Games of Incomplete Information

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- Auction Theory
- Administrative stuff

Games of Incomplete Information

Several basic concepts:

- To say that a game is of **complete or incomplete information** is to say something about what is known about the *circumstances* under which the game is played.

- All the games we studied so far are games of complete information. Roughly speaking, enough information has been supplied to allow the game to be analyzed. For this to be true in general, various things are taken to be common knowledge.
  - In online auctions, it is not realistic to assume that bidders know the other bidders' valuations.
  - Consider preferences: in the auction experiment, it is not realistic to assume that players' risk attitudes are common knowledge.
  - In a game like Russian Roulette, both players have an incentive to pretend to be more reckless than they actually are.
  - In a Cournot duopoly game, it is unrealistic to assume that one firm knows the cost structure of the other firm.

- Harsanyi's theory of incomplete information offers a means to get a handle on such matters: a technique for completing a structure in which information is incomplete.
- Main techniques: expected utility calculation.
- Solution concepts: a comparison

	Strategic Form	Extensive Form
Complete Info.	DSE; NE	SPNE
Incomplete Info.	Bayesian Nash	Perfect Bayesian

Strategic Form Games of Incomplete Information

Situation: some parties are uncertain about the characteristics of some other party.

1. A Bayesian game consists of
   - A finite set of players, N;
   - A set of states, Ω, each of which is a description of all the players' characteristics. For each player $i \in N$:
   - The set of actions available to each player i, A_i;
   - the set of signals observed by i;
   - a probability measure on Ω, which denotes the prior belief of player i.
   - a preference relation, $\succeq_i$.

2. A **Bayesian-Nash equilibrium** is a Nash equilibrium of the Bayesian game, in which each player chooses the best action available to him given the signal that he receives and his belief about the state and the other players' actions that he deduces from this signal. I.e., each player chooses an action to maximize expected utility given the signals and his beliefs.

3. Three differences of Bayesian Nash equilibrium from Nash equilibrium:
   - You can be more than one type;
   - You do not know what types other players are;
   - Strategies: plans of actions for every types you and others might be.
4. Note: The mention of Bayes here does not mean that Bayes' Rule is necessarily going to be invoked somewhere along the way. It simply refers to the incomplete information part.

Auctions: Background

- What's so interesting about auctions?

  An alternative to bargaining for selling a fixed supply of a commodity for which there is no well-established on going market

- Applications
  - Real estate, art, flower, oil lease
  - Privatization and deregulation
    * Government contract
    * Electricity
    * Airwaves: FCC spectrum auction
  - Allocation of common resources
  - E-commerce: eBay

- Auction Institutions
  - English
  - Dutch
  - First price sealed bid
  - Vickrey
  - Many other kinds

Types of Auctions

- Private value auctions

  Bidders' valuations for the auctioned item(s) are independent from one another and are their private information.

  E.g., flowers, art, antiques

- Common value auctions

  Bidders are uncertain about the ultimate value of the item, which is the same for all bidders

  E.g., oil leases, Olympic broadcast rights

- Affiliated (correlated) value auctions

  Bidders' valuations for the auctioned item(s) are correlated, but not necessarily the same for all. In between private and common value auctions.

Auction Research

- Research Questions
  - Efficiency comparison of auction institutions
  - Revenue comparison
  - Bidder earning comparison
  - Collusion?
  - Transparency?
- What do we know?
  - Single item: well
  - Multiple items: little
    - Substitutes
    - Complements
- Agenda for theoretical research
  - Multi-item auctions
- Agenda for experimental research
  - Test/discriminate among theories
  - Design and test new institutions

English Auction

- Background

  oral auctions in English-speaking countries.
  Originally “Roman”.

- Commodities

  antiques, artworks, cattle, horses, real estate, wholesale fruits and vegetables, old books, etc.

- Rules for Experiment

  – Auctioneer first solicits an opening bid from the group.
  – Anyone who wants to bid should call out a new price at least $1 higher than the previous high bid.
  – The bidding continues until all bidders but one have dropped out.
  – The highest bidder gets the object being sold for a price equal to the final bid.
  – Winner's profit = Buyer Value - price;
  Everyone else's profit = 0.
  – Your Buyer Value = Last two digits of your SSN.

- Optimal strategy

  participate till price = buyer value, then drop out.

- Equilibrium Outcome

  The highest bidder gets the object at a price close to the second highest Buyer Value.

- Comparative statics

  As n increases, the winning bid is closer to the highest BV. The more spread-out the different bidders' valuation are, the larger $|v^{max} - v^{2nd}|$. This means that if there is wide disagreement about the item's value, the winner might be able to get it cheaply.

- Problems

  collusion; bidding rings.

The Dutch Auction

- Background

  whole sale produce, cut-flower markets in the Netherlands.

- Commodities

  – flowers in the Netherlands
  – fish market in England and Israel
  – tobacco market in Canada

- Rules

  – Auctioneer starts with a high price.
  – Auctioneer lowers the price gradually until some buyer shouts "Mine!"
  – The first buyer to shout "Mine!" gets the object at the price the auctioneer just called.
  – Winner's profit = Buyer Value - price;
  Everyone else's profit = 0.
  – Your Buyer Value = 100 - Last two digits of your SSN.

First-Price Sealed-bid Auctions

- Background

  used to award construction contracts (lowest bidder), real estate, art treasures;

- Rules

  – Bidders write their bids for the object and their names on slips of paper and deliver them to the auctioneer.
  – The auctioneer opens the bid and find the highest bidder.
  – The highest bidder gets the object being sold for a price equal to her own bid.
  – Winner's profit = Buyer Value - price;
  Everyone else's profit = 0.
  – Your Buyer Value = First and second number of the last four digits of your SSN.

- Set up the problem

  In a sealed-bid, first price auction in a private values environment with n bidders, each bidder has a private valuation, v_i, which is his private information. The distribution of v_i is common knowledge. Let B_i denote the bid of player i. Let π_i denote the profit of player i. If $v_i \sim u[0, 100]$, what is the Bayesian Nash equilibrium bidding strategy for the players?

- Optimal bidding strategies If $B_i \geq v_i$, then $\pi_i \leq 0$. Therefore, $B_i < v_i$, which gives

$$\pi_i = \begin{cases} 0 & , \quad \text{if } B_i \neq \max_j\{B_j\} \\ v_i - B_i & , \quad \text{if } B_i = \max_j\{B_j\} \end{cases}$$

  The question is how much below v_i should his bid be? The less B_i is, the less likely he will win the object, but the more profit he makes if he wins the object.

- $n = 2$

  you are characterized by the strategy-type two tuple, (B, v). Suppose the other bidder's value is X, and she is characterized by $(\alpha X, X)$, where $\alpha \in (0, 1)$. Your expected profit is

$$E\pi = P(\text{Your bid is higher}) \cdot (v - B) + P(\text{Your bid is lower}) \cdot 0$$

With uniform distribution, $P(X < B/\alpha) = \frac{1}{100}\frac{B}{\alpha}$.
Therefore,

$$E\pi = \frac{1}{100}\frac{B}{\alpha}(v - B),$$

assuming risk neutrality, you choose B to

$$\max_B B(v - B) = Bv - B^2.$$

It follows that $B = v/2$.

- With n bidders

$$P(\text{Your bid is the highest}) = (\frac{1}{100}\frac{B}{\alpha})^{n-1}.$$

$$\max_B B^{n-1}(v - B) \Rightarrow B = \frac{n-1}{n}v.$$

Note: as n increases, $B \to v$. I.e., increased competition drives bids close to the valuations.

- Equivalence of Dutch and First-price, sealed-bid auctions: same reduced form.
  - The object goes to the highest bidder at the highest price.
  - A bidder must choose a bid without knowing the bids of any other bidders.
  - Optimal bidding strategies are the same.

Sealed-bid Second-price Auctions

- Background: Vickrey (1961).
- Commodities
  - stamp collectors' auctions
  - US Treasury's long-term bonds
  - Airwaves auction in New Zealand
  - e-Bay and Amazon
- Rules
  - Bidders write their bids for the object and their names on slips of paper and deliver them to the auctioneer.
  - The auctioneer opens the bid and find the highest bidder.
  - The highest bidder gets the object being sold for a price equal to the second highest bid.
  - Winner's profit = Buyer Value - price;
    Everyone else's profit = 0.
  - Your Buyer Value = 100 - First and second number of the last four digits of your SSN.

- Equilibrium bidding strategy

  It is a weakly dominant strategy to bid your true value.

  Let V be your Buyer Value, let B be your bid, and let X be the highest bid made by anybody else in the auction. We want to show that overbidding or underbidding cannot increase your profit and might decrease it. Let π^t be your profit when $B = V$. Let π be your profit otherwise.

  Proof: First consider the case of overbidding, $B > V$.

  1. $X > B > V$: You don't get the object either way. $\pi = \pi^t = 0$.
  2. $B > X > V$: $\pi = V - X < 0$, but $\pi^t = 0$.
  3. $B > V > X$: $\pi = V - X = \pi^t > 0$.

  Next consider the case of underbidding, $B < V$.

  1. $X < B < V$: $\pi = V - X = \pi^t > 0$.
  2. $B < X < V$: $\pi = 0$, but $\pi^t = V - X > 0$.
  3. $B < V < X$: You don't get the object either way. $\pi = \pi^t = 0$.

- Equivalence of English and sealed-bid, 2nd-price

  - The object goes to the highest bidder.
  - Price is close to the second highest BV.