate the steps of the Boston mechanism when student i_4 reports P'_{i_4}, as opposed to P_{i_4}.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_4}, i_1, i_2$	$\boxed{i_3}$	
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 2)$
Step 2		i_1	$\boxed{i_2}$
Round 3	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3			$\boxed{i_1}$

Now student i_4 ends up at school s_1. Thus, the Boston mechanism is also manipulable at problem $(\succ, P)$. ∎

Example 2b. (Shanghai mechanism need not be manipulable when the Boston mechanism is) Consider the following example with the given priority structure and the profile of preferences. Each school, s_1, s_2, and s_3, has a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_1	i_2	$\vdots$
i_2	i_3	
$\vdots$	$\vdots$	

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_2	s_2
$\vdots$	s_2	$\vdots$	s_3
	s_3		$\vdots$

Clearly, at problem $(\succ, P)$ under the Boston mechanism, student i_2 can successfully obtain a seat at s_2 by submitting P'_{i_2} as opposed to P_{i_2} which places her at s_3. Note, however, that under the Shanghai mechanism no student can ever gain by lying at problem $(\succ, P)$. ∎

Due to the interdependence of some of the proofs, our order of proving the next results differs from the order in which they appear in the main text. In the remainder of the Appendix A, for brevity, we will suppress student priorities and denote a school choice problem $(\succ, P)$ by the profile $P = (P_i)_{i \in I}$.

Proof of Proposition 2: (Part i). Lemma 1 implies that DA is more stable than SH and that BOS is more efficient than Shanghai. For the problem in Example 2b, SH is stable while BOS is not. We show that if SH is unstable at a problem, then so is BOS. We prove the contrapositive of this statement. Let P be a problem at which $BOS(P)$ is stable. We show that $SH(P) = BOS(P)$. Consider the two mechanisms applied to problem P. We first argue that the set of students who obtain their assignments at steps 1 or 2 of BOS is the same as the set of students who obtain their assignments at the end of step 2 of SH. To see this, first note that the same students are rejected from the same schools at step 1 of both BOS and SH. Then in the second step of both mechanisms the same students apply to the same schools. Since $BOS(P)$ is stable, under BOS any student rejected from any school s at step 2 has lower $s-$priority than every student who obtained a seat at s at step 1. Then the previous two observations imply that at the end of step 2 of both mechanisms, the same students get rejected from the same schools. Iterating this reasoning for the remaining steps, we conclude that $SH(P) = BOS(P)$.

(Part ii). Consider the following problem with seven students and seven schools each with a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{s_5}$	$\succ_{s_6}$	$\succ_{s_7}$
i_1	i_2	i_4	i_5	i_6	i_7	$\vdots$
$\vdots$	i_3	i_3	i_4	i_4	i_4	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	P_{i_7}
s_1	s_1	s_2	s_4	s_4	s_5	s_6
$\vdots$	s_2	s_3	s_5	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	s_7	s_6			
		$\vdots$	s_3			
			s_7			

The outcome of the application-rejection (e) mechanism is the unstable matching $\mu = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_1 & s_2 & s_3 & s_7 & s_4 & s_5 & s_6 \end{smallmatrix}\right)$ when $e = 3$, and the stable matching $\mu' = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_1 & s_2 & s_7 & s_3 & s_4 & s_5 & s_6 \end{smallmatrix}\right)$ when $e = 2$.

(Part iii). Consider the following problem with seven students, four schools each with a quota of one and one school (school s_5) with a quota of three.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{s_5}$
i_6	i_2	i_5	i_7	$\vdots$
i_4	i_3	i_6	i_6	
i_1	i_4	$\vdots$	$\vdots$	
i_2	$\vdots$			
$\vdots$				

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	P_{i_7}
s_1	s_1	s_2	s_2	s_3	s_3	s_4
s_5	s_2	s_5	s_1	$\vdots$	s_4	$\vdots$
$\vdots$	$\vdots$	$\vdots$	s_5		s_1	
			$\vdots$		s_5	
					$\vdots$	

The outcome of the application-rejection (e) mechanism is the Pareto efficient matching $\mu = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_5 & s_2 & s_5 & s_5 & s_3 & s_1 & s_4 \end{smallmatrix}\right)$ when $e = 3$, and the inefficient matching $\mu' = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_5 & s_2 & s_5 & s_1 & s_3 & s_5 & s_4 \end{smallmatrix}\right)$ when $e = 2$.

Proof of Theorem 2: (Part i). Ergin and Sönmez (2006) establish this result for $e = 1$. So suppose $e = 2$. Let $Q = (Q_i)_{i \in I}$ be an arbitrary strategy profile and let μ be the resulting outcome of SH. Suppose μ is unstable, i.e., by Remark 2 there is a pair (i, s) such that $s\ P_i\ \mu(i)$ and $i \succ_s j$ for some $j \in \mu(s)$. This implies that s is not the first choice of student i at Q_i. Let $a \in S \backslash \{s\}$ be the first choice of i at Q_i and Q'_i be any strategy where student i ranks s as her first choice. Now we compare the first two steps of SH applied to problems (Q'_i, Q_{-i}) and Q. Note that each student $k \in I \backslash \{i\}$ applies to the same school at the first step for both problems. Then at problem (Q'_i, Q_{-i}) student i is tentatively placed to school s at the end of the first step. Since the set of students who apply to each school $s' \in S \backslash \{s, a\}$ at the first step are the same for both problems, the set of students who are rejected from each $s' \in S \backslash \{s, a\}$ must also be the same. Moreover, at problem (Q'_i, Q_{-i}), there is one less applicant to a at the first step compared to problem Q. This means if a student (other than i) has not applied to school s at the first two steps at problem Q, the she does not apply to it at the first two steps at problem (Q'_i, Q_{-i}) either.[24] Then student i

[24] In particular, note that since $e = 2$, student i cannot initiate a rejection chain that can possibly hurt herself.

must be (permanently) assigned to school s at (Q'_i, Q_{-i}). Hence, Q cannot be a Nash equilibrium profile, and μ is not a Nash equilibrium outcome.

Conversely, let μ be a stable matching under true preferences P_I . Consider a preference profile $Q = (Q_i)_{i \in I}$ where each student i ranks $\mu(i)$ as her first choice at Q_i. Clearly, the Shanghai mechanism terminates at the end of the first step when applied to Q and μ is the resulting outcome. We claim that Q is an equilibrium profile. Consider a student i and a school s such that $s \; P_i \; \mu(i)$. Let Q'_i be an alternative strategy for i. Since μ is stable, $|\mu(s)| = q_s$ and for each $j \in \mu(s), j \succ_s i$. Since each $j \in \mu(s)$ applies to s at the first step at problem (Q'_i, Q_{-i}), student i cannot obtain a seat at school s regardless of which step she applies to it.

(Part ii). Consider the following priority profile and true preferences of students for a problem with three students and three schools each with a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_3	i_2	i_2
i_2	$\vdots$	i_1
i_1		i_3

P_{i_1}	P_{i_2}	P_{i_3}
s_1	s_1	s_3
s_3	s_2	s_1
s_2	s_3	s_2

Consider a strategy profile $Q = (Q_1, Q_2, Q_3)$ where $Q_1 = P_{i_1}$, $Q_3 = P_{i_3}$, and Q_2 is a strategy in which student i_2 ranks school s_2 as her first choice. It is easy to check that for any preference revelation game induced by an application-rejection mechanism (e) with $e \geq 3$, Q is a Nash equilibrium profile which leads to the unstable matching $\mu = \binom{i_1 \;\; i_2 \;\; i_3}{s_1 \;\; s_2 \;\; s_3}$.[25]

(Part iii). Consider the following priority profile and true preferences of students $P = (P_{i_1}, P_{i_2}, P_{i_3})$ for a problem with three students and three schools each with a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_3	i_2	i_1
i_2	$\vdots$	i_3
i_1		i_2

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_3	s_3
s_3	s_2	s_1	s_1
s_2	s_3	s_2	s_2

Suppose $e = 3$. The outcome of the mechanism for the profile $P = (P_{i_1}, P_{i_2}, P_{i_3})$ is the stable matching $\mu = \binom{i_1 \;\; i_2 \;\; i_3}{s_3 \;\; s_2 \;\; s_1}$. However, P is not an equilibrium since student i_2 can secure a seat at school s_1 by deviating to P'_{i_2}. A similar example can easily be constructed for any fixed e with $3 < e < \infty$ ∎

[25]Clearly, (i_2, s_1) is a blocking pair for μ. To see that Q is indeed an equlibrium profile, it suffices to consider possible deviations by student i_2. For any preferences in which she ranks s_1 first, she gets rejected from s_1 at the third step. If she ranks s_2 first, clearly her assignmnet does not change. If she ranks s_3 first, she is assigned to s_3.

Proof of Theorem 1:

(Part i). Since DA is strategy-proof, it suffices to show that BOS is more manipulable than SH. Suppose SH is manipulable at a problem P. Then if students true preferences are given by the profile P, P is not an equilibrium profile under SH. By Theorem 2, $SH(P)$ is unstable with respect to profile P. By Proposition 2, $BOS(P)$ is also unstable with respect to profile P. Then by Theorem 2, if students true preferences are given by the profile P, P is not an equilibrium profile under BOS. Thus BOS is also manipulable at problem P. Note that Example 2b illustrates a problem at which BOS is manipulable whereas SH is not.

(Part ii, $\mathcal{R}^*$). We first prove the following lemma:

Lemma 2 *Given problem $(\succ, P) \in \mathcal{R}^*$, if the outcome of φ^e leads to a priority violation, then any student whose priority is violated can manipulate at $(\succ, P)$.*

Proof of Lemma 2: Clearly, $e \neq \infty$. Suppose there exists $i, j \in I$ and $s \in S$ such that $\varphi^e_j(P) \equiv s$ P_i $\varphi^e_i(P)(P)$ and $i \succ_s j$. Let P'_i be the preference in which all first $e-1$ choices of student i are unfilled and that her e-th choice is school s. By comparing the first round of φ^e for problem P with its round for problem (P'_i, P_{-i}), it is easy to see that $\varphi^e_i(P'_i, P_{-i}) \equiv s$.

Let φ^e and $\varphi^{e'}$ denote the two application-rejection mechanisms with $e' < e$. Suppose there exists $i \in I$ with P'_i such that $\varphi^e_i(P'_i, P_{-i})$ P_i $\varphi^e_i(P)$. Suppose by contradiction that no student can manipulate $\varphi^{e'}$ at problem P. Then the following two claims must be true.

Claim 1: for all $j \in I$ $\varphi^{e'}_j(P)$ R_j $\varphi^e_j(P) \equiv s$.

Proof: Suppose that at problem P student j applies to school s at some step of some round t. Let P'_j be the preference in which all first $e-1$ choices of student j are unfilled and her e-th choice is school s. We first show that $\varphi^e_j(P'_j, P_{-j}) = s$. To see this, note that when we compare the steps of the first round of φ^e for problems P and (P'_j, P_{-j}), the only difference is that for the latter problem student i does not make any applications until the last step of the first round. We consider two cases. Case (1): at (P'_j, P_{-j}) student applies to school s in some round $t > 1$. In this case school s does not fill its quota at the end of the first round. Then in the first round of φ^e, at (P'_j, P_{-j}) each student applies to a (weakly) smaller number of schools than at P. Therefore, the claim is true. Case (2): at (P'_j, P_{-j}) student j applies to school s in round $t = 1$. In this case since student j does not apply to any school before the last step, the number of applicants to school s at (P'_j, P_{-j}) is (weakly) smaller than that at P. Then student j cannot be rejected from school s at (P'_j, P_{-j}) either.

We now turn to the proof of Claim 1. We show that under $\varphi^{e'}$ at problem P, student j can secure a seat at school s by submitting P^*_j (as opposed to P_j) in which all her first $e'-1$ choices are unfilled and her e'-th choice is s, i.e., $\varphi_j^{e'}(P_j^*, P_{-j}) = s$. Note that the first $e'-1$ steps of of φ^e for (P_j^*, P_{-j}) are identical to the first $e'-1$ steps of $\varphi^{e'}$ for (P_j^*, P_{-j}). Similarly to the above Case (2), student j cannot be rejected from school s at the e'-th step of $\varphi^{e'}$ when applied to (P_j^*, P_{-j}). Then non-manipulability of $\varphi^{e'}$ at problem P implies $\varphi_j^{e'}(P)\ R_j\ s$.

Claim 2: $\varphi_i^{e'}(P)\ P_i\ \varphi_i^e(P)$.
Proof: This can be shown similarly to the proof of Claim 1. Let $s \equiv \varphi_i^e(P_i', P_{-i})$. Since $s\ P_i\ \varphi_i^e(P)$, we have $\varphi_i^e(P_i^{\tilde{}}, P_{-i}) = s$ where $P_i^{\tilde{}}$ is preference in which all first $e-1$ choices of student i are unfilled and her e-th choice is s. We show that under $\varphi^{e'}$ at problem P, student i can secure a seat at school s by submitting $P_i^{\tilde{}}$ (as opposed to P_i) in which all her first $e'-1$ choices are unfilled and her e'-th choice is s. One can then see that $\varphi^{e'}(P_i^{\tilde{}}, P_{-i}) = s$ since the first $e'-1$ steps of φ^e for $(P_i^{\tilde{}}, P_{-i})$ are identical to the first $e'-1$ steps of $\varphi^{e'}$ for $(P_i^{\tilde{}}, P_{-i})$. Thus, student i cannot be rejected from school s at the e'-th step of $\varphi^{e'}$ when applied to $(P_i^{\tilde{}}, P_{-i})$.

Claims 1 and 2 together imply that allocation $\varphi^{e'}(P)$ Pareto dominates allocation $\varphi^e(P)$, i.e., no student is worse off, and there is at least one student (namely i) who is better off at $\varphi^{e'}(P)$ compared to $\varphi^e(P)$. We show that this leads to a contradiction. Let $\varphi_i^{e'}(P) = a \neq \varphi_i(P)$. Then there is $j \in I\backslash\{i\}$ with $j \in \varphi^e(P)(a)\backslash\varphi^{e'}(P)(a)$ who is also better off at $\varphi^{e'}(P)$. Suppose $\varphi_j^{e'}(P) = b \neq a = \varphi_j^e(P)$. Then there is $k \in I\backslash\{i, j\}$ with $j \in \varphi^e(P)(b)\backslash\varphi^{e'}(P)(b)$ who is also better off at $\varphi^{e'}(P)$. Iterating this argument, we obtain a set of students $N_1 = \{i_1^1, i_2^1, \ldots, i_m^1\}$ with $m \geq 2$ such that $\varphi_{i_t}^{e'}(P) = \varphi_{i_{t+1}}^e(P)$ for all $i_t \in N_1$ [where $m+1 \equiv 1$]. Now considering the φ^e algorithm applied to problem P, this is possible only if each $i \in N_1$ applies to the corresponding school $\varphi_i^e(P)$ within the same round since the assignments are made permanent at the end of each round. Let $i_{k^*}^1 \in N_1$ be the *last* student among those in the set N_1 to apply to her permanent assignment at problem P. Suppose this application takes place at some step $r > 1$ of the φ^e algorithm. By the choice of student $i_{k^*}^1$, student $i_{k^*-1}^1 \in N_1$ must be rejected from $\varphi_{i_{k^*}^1}^e(P)$ at an earlier step than r. This means that the quota of school $\varphi_{i_{k^*}^1}^e(P)$ is full at the beginning of step r of the φ^e algorithm applied to problem P. Then there is $j_1 \in I\backslash N_1$ with $j_1 \succ_{\varphi_{i_{k^*}^1}^e(P)} i_{k^*-1}^1$ who is rejected from school $\varphi_{i_{k^*}}^e(P)$ when student i_{k^*} is permanently admitted at step r. This also implies that student j_1 applies to school $\varphi_{j_1}^e(P)$ at some step $r' > r$.

Now consider the following two possibilities: if $\varphi_{i_{k^*-1}}^{e'}(P)\ P_{j_1}\ \varphi_{j_1}^{e'}(P)$, then student j_1 is envious of student $i_{k^*-1}^1$ at $\varphi^{e'}(P)$ leading to a contradiction by Lemma 2. If $\varphi_{j_1}^{e'}(P)\ R_{j_1}\ \varphi_{i_{k^*-1}}^{e'}(P)$, then student j_1 is also better off at $\varphi^{e'}(P)$ compared to $\varphi^e(P)$. Repeating the argument in the previous paragraph, we obtain a set $N_2 = \{i_1^2, i_2^2, \ldots, i_{m'}^2\} \subset I$ of students with $j_1 \in N_2$ who are each better off at $\varphi^{e'}(P)$ compared to $\varphi^e(P)$. Let $i_{k^*}^2 \in N_2$ be the *last* student among those in the set N_2 to apply to her permanent assignment at problem P. Then since $j_1 \in N_2$, student $i_{k^*}^2$ applies to school $\varphi_{i_{k^*}^2}^e(P)$ at some step $r'' \geq r'$. This means that the quota of school $\varphi_{i_{k^*}^2}^e(P)$ is full at the beginning of step r'' of the φ^e algorithm applied to problem P. Then there is $j_2 \in I \backslash N_2$ with $j_2 \succ_{\varphi^e_{i^2_{k^*}}(P)} i^2_{k^*-1}$ who is rejected from school $\varphi^e_{i^2_{k^*}}(P)$ when student $i^2_{k^*}$ is permanently admitted at step r''. This also implies that student j_2 applies to school $\varphi^e_{j_2}(P)$ at some step $r''' > r''$. Since $r''' > r'$, clearly $j_1 \neq j_2$. Similar to before, we now check wheher j_2 is envious of $i^2_{k^*-1}$ at $\varphi^{e'}(P)$. Avoiding a contradiction to Lemma 2 implies there is some $j_3 \notin \{j_1, j_2\}$ who is better off at $\varphi^{e'}(P)$ compared to $\varphi^e(P)$. Iterating this argument we finally reach a contradiction to the finiteness of I.

The proof of parts (i) & (iii) of Lemma 1 gives an example of a problem where φ^e is manipulable when $\varphi^{e''}$ with $e'' > e$ is not. Clearly, φ^e is manipulable by student i_{e+1} at this problem. It is easy to see that no student can manipulate $\varphi^{e''}$ via a preference misreport (which can possibly include unfilled choices) at this problem.

(Part ii, $\mathcal{R}^{**}$). When each school has the same priority order, within-round manipulations are no longer possible. This is because in this case a student rejected from a school can never displace and free up a student with higher priority than herself. Consequently, within a given round of the application-rejection algorithm, a student cannot affect the assignment of a higher priority student, i.e., early applications within a round no longer have any detrimental effects. Therefore, this part can be proven identically to above part upon replacing each preference list used in the proof that involve unfilled choices of a student by a preference list where the student ranks the "target" school as her first choice, e.g., if P'_j is defined in the above proof as the preferences in which all first $e-1$ choices of student j are unfilled and her e-th choice is school s, now consider P'_j as a preference list in which school s is ranked first by student j.

An example of a problem where φ^e is manipulable when $\varphi^{e''}$ with $e'' > e$ is not can be obtained by slightly modifying the problem given in the proof of parts (i) & (iii) of Lemma 1. Assume that student preferences are as before but there is a single priority order for each school given as follows: for each $s \in S$, suppose $i_k \succ_s i_{k'}$ whenever $k < k'$, i.e., i_1 has the highest priority, i_2 has the second highest priority and so on. As before, φ^e is manipulable by student i_{e+1} at this problem but no student can manipulate $\varphi^{e''}$ via a preference misreport.

∎

Proof of Proposition 1: Fix a problem. Take any two application-rejection mechanisms φ^e and $\varphi^{e'}$ with $e' > e$. The first e steps of both mechanisms are identical. This means any student who has been assigned to his first choice at the end of step e under φ^e is permanently assigned to this school. Under $\varphi^{e'}$ however, such a student may be rejected from her first choice at some step

$r \in \{e+1, \ldots, e'\}$.[26] ■

[26] For example, consider the problem given in the proof of part (i) of Lemma 1. At this problem, two students obtain their first choices under BOS whereas none do under SM.

Appendix B: Nash Equilibrium Outcomes in the 6-School Environment (For Online Publication)

We first rewrite Table 3 as a preference profile, where, for each student, the underlined school is her district school:

P_1	P_2	P_3	P_4	P_5	P_6
b	a	b	a	a	a
d	d	f	$\underline{d}$	b	b
c	$\underline{b}$	d	f	c	c
$\underline{a}$	e	a	c	$\underline{e}$	$\underline{f}$
e	f	$\underline{c}$	b	d	e
f	c	e	e	f	d

We now examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. Since the outcomes are stable, the analysis is simplified by first computing the student optimal DA outcome μ^S and the college optimal μ^C and checking if there are any stable allocations in between the two in case they are different. Note that since school e is worse for each student than his district school, student 5 always gets matched to school e in all stable matchings. An allocation below μ^C is always the same regardless of the priority order since it simply assigns each student to his district school.

Every stable matching (with respect to the given profile and the corresponding priority order) is a Nash equilibrium outcome of DA. That is, the Nash equilibrium outcomes of DA is a superset of the stable set. This means any Nash equilibrium we compute for Boston (or Shanghai) is also a Nash equilibrium of DA. But there may be other unstable Nash equilibrium outcomes. In what follows, we present the Nash equilibrium outcomes for each block.

Block 1: $f = 1 - 2 - 3 - 4 - 5 - 6$.

There are two Nash equilibrium outcomes that are stable:

$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}$ and $\mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$

There are three unstable Nash equilibrium outcomes:

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}$, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}$, and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}$.

Block 2: $f = 6 - 1 - 2 - 3 - 4 - 5$

There are three Nash equilibrium outcomes that are stable:

$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & f & d & e & a \end{pmatrix}$, $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}$, and $\mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$

There are three other unstable Nash equilibrium outcomes:

$\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{d\ \ b\ \ c\ \ a\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{c\ \ b\ \ a\ \ d\ \ e\ \ f}$, and $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{b\ \ a\ \ c\ \ d\ \ e\ \ f}$.

Block 3: $f = 5 - 6 - 1 - 2 - 3 - 4$
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ c\ \ d\ \ e\ \ f}$

There are four other unstable Nash equilibrium outcomes:

$\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{d\ \ b\ \ c\ \ a\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{c\ \ b\ \ a\ \ d\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ f\ \ d\ \ e\ \ c}$, and $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{b\ \ a\ \ c\ \ d\ \ e\ \ f}$.

Block 4: $f = 4 - 5 - 6 - 1 - 2 - 3$.
There are two stable Nash equilibrium outcomes:

$\mu^S = \binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{d\ \ b\ \ c\ \ a\ \ e\ \ f}$ and $\mu^C = \binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ c\ \ d\ \ e\ \ f}$

There are three other unstable Nash equilibrium outcomes:

$\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ f\ \ d\ \ e\ \ c}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{c\ \ b\ \ a\ \ d\ \ e\ \ f}$, and $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{b\ \ a\ \ c\ \ d\ \ e\ \ f}$.

Block 5: $f = 3 - 4 - 5 - 6 - 1 - 2$.
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ c\ \ d\ \ e\ \ f}$

There are three other unstable Nash equilibrium outcomes:

$\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{d\ \ b\ \ c\ \ a\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{c\ \ b\ \ a\ \ d\ \ e\ \ f}$, and $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{b\ \ a\ \ c\ \ d\ \ e\ \ f}$.

Block 6: $f = 2 - 3 - 4 - 5 - 6 - 1$
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ c\ \ d\ \ e\ \ f}$

There are four other unstable Nash equilibrium outcomes:

$\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{d\ \ b\ \ c\ \ a\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{c\ \ b\ \ a\ \ d\ \ e\ \ f}$, $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{a\ \ b\ \ f\ \ d\ \ e\ \ c}$, and $\binom{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}{b\ \ a\ \ c\ \ d\ \ e\ \ f}$.

Appendix C: Experimental Instructions (For Online Publication)

Instructions for the SH_4 treatment (Type 1) is presented first. Instructions for the BOS_4 and DA_4 treatments are identical except for the subsection, "The allocation of schools . . .," and the work sheet for Review Question #1. Thus, only this subsection is presented. Instructions for the 6-school treatments are identical except for the number of schools and players. Hence they are omitted, but are available from the authors upon request.

C.1: Instructions for the Shanghai Mechanism (SH_4, Type 1)

Instructions - Mechanism MB

(Please turn off your cell phone. Thank you.)

This is an experiment in the economics of decision making. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. The amount of money you earn will depend upon the decisions you make and on the decisions other people make. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you. At the end of the instructions, you will be asked to provide answers to a series of review questions. Once everyone has finished the review questions, we will go through the answers together.

Procedure

- There are 12 participants of four different types in this experiment. You are type 1. Your type remains the same throughout the experiment.
- You will be randomly matched into groups of four at the beginning of each period. Each group contains one of each of the four different types.
- In this experiment, four schools are available for each group. Each school has one slot. These schools differ in geographic location, specialty, and quality of instruction in each specialty. Each school slot is allocated to one participant.
- **Your payoff** amount depends on the school you are assigned to at the end of each period. Payoff amounts are outlined in the following table. These amounts reflect the desirability of the school in terms of location, specialty and quality of instruction.

Slot received at School:	A	B	C	D
Payoff to Type 1	[**11**]	7	5	16

The table is explained as follows:

You will be paid	11	points if you hold a slot of School A at the end of a period.
You will be paid	7	points if you hold a slot of School B at the end of a period.
You will be paid	5	points if you hold a slot of School C at the end of a period.
You will be paid	16	points if you hold a slot of School D at the end of a period.

- ***NOTE* different types have different payoff tables. This is a complete payoff table for each of the four types:**

	A	B	C	D
Payoff to Type 1	[**11**]	7	5	16
Payoff to Type 2	5	[**11**]	7	16
Payoff to Type 3	7	16	[**11**]	5
Payoff to Type 4	5	16	7	[**11**]

The square brackets, **[ ]**, indicate the resident of each school district, who has higher priority in that school than other applicants. We will explain this in more detail in the next section.

- In this experiment, participants are defined as belonging to the following school districts:

Participant Type 1	lives within the school district of school	A,
Participant Type 2	lives within the school district of school	B,
Participant Type 3	lives within the school district of school	C,
Participant Type 4	lives within the school district of school	D.

- The experiment consists of 20 periods. In each period, you will be randomly matched with 3 other people in the room to form a group of four, which has one of each type. Your earnings for each period depend on your choices as well as the choices of the three other people you are matched with.

- Every period, each participant will rank the schools. Note that you need to rank all four schools in order to indicate your preferences.

- After all participants have submitted their rankings, the server will allocate the schools in each group and inform each person of his/her school allocation and respective payoff. Note that your allocation in each period is independent of your allocations in the previous periods.

- Your total payoff equals the sum of your payoffs in all 20 periods. Your earnings are given in points. At the end of the experiment you will be paid based on the exchange rate,

  $1 = 20 points.

  In addition, you will be paid $5 for participation, and up to $3.5 for answering the Review Questions correctly. Everyone will be paid in private and you are under no obligation to tell others how much you earn.

Allocation Method

- **The priority order for each school is separately determined as follows**:
  - **High Priority Level:** Participant who lives within the school district.
  - **Low Priority Level:** Participants who do not live within the school district.

    The priority among the Low Priority Students is based on their respective position in a lottery. The lottery is changed every five periods. In the first five periods, your lottery number is the same as your type number. In each subsequent block of five periods, your lottery number increases by one per block. Specifically, the lottery number for each type in each five-period block is tabulated below:

	Type 1	Type 2	Type 3	Type 4
Periods 1-5	1	2	3	4
Periods 6-10	2	3	4	1
Periods 11-15	3	4	1	2
Periods 16-20	4	1	2	3

- **The allocation of schools is obtained as follows**:
  - An application to the first ranked school is sent for each participant.
  - Throughout the allocation process, a school can hold no more applications than its capacity.

    If a school receives more applications than its capacity, then it retains the student with the highest priority and rejects the remaining students.
  - Whenever an applicant is rejected at a school, his/her application is sent to the next highest ranked school.
  - Whenever a school receives new applications, these applications are considered together with the retained application for that school. Among the retained and new applications, the one with the highest priority is retained.

- The allocation is finalized every two steps. That is, in steps 2, 4 and 6 etc., each participant is assigned a school that holds his or her application in that step. These students and their assignments are removed from the system.
- The allocation process ends when no more applications can be rejected.

Note that the allocation is finalized every two steps.

An Example:

We will go through a simple example to illustrate how the allocation method works. This example has the same number of students and schools as the actual decisions you will make. You will be asked to work out the allocation of this example for Review Question 1.

Students and Schools: In this example, there are four students, 1-4, and four schools, A, B, C and D.

Student ID Number: $1, 2, 3, 4$ Schools: A, B, C, D

Slots and Residents: There is one slot at each school. Residents of districts are indicated in the table below.

School	Slot	District Residents
A	☐	1
B	☐	2
C	☐	3
D	☐	4

Lottery: The lottery produces the following order.

$\boxed{1 - 2 - 3 - 4}$

Submitted School Rankings: The students submit the following school rankings:

	1st Choice	2nd Choice	3rd Choice	Last Choice
Student 1	D	A	C	B
Student 2	D	A	B	C
Student 3	A	B	C	D
Student 4	A	D	B	C

Priority : School priorities first depend on whether the school is a district school, and next on the lottery order:

	Resident	Non-Resident
Priority order at A:	**1**	– 2 – 3 – 4
Priority order at B:	**2**	– 1 – 3 – 4
Priority order at C:	**3**	– 1 – 2 – 4
Priority order at D:	**4**	– 1 – 2 – 3

The allocation method consists of the following steps: Please use this sheet to work out the allocation and enter it into the computer for Review Question #1.

Step 1 (temporary): Each student applies to his/her first choice. If a school receives more applications than its capacity, then it holds the application with the highest priority and rejects the remaining students.

Applicants		School		Accept	**Hold**	**Reject**
3, 4	⟶	A	⟶	N/A	☐	
	⟶	B	⟶	N/A	☐	
	⟶	C	⟶	N/A	☐	
1, 2	⟶	D	⟶	N/A	☐	

Step 2 (final): Each student rejected in Step 1 applies to his/her next choice. When a school receives new applications, these applications are considered together with the application on hold for that school. Among the new applications and those on hold, the one with the highest priority is accepted, while the rest are rejected.

Accepted	Held	New Applicants		School		**Accept**	Hold	**Reject**
	☐		⟶	A	⟶	☐	N/A	
	☐		⟶	B	⟶	☐	N/A	
	☐		⟶	C	⟶	☐	N/A	
	☐		⟶	D	⟶	☐	N/A	

Step 3 (temporary): Each student rejected in Step 2 applies to his/her next choice. If a school still has vacancy, it holds the application with the highest priority and rejects the rest. If a school is already full, it rejects all new applications.

Accepted	Held	New applicants		School		Accepted	**Hold**	**Reject**
☐			⟶	A	⟶	☐		
☐			⟶	B	⟶	☐		
☐			⟶	C	⟶	☐		
☐			⟶	D	⟶	☐		

Step 4 (final): Each student rejected in Step 3 applies to his/her next choice. If the next choice has a vacancy, it accepts the application. Furthermore, all applications on hold are accepted in this step.

Accepted	Held	New Applicants		School		**Accept**	Hold	Reject
☐			⟶	A	⟶	☐	N/A	N/A
☐			⟶	B	⟶	☐	N/A	N/A
☐			⟶	C	⟶	☐	N/A	N/A
☐			⟶	D	⟶	☐	N/A	N/A

The allocation ends at Step 4.

- Please enter your answer to the computer for Review Question 1. After everyone has entered their answers, we will distribute an answer sheet and go through the answer together.
- Afterwards, you will be asked to answer another 10 review questions. When everyone is finished with them, we will again go through the answers together.
- Feel free to refer to the experimental instructions before you answer any question. Each correct answer is worth 25 cents, and will be added to your total earnings. You can earn up to $3.5 for the Review Questions.

Review Questions 2 - 11

2. How many participants are there in your group each period?
3. True or false: You will be matched with the same three participants each period.
4. True or false: Participant living in a school district has higher priority than any other applicants for that school.
5. True or false: The priority for non-residents of a school district is determined by a lottery.
6. True or false: The lottery is fixed for the entire 20 periods.
7. True or false: A lottery number of 1 means that I have the highest priority among the other non-resident applicants in a school.
8. True or false: Other things being equal, a low lottery number is better than a high lottery number.
9. True or false: If you are accepted by a school of your choice, the schools ranked below are irrelevant.
10. True or false: If you are not rejected at a step, then you are accepted into that school.
11. True or false: The allocation is final at the end of each step.

You will have 5 minutes to go over the instructions at your own pace. Feel free to earn as much as you can. Are there any questions?

C.2: Instructions for the Boston Mechanism (BOS$_4$)

. . . . . .

- **The allocation of schools is described by the following method**:

Step 1.
a. An application to the first ranked school is sent for each participant.
b. Each school accepts the student with highest priority in that school. These students and their assignments are removed from the system. The remaining applications for each respective school are rejected.

Step 2.
a. The rejected applications are sent to his/her second choice.
b. If a school is still vacant, then it accepts the student with the highest priority and rejects he remaining applications.

Step 3.
a. The application of each participant who is rejected by his/her top two choices is sent to his/her third choice.
b. If a school is still vacant, then it accepts the student with the highest priority and rejects the remaining applications.

Step 4. Each remaining participant is assigned a slot at his/her last choice.

Note that the allocation is final in each step.

. . . . . .

C.3: Instructions for the Deferred Acceptance Mechanism (DA$_4$)

. . . . . .

The allocation of schools is described by the following method:

- An application to the first ranked school is sent for each participant.
- Throughout the allocation process, a school can hold no more applications than its capacity.

  If a school receives more applications than its capacity, then it temporarily retains the student with the highest priority and rejects the remaining students.
- Whenever an applicant is rejected at a school, his or her application is sent to the next highest ranked school.
- Whenever a school receives new applications, these applications are considered together with the retained application for that school. Among the retained and new applications, the one with the highest priority is temporarily on hold.

- The allocation is finalized when no more applications can be rejected.

  Each participant is assigned a slot at the school that holds his/her application at the end of the process.

Note that the allocation is temporary in each step until the last step.

. . . . . .

Appendix D: Additional Tables and Analysis (For Online Publication)

This appendix contains additional tables and data analysis, including the probit analysis of factors affecting truthtelling, Nash equilibrium outcomes, as well as session-level efficiency and stability results.

Table 7: Probit: Truthful Preference Revelation

Dependent Variable: Truthtelling						
Environments:	4-School Environment			6-School Environment		
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)
Mechanisms:	BOS	SH	DA	BOS	SH	DA
Lottery Position	-0.128***	-0.056**	-0.013	-0.098***	-0.071***	-0.028***
	(0.027)	(0.026)	(0.018)	(0.010)	(0.009)	(0.005)
Period	-0.009**	0.002**	0.002	-0.004	-0.007***	-0.005**
	(0.004)	(0.001)	(0.003)	(0.003)	(0.002)	(0.002)
Log Likelihood	-619.97	-650.10	-538.00	-986.75	-1233.94	-1475.81
Observations	960	960	960	2160	2160	2160

Notes:
1. Robust standard errors are adjusted for clustering at the session level.
2. Coefficients are probability derivatives.
3. Significant at the: ** 5 percent level; *** 1 percent level.

To investigate factors affecting truthtelling, we use probit regressions for each treatment. In Table 7, we present six probit specifications. The dependent variable is a dummy variable indicating whether a participant reveals her preferences truthfully. The independent variables include lottery position (1 being the best, and 6 being the worst), and a period variable to capture any effects of learning. In the 4-school environment, participants are 12.8% (resp. 5.6%) less likely to tell the truth under BOS (resp. SH) for each increase in the lottery position, while such an effect is absent under DA, where truthtelling is a dominant strategy. We also observe a small but significant effect of learning to manipulate (resp. tell the truth) under BOS (resp. SH). In comparison, in the 6-school environment, we observe a similar lottery position effect on truthtelling, but for all three mechanisms. The 2.8% marginal effect of lottery position on truthtelling under DA indicates that some participants might not understand the incentives in DA in the 6-school environment, consistent with the significantly lower level of truthtelling in this environment compared to the 4-school environment (Figure 1). Again, we observe a small but significant effects of learning but on preference manipulation under SH and DA.

Table 8 reports session-level statistics for each mechanism and pairwise comparisons between mechanisms and outcomes, using each session as an observation.

Table 8: Proportion of Nash Equilibrium Outcomes

4-School	BOS ($\mu^{C/S}$)	SH ($\mu^{C/S}$)	DA	DA ($\mu^{C/S}$)	DA (μ^{*})	H_a	p-value
Session 1	0.683	0.717	0.967	0.950	0.017	BOS $\neq$ SH	0.143
Session 2	0.600	0.700	0.850	0.717	0.133	BOS $<$ DA	0.014
Session 3	0.600	0.633	0.817	0.800	0.017	SH $<$ DA	0.029
Session 4	0.533	0.650	0.950	0.833	0.117	DA(μ^{*}) $<$ DA($\mu^{C/S}$)	0.063
6-School	**BOS**	**SH**	**DA**	**DA(Stable)**	**DA(Unstable)**	**H_a**	**p-value**
Session 1	0.011	0.144	0.822	0.811	0.011	BOS $<$ SH	0.014
Session 2	0.011	0.189	0.778	0.778	0.000	BOS $<$ DA	0.014
Session 3	0.033	0.100	0.844	0.789	0.056	SH $<$ DA	0.014
Session 4	0.078	0.211	0.711	0.644	0.067	DA(unstable) $<$ DA(stable)	0.063

Table 9: Normalized Efficiency: First Block, Last Block and All Periods

	First Block (periods 1-5)			Last Block			All Periods		
4-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.733	0.733	0.750	0.733	0.758	0.767	0.721	0.752	0.752
Session 2	0.742	0.733	0.733	0.758	0.792	0.808	0.744	0.750	0.777
Session 3	0.742	0.725	0.750	0.742	0.792	0.775	0.733	0.733	0.748
Session 4	0.683	0.708	0.742	0.767	0.750	0.833	0.727	0.732	0.777
6-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.870	0.893	0.800	0.773	0.673	0.567	0.849	0.774	0.676
Session 2	0.850	0.857	0.807	0.780	0.760	0.593	0.850	0.768	0.685
Session 3	0.910	0.910	0.850	0.740	0.753	0.560	0.810	0.836	0.679
Session 4	0.890	0.823	0.817	0.767	0.717	0.717	0.828	0.779	0.720

Table 10: Stability: First Block, Last Block and All Periods

	First Block (periods 1-5)			Last Block			All Periods		
4-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.733	0.867	1.000	0.533	0.467	0.867	0.683	0.717	0.950
Session 2	0.533	0.800	0.733	0.333	0.600	0.733	0.600	0.700	0.717
Session 3	0.800	0.800	0.933	0.400	0.467	0.600	0.600	0.633	0.800
Session 4	0.467	0.600	0.933	0.400	0.667	0.667	0.533	0.650	0.833
6-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.000	0.000	0.800	0.000	0.067	0.867	0.011	0.144	0.811
Session 2	0.000	0.133	0.600	0.000	0.133	0.867	0.011	0.189	0.778
Session 3	0.000	0.067	0.333	0.000	0.000	0.933	0.033	0.100	0.789
Session 4	0.133	0.333	0.467	0.000	0.067	0.467	0.078	0.211	0.644